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A void ratio dependent water retention curve model including hydraulic hysteresis

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Abstract. Past experimental evidence has shown that *Water Retention Curve* (WRC) evolves with mechanical stress and structural changes in soil matrix. Models currently available in the literature for capturing the volume change dependency of WRC are mainly empirical in nature requiring an extensive experimental programme for parameter identification which renders them unsuitable for practical applications. In this paper, an analytical model for the evaluation of the void ratio dependency of WRC in deformable porous media is presented. The approach proposed enables quantification of the dependency of WRC on void ratio solely based on the form of WRC at the reference void ratio and requires no additional parameters. The effect of hydraulic hysteresis on the evolution process is also incorporated in the model, an aspect rarely addressed in the literature. Expressions are presented for the evolution of main and scanning curves due to loading and change in the hydraulic path from scanning to main wetting/drying and vice versa as well as the WRC parameters such as air entry value, air expulsion value, pore size distribution index and slope of the scanning curve. The model is validated using experimental data on compacted and reconstituted soils subjected to various hydro-mechanical paths. Good agreement is obtained between model predictions and experimental data in all the cases considered.

1 Introduction

A crucial step in many constitutive modellings of unsaturated soils is the determination of the *water retention curve* (WRC) or alternatively called *soil water characteristic curve* (SWCC). Conventionally, WRC has been shown in terms of degree of saturation versus suction. Experimental evidence have shown that WRC is affected not only by matric suction but also by void ratio; that is, the hydraulic and mechanical states of the soil both have significant effects on the degree of saturation. Hence, at equilibrium conditions, the interplay of degree of saturation-suction-void ratio of a soil constitutes a state surface [1-3]. The state surface for the main drying path is different from that for the main wetting path, and these two unique surfaces serve as the upper and lower boundaries of all the possible states attainable for a soil.

Numerous relationships have been presented in the literature for the mathematical representation of WRC (e.g. [4-6]). However, a vast majority has been for non-deformable soils ignoring the impact of volume change on WRC [7].

The main contributions to the evolution of WRC with volume change have been due to [1, 8-13]. The available models are empirical in nature and mainly rely on fitting parameters. Excessive experimental time and cost related to the quantification of the embedded parameters in the existing models is a major issue in the process of the application of such models in the geotechnical engineering practice. More importantly, the influence of volumetric deformations on the evolution of the scanning path of WRC, i.e. the hysteresis effect, has been rarely investigated. This is despite the fact that essentially in the majority of cases, either in laboratory investigations on unsaturated soils or in real-life, the loading/unloading of soils are being done on the scanning path of WRC.

In this paper, we present the outline of an analytical model for capturing the evolution of the whole WRC hysteretic loop. The model has been developed based on the theoretical work of [14] that requires a few input parameters with clear physical meanings and more importantly the hysteresis effect in water retention behavior is incorporated. Particular attention is given to the scanning behaviour of the soil and the dependency of pore size distribution to void ratio. The approach proposed is general in nature and allows prediction of the shift in WRC with volume change entirely based on the form of WRC at a reference void ratio with no additional parameters required. The applicability of the model to experimental data is demonstrated.

2 Outline of the model

The model is developed based on the fundamental contribution of [14], which relates the water volumetric strain ($\dot{\varepsilon}_{vw} = \frac{dV_w}{V}$ where V_w and V are water volume and

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total volume of soil, respectively) to the total volumetric strain ($\dot{\varepsilon}_{vw} = \frac{dV}{V} = \frac{dV_v}{V}$ where V_v is the volume of voids) in constant suction loading conditions through the effective stress parameter in incremental from, ψ , as

$$\dot{\varepsilon}_{vw} = \psi \dot{\varepsilon}_v \tag{1}$$

Using the definition of the degree of saturation combined with (1) gives (assuming solids are incompressible)

$$\dot{S}_r = (\psi - S_r)\frac{\dot{e}}{e} \tag{2}$$

where e is void ratio. ψ can be calculated in (1) and (2) from

$$\psi = \frac{\partial(\chi s)}{\partial s} = \left(1 - \frac{\partial ln\chi}{\partial lns}\right)\chi \tag{3}$$

where χ is the effective stress parameter in total form which describes the contribution of suction to the effective stress of the soil solid skeleton. Note that in (3), $\frac{\partial ln\chi}{\partial lns}$ is taken as positive when χ decreases with an increase in suction. Earlier definitions of the effective stress parameter assumed direct correlation with the degree of saturation, $\chi = S_r$ [15, 16]. However, it was shown in many recent works (e.g. [17-20]) that only when the work of air-water interface is ignored, the effective stress parameter may be taken as the degree of saturation. Based on back-calculation of unsaturated shear test data for many soil types, Khalili and Khabbaz [21] proposed an equation for the effective stress parameter. In this work, The hysteretic effective stress parameter, χ, is quantified based on the contributions of [21-23].

For main wetting and drying paths

$$\chi = \begin{cases} 1, & s < s_e \\ \left(\frac{s_e}{s}\right)^{\Omega}, & s \geq s_e \end{cases} \tag{4-1}$$

For drying path reversals

$$\chi = \left(\frac{s_{ae}}{s_{rd}}\right)^{\Omega}\left(\frac{s_{rd}}{s}\right)^{\zeta}, \quad \left(\frac{s_{ex}}{s_{ae}}\right)^{\frac{\Omega}{\Omega-\zeta}} s_{rd} \leq s \leq s_{rd} \tag{4-2}$$

For wetting path reversal

$$\chi = \left(\frac{s_{ex}}{s_{rw}}\right)^{\Omega}\left(\frac{s_{rw}}{s}\right)^{\zeta}, \quad s_{rw} \leq s \leq \left(\frac{s_{ae}}{s_{ex}}\right)^{\frac{\Omega}{\Omega-\zeta}} s_{rw} \tag{4-3}$$

where the air entry, s_{ae}, and air expulsion, s_{ex}, values denote the points of transition from saturation to unsaturation and vice versa along the main drying and main wetting paths, respectively. Ω is the material parameter, with the best fit value of 0.55, ζ is the slope of the transition path between the main drying and wetting paths in a lnχ~ln$s$ plane. s_{rd} and s_{rw} are the points of suction reversal on the main drying and main wetting paths, respectively. A schematic representation of (4) is shown in Figure 1.

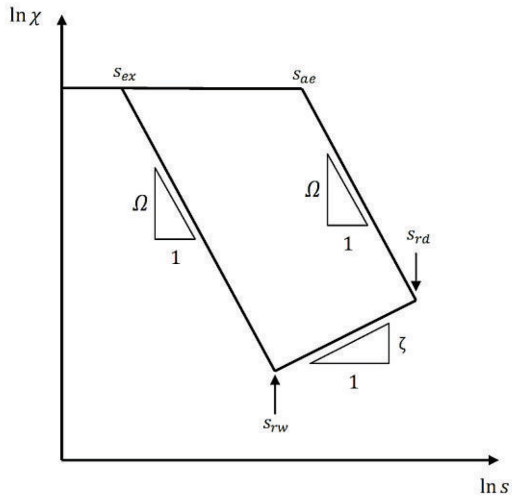

Figure 1. Effective stress parameter in terms of suction including hydraulic hysteresis

The step by step procedure in the model for quantifying the evolved form of the hysteretic WRC for a change in void ratio starting from a reference state is described below (all updated parameters are denoted by asterisk). All WRC parameters used in the model and the evolution with void ratio in $lnS_r - lns - e$ space is shown in Figure 2. The details of the derivations can be found in [24].

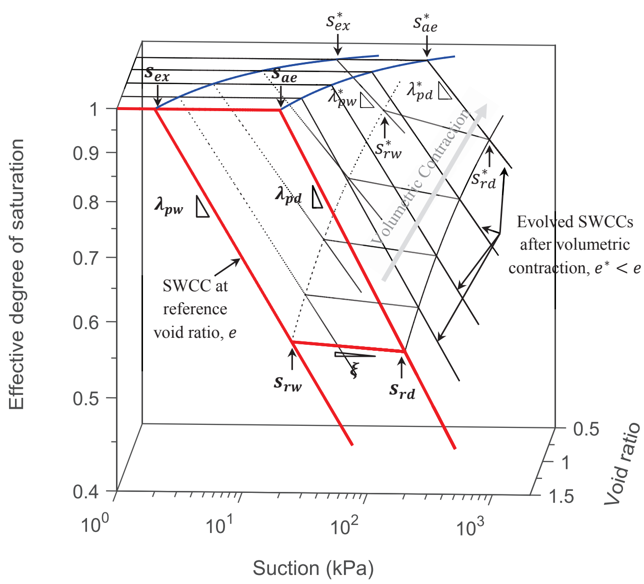

Figure 2. Evolution of water retention curve with void ratio (parameters at the evolved states are denoted with asterisk)

STEP 1. Evaluate the input parameters of the model including the WRC parameters at the reference void ratio, s_{ae}, s_{ex}, λ_{pd}, λ_{pw}, and ξ (see Figure 2), and the effective stress parameters, Ω and ζ (see Figure 1). Since compression is related to wetting and expansion is related to drying, for simulations including volumetric contraction, only use s_{ex0} and λ_{pw0}, and for simulations including volumetric extension, only use s_{ae0} and λ_{pd0}. The parameters associated with the WRC at the reference void ratio can be obtained by fitting the Brooks and Corey model [4] model extended by [14] to include hydraulic hysteresis effects as follows to the WRC at the reference void ratio.

For main wetting and drying paths

$$S_{eff} = \begin{cases} 1, & s < s_e \\ \left(\frac{s_e}{s}\right)^{\lambda_p}, & s \geq s_e \end{cases} \tag{5-1}$$

For drying path reversals

$$S_{eff} = \left(\frac{s_{ae}}{s_{rd}}\right)^{\lambda_p}\left(\frac{s_{rd}}{s}\right)^{\xi}, \quad \left(\frac{s_{ex}}{s_{ae}}\right)^{\frac{\lambda_p}{\lambda_p-\xi}} s_{rd} \leq s \leq s_{rd} \tag{5-2}$$

For wetting path reversal

$$S_{eff} = \left(\frac{s_{ex}}{s_{rw}}\right)^{\lambda_p}\left(\frac{s_{rw}}{s}\right)^{\xi}, \quad s_{rw} \leq s \leq \left(\frac{s_{ae}}{s_{ex}}\right)^{\frac{\lambda_p}{\lambda_p-\xi}} s_{rw} \tag{5-3}$$

where S_{eff} = effective degree of saturation, is the normalised form of the total degree of saturation, S_r, with respect to residual degree of saturation, S_{res}, ($S_{eff} = \frac{S_r - S_{res}}{1 - S_{res}}$).

STEP 2. Update the air entry value, s_{ae}, or air expulsion value, s_{ex}, for an infinitesimal change in void ratio, $\dot{e}$, using

$$s_e^* = s_e\left(1 - \frac{\Omega}{(1-S_{res})\lambda_p}\frac{\dot{e}}{e}\right) \tag{6}$$

where s_e is either air entry or air expulsion value.

STEP 3. Update the pore size distribution index pertaining to main drying, λ_{pd}, and/or main wetting, λ_{pw}, from

$$\lambda_p^* = \lambda_p[1 - \frac{3\left((1-\Omega)\left(2^{(1-\Omega/\lambda_p)}-1\right)-S_{res}\right)}{2(1-S_{res})}\frac{\dot{e}}{e}] \tag{7}$$

STEP 4. For the impact of void ratio on the scanning path, update the slope, ξ, using

$$\xi^* = \xi[1 + \frac{(1-\zeta)(\zeta-\xi)}{\xi} S_{eff,r}^{\frac{\Omega}{\lambda_p}-1}\frac{\dot{e}}{e}] = \xi[1 + \frac{(1-\zeta)(\zeta-\xi)}{\xi}\left(\frac{s_e}{s_r}\right)^{\Omega-\lambda_p}\frac{\dot{e}}{e}] \tag{8}$$

where s_r and $S_{eff,r}$ are the suction and effective degree of saturation at the point of suction reversal. If the soil is subjected to volumetric contraction, use s_{rw} and the respective effective degree of saturation, otherwise use s_{rd}.

STEP 5. Use (2) - (4) to calculate the updated degree of saturation at each step.

STEP 6. If the point of interest is located on the scanning path, use the following equation to update the suction at the point of suction reversal and use this to check whether the transition from scanning to main path of WRC due to the change in void ratio has occurred

$$s_r^* = \left(\frac{(s_e^*)^{\lambda_p^*}}{s^{\xi^*} S_{eff}^*}\right)^{\frac{1}{(\lambda_p^*-\xi^*)}} \tag{9}$$

where S_{eff}^* is the updated effective degree of saturation at the point of interest on the scanning path. Comparing the suction at the point of interest, s, to s_r^* from (9) at each step determines whether the soil hydraulic state has remained on the scanning path or it has been pushed to the main path due to the change in void ratio.

STEP 7. Repeat the procedure for incremental void ratio steps until reaching the target void ratio.

3 Model predictions

3.1 Evolution of main path of WRC with void ratio

***Municipal Boom clay* [25]**

Drying WRC test results on municipal Boom clay was presented by [25]. Four tests at different void ratios 0.59, 0.69, 0.8, and 0.93 were performed as replotted in Figure 3. For the purpose of prediction, the following parameters are evaluated from the reference WRC at the void ratio of 0.59: s_{e0} = 600 kPa; λ_{p0} = 0.23; S_{res} = 0; $\Omega = 0.55$; e_0 = 0.59. The water retention behavior at the other three void ratios is predicted using these parameters and plotted in Figure 3 along with the experimental data points. The figure shows that the model has yielded good prediction of the evolved main drying path of WRC with void ratio. It is also inferred from these results that the air entry value and the pore size distribution index of the soil are well-captured through the equations of the presented framework.

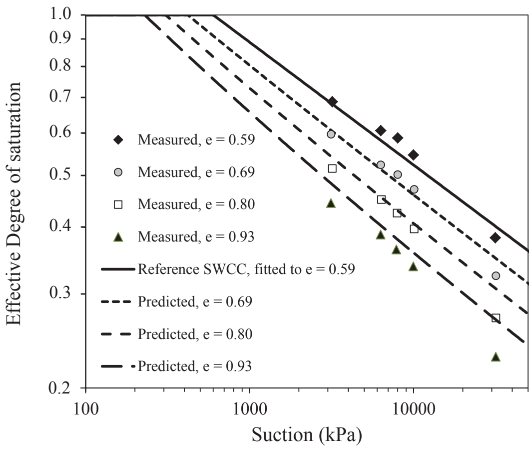

Figure 3. Measured and predicted evolution of the main drying path of WRC with void ratio for municipal Boom clay – data after [25]

3.2 Variation of degree of saturation at constant-suction loading/unloading tests

Highly expansive sand-bentonite mixture **[26]**
Results from constant suction compression tests on four specimens of a mixture of bentonite with Toyoura sand reported by [26] and [27] were used here for further model validation. The specimens were prepared by static compaction with nearly similar initial conditions at void ratios between 0.55 and 0.56, and the water contents in the range of 9.2% to 10%. The reported initial suction of the samples was around 2500 kPa. The specimens were first wetted to different target suction values of 300 kPa, 600 kPa, 1200 kPa, and 1500 kPa, for Tests 1 to 4, respectively, under a constant vertical stress of 10 kPa. The process resulted in swelling of all four samples. After wetting, the suction was held constant and the specimens were subjected to vertical loading.

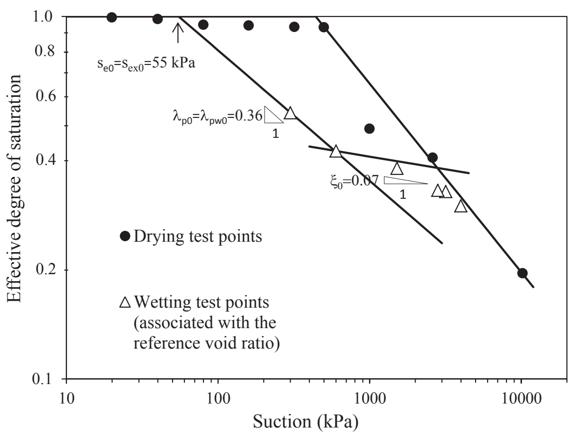

Figure 4. Measured WRC and the calibrated parameters (Data after [26])

To predict the variation of degree of saturation with void ratio during the loading, the measured reference WRC, replotted in Figure 4, were utilised with the following model parameters: s_{e0}=55.0 kPa; λ_{p0} = 0.36; ξ_0 = 0.07; S_{res} = 0.1; e_0 = 0.587. Note that, due to the lack of sufficient data points reported for the main wetting path of WRC (see Figure 4), the main drying data points of the WRC test are also plotted in Figure 4. This enabled us to compare the line fitted to the main wetting path with that fitted to the main drying path and therefore avoid possible errors in evaluation of the input parameter λ_{p0} as the pore size distribution index of the main wetting path (the values of λ_{pd} and λ_{pw} are generally close for most of the soils). The effective stress parameters used in the model are $\Omega = 0.55$ and ζ= -0.1. The value of ζ= -0.1 was back-calculated from Test 4 and the same value was used in predicting other tests. To ascertain the initial hydraulic state of the samples; i.e. at the start of the loading test, the main wetting curve of the reference WRC in Figure 4 was adjusted for the volumetric swell/shrinkage experienced by the samples during the preparation stage as shown in Figure 5. By comparing the location of the points related to the start of the loading tests to the corresponding adjusted main wetting WRCs in Figure 5, all the samples were found to be located on the scanning wetting path.

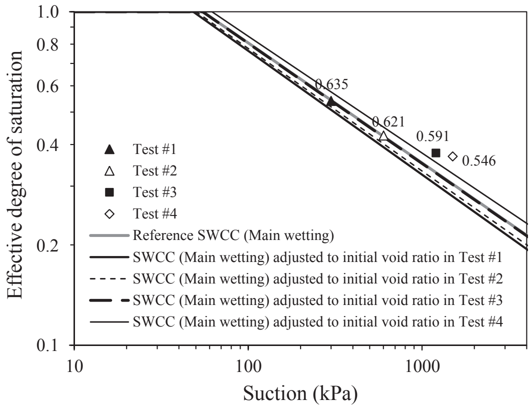

Figure 5. WRC main wetting curves at the reference void ratio and adjusted to the void ratio at the start of the constant suction loading tests (shown as numbers). Points are related to the start of the loading tests shown to compare the samples state with respect to the corresponding main wetting paths at the start of loading

Presented in Figure 6 are the model predictions together with the corresponding experimental results. Arrows on the plots depict the predicted points of transition from scanning wetting to main wetting path for the tests. The initial hydraulic state and the hydraulic path(s) followed in each loading test are also shown on the figure. As can be seen, the model has accurately predicted the variation of degree of saturation with void ratio for all four tests performed at different suctions and void ratios using a single set of reference model parameters. The results show that an increase in the rate of change in degree of saturation during a test typically relates to transition of the hydraulic path from scanning to main wetting. For tests conducted at constant suctions of 300 kPa and 600 kPa this transition occurs close to the start of the test with a clear increase in the rate of increase in the degree of saturation with decrease in the void ratio, whereas it is not detected in tests performed at suctions of

1200 kPa and 1500 kPa. These are reasonable as the initial hydraulic state of the tests performed at smaller suctions was relatively close to the main wetting path compared to the tests performed at higher suction levels. The ability of the model to capture such a subtle transition from scanning to wetting clearly shows the robustness of the approach proposed. The results also indicate that accurate assessment of the soil's initial hydraulic state (i.e. main or scanning) based on sample preparation prior to loading is important in subsequent prediction of the behavior as this directly affects the rate of change in degree of saturation with void ratio. This is particularly seen in this example for test Nos. 1 and 2.

4 Conclusions

The outline of an analytical model for taking into account the dependency of the WRC to void ratio was presented which includes both branches of main wetting and main drying as well as transition from wetting to drying and vice versa. Unlike similar models presented in the literature being mainly empirical, in the proposed model the void ratio dependency of the WRC is captured adopting energy approach; hence no additional soil parameter is needed in order to characterize the dependency of WRC on volume change. Good agreement was observed between the results of simulations and the experimental data showing that the proposed model is able to effectively capture the volume change effect on the WRC and can be incorporated into numerical simulations of unsaturated soils.

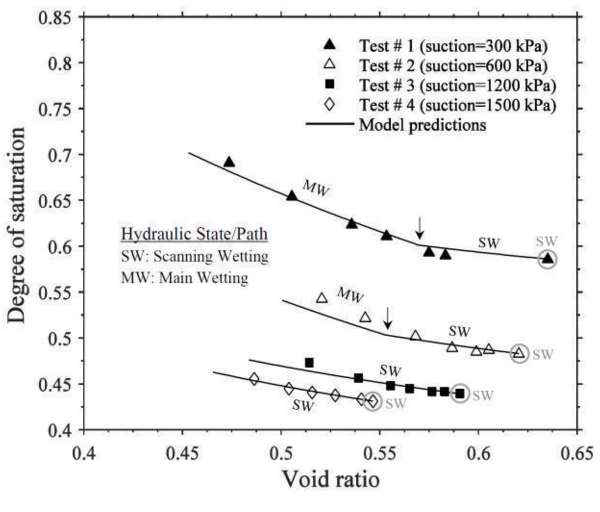

Figure 6. Measured and predicted variation of degree of saturation with void ratio at constant suction oedometric compression paths on highly expansive sand-bentonite mixture – data after [26]. Arrows show the predicted points of transition from scanning to main wetting path

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