Calculation of seismic regime parameters on the basis of a probabilistic model of Kamchatka earthquake catalogue

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Abstract. In the present paper on the basis of a probabilistic interpretation of the earthquake repetition law, simple formulas were obtained which allow us from the known number of earthquake events in a representative interval of energy class (magnitude) to estimate the number of seismic events falling within an unrepresentative interval and the expected time of an earthquake with a magnitude exceeding the maximum registered values for the period of instrumental observations.

1 Introduction

The character of Kamchatka seismicity is determined by interaction of several lithospheric plates: Pacific, Okhotsk, Eurasian, North-American and Bering. The majority of earthquakes and the strongest events are associated with Pacific plate subduction. This process is accompanied by tectonic stress accumulation and relaxation and manifests in a large number of earthquakes, the hypocenters of which are associated with the descending oceanic plate. Earthquakes occurring in Kamchatka have different tectonic nature [1]. Figure 1 illustrates six tectonic-geographical zones (structures) of Kamchatka region which have significantly different characteristics of seismicity. S_1, S_2, S_3 and S_4 zones have complicated three-dimensional configuration and sloping edges in depth. S_5 and S_6 zones do not contain deep earthquakes and have vertical edges.

On the basis of a probabilistic approach proposed in [2, 3], it is possible to analyze the seismic regime for tectonic-geographical zones in Fig. 1. On the basis of probabilistic interpretation of earthquake repetition law graph suggested in [2] and considered in detail in [3, 4], it is possible to obtain simple relations allowing:

• from the known number of earthquakes in a representative energy class (magnitude) to calculate the number of events falling within a unrepresentative interval;

• on the considered area to estimate the expected time of an earthquake with magnitude exceeding maximum registered values for the instrumentation period of observations.
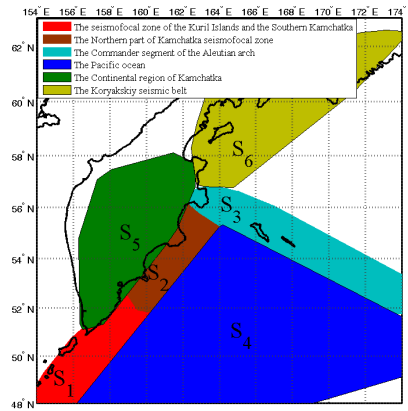
Earthquake repetition law in a probabilistic representation for energy class K distribution density of earthquake events has the following form [2]

$$f(K^{1,2}) = f(K_0^{1,2}) \cdot 10^{-\gamma_{1,2}(K^{1,2}-K_0^{1,2})} \tag{1}$$

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Figure 1. Surface-section of seismicity zones for Kamchatka region

or in a double logarithmic scale the dependence (1) may be presented as

$$\lg f(K^{1,2}) = \lg f(K_0^{1,2}) - \gamma_{1,2}(K^{1,2} - K_0^{1,2}) \tag{2}$$

Expressions (1) and (2) are written for the following ranges of changing of K: $9.5 \leq K_1 < 12.5$ with γ_1 ($K_0^1 = 9.5$) and $12.5 \leq K_2 < 15.5$ with γ_2 ($K_0^2 = 12.5$) and $f(K_0)$ denotes initial distribution density calculated in the middle of the interval $K_0 \pm \Delta K/2$.

2 Calculation of unrepresentative energy class event number

It was shown by the calculations on the example of Kamchatka earthquake catalogue [3], that it does not depend on the fact if a catalogue is cleared from aftershocks or not, for the fixed period of time T and a given initial interval $K_0 \pm \Delta K/2$, the distribution densities $f(K)$, determined by (1), almost do not depend on the considered area where epicenters of events are located. Spread of values appears only in the result of different number of events falling within the considered area. The larger is the number of events, the more stable is the distribution $f(K)$. It is achieved either by the increase of an area for a given time period T or by the increase of a period itself for a fixed area. Table 1 shows the probability values for the earthquakes to fall within the interval of the energy class K, calculated for the period T from 01.01.1962 to 01.01.2008 and averaged for $S_1, S_2, \ldots, S_6$ zones for the uncleared catalogue of Kamchatka earthquakes [5].
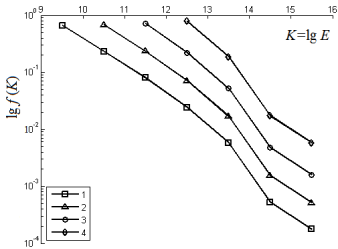
For different initial K_0 the following values $P(K_0)$ were obtained when $\Delta K = 1$ (Table 1): $P(K_0 = 9.5 \pm 0.5) = 0.658$, $P(K_0 = 10.5 \pm 0.5) = 0.692$, $P(K_0 = 11.5 \pm 0.5) = 0.713$, $P(K_0 = 12.5 \pm 0.5) = 0.716$. The average value of the probability $P_{av}(K_0)$ for different initial K_0 is equal to 0.6947.

Figure 2 shows the dependences (2) for different K_0, equal to 9.5, 10.5, 11.5 and 12.5, calculated for the tectonic-geographic S_2 zone. On the basis of the dependence (2) the values of incline angles γ_1 and γ_2 of the earthquake repetition law were calculated (Table 2) for two ranges of the energy class $9 \leq K < 13$ and $12 \leq K < 16$, correspondingly. The values of γ (Table 2) presented in the paper [1] and the values of γ_1 calculated on the basis of probability distributions almost coincide.

Comparing different values of probabilities showed in Table 1, one may note that with the increase of initial K_0 the values of the obtained probabilities have the tendency of gradual increase. It is

Table 1. Averaged distributions of probabilities for different initial values K_0

Intervals of energy class K_i	Average $P(K_i)$ for the areas of $S_1, S_2, \ldots, S_6,$ $K \geq 9$	Average $P(K_i)$ for the areas of $S_1, S_2, \ldots, S_6,$ $K \geq 10$	Average $P(K_i)$ for the areas of $S_1, S_2, \ldots, S_6,$ $K \geq 11$	Average $P(K_i)$ for the areas of $S_1, S_2, \ldots, S_6,$ $K \geq 12$
9.5 ± 0.5	0.658			
10.5 ± 0.5	0.234	0.692		
11.5 ± 0.5	0.076	0.218	0.713	
12.5 ± 0.5	0.023	0.064	0.205	0.716
13.5 ± 0.5	0.007	0.019	0.059	0.208
14.5 ± 0.5	0.0017	0.005	0.017	0.0602
15.5 ± 0.5	0.00072	0.002	0.00523	0.016

Figure 2. $f(K)$ dependence in a double logarithmic scale (relation (2)) for S_2 zone for different initial values K_0: 1) $K_0 = 9.5 \pm 0.5$, 2) $K_0 = 10.5 \pm 0.5$, 3) $K_0 = 11.5 \pm 0.5$, 4) $K_0 = 12.5 \pm 0.5$.

Table 2. Inclination angle values of the graph of earthquake repetition law

	S_1	S_2	S_3	S_4	S_5	S_6
$\gamma, 9 \leq K < 13$	-0.5	-0.43	-0.45	-0.7	-0.53	–
$\gamma_1, 9 \leq K < 13$	-0.476	-0.425	-0.456	-0.623	-0.589	-0.434
$\gamma_2, 12 \leq K < 16$	-0.745	-0.726	-0.595	–	–	-0.309

explained by the fact that with the decrease of K_0, some difficulties on registration of events appear. They are determined by unbranched network of seismic stations, external disturbing factors and so on, which eventually result in the gaps of occurred earthquakes. Thus, the graph fall of the repetition law for small values of K, and, correspondingly, the probability decrease occur due to the decrease of the registered number of events falling within the unrepresentative region of energy class values. On the other side, with the increase of initial value of the complete group of real events decreases and the part of events in the initial chosen interval increases and the probability in frequency presentation grows.

Calculation of distribution density on the basis of real registered events with energy less than a representative one and its comparison with $f(K_i)$, calculated on the basis of the relation (1), give disagreement. Thus, the task appears to obtain a formula for calculation of event number in energy class unrepresentative interval on the basis of known event number and, consequently, distribution density in representative class. Hence, if for period T and for area S within the energy class representative interval $K_0 \pm 0.5$, $n_0(\Delta K)$ earthquakes were registered when their total number is N_Σ, forming an

event complete group with energy classes from $K_0 \pm \Delta K$ to $K_{max} \pm \Delta K$, than the number of events n_{-1}, occurred in the next unrepresentative interval with $K_{-1} < K_0$, may be determined by the relation (3).

$$n_{-1} = \frac{N_\Sigma \cdot P_0(K_0 \pm \Delta K/2)}{1 - P_0(K_0 \pm \Delta K/2)} \tag{3}$$

It is assumed that with K value decrease in unrepresentative class region, γ_1 does not change. Now we determine the number of earthquakes which occurred in the northern part of Kamchatka seismofocal zone (S_2 zone) for the period T (from 01.01.1962 to 01.01.2008) and falling within the unrepresentative interval of the energy class 8.5 ± 0.5. Since the total number of events with $K \geq 9$, which occurred in S_2 zone for the period T, is $N_\Sigma = 11323$, and the probability to fall within the initial interval is $P(9.5 \pm 0.5) = 0.6582$, then according to the recurrent relation (3) we obtain $n_{-1} = 21805$. At the same time according to the earthquake catalogue, the total number of registered events for $T = 46$ years on the area S_2 and falling within the interval 8.5 ± 0.5 equals 16893. Similarly, on the basis of (3) the number of events was calculated that fall into energy class interval 8.5 ± 0.5 for other considered zones S_i. The calculation results are shown in Table 3.

Table 3. The number of events in an unrepresentative range of energy class

Zone	S_1	S_2	S_3	S_4	S_5	S_6
$n(8.5 \pm 0.5)$, (catalogue)	7308	11323	3177	1534	2348	710
$n(8.5 \pm 0.5)$, (relation (3))	10566	21805	6361	3912	3698	1000

## 3 Calculation of the expectance period of a strong earthquake

The following expression was obtained in the paper [2] on the basis of relation (1)

$$\lg N = A - bM \tag{4}$$

where $A = \lg N(M_0) + bM_0 = \lg[10^{bM_0} \cdot N(M_0)]$ and $b = 1.5\gamma$, $N(M_0)$ is the number of events falling within the interval $M_0 \pm \Delta M/2$, $M_0 = (K_0 - 4.6)/1.5$ ($M_0^1 = 3.27$ for $K_0 = 9.5$, $M_0^2 = 5.27$ for $K_0 = 12.5$, $\Delta M = 0.67$ for $\Delta K = 1$). The relation between K class and M magnitude for Kuril-Kamchatka earthquakes was specified by $K = 4.6 + 1.5M$. When there is no enough statistics in the region of high M values, as $\lg N$ becomes negative, and the time of expectance exceeds the period of instrumentation observations, it is possible to estimate this time by relation (4) for magnitude higher values compared to already registered ones for the period of instrumentation observations. We suppose that spatial and energy characteristics of Kamchatka earthquakes for 01.01.1962–01.04.2016 at the area reflect, in average, its seismic regime. Thus, the average number of events, the epicentres of which are located on area S_7 (Fig. 3), calculated for a long period of observations (several decades, for example), describes quite well the average regime for a longer period. In this case, from the equality of average calculated for the periods of instrumentation observations T_{ins} and the required period T_{req} during which one event may occur with maximum magnitude M_{max}, we obtain

$$T_{req} = \frac{10^{1.5\gamma(M_{max} - M_0^2)}}{N(M_0^2)} T_{ins} \tag{5}$$

According to (5) the expected time for Kamchatka region for three magnitudes $M_{max}^1 = 8.5$, $M_{max}^2 = 9.0$ and $M_{max}^3 = 9.5$ is estimated. The results of data processing from the catalogue for
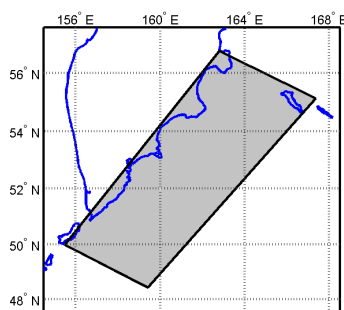
Figure 3. S_7 zone

Kamchatka region for zone S_7 for the period of T_{ins} = 54 years (from 01.01.1962 to 01.04.2016): N_Σ = 27117 is the summary number of events of the catalogue with the energy class within the interval from 9.5 ± 0.5 to 15.5 ± 0.5 (for the magnitudes from 3.27 ± 0.335 to 7.27 ± 0.335); $P(5.27 \pm 0.335)$ = 0.024 is the probability for seismic events to fall within the interval $M_0^2 \pm 0.335$; $N(M_0^2)$ = $P(5.27 \pm 0.335) \times N_\Sigma$ = 0.024×27117 = 662 is the number of events registered on S_7 area for T_{ins} within the magnitude interval $M_0^2 \pm 0.335$; $|\gamma|$ = 0.64 is the incline coefficient calculated by the least square method. Calculation on the basis of the given data results in the following values of the expected periods with corresponding magnitudes of events: for M_{max}^1 = 8.5 expected period $T_{req}^{8.5} \approx 100$ years; for M_{max}^2 = 9.0 expected period $T_{req}^{9.0} \approx 300$ years; for M_{max}^3 = 9.5 expected period $T_{req}^{9.5} \approx 960$ years.

4 Conclusion

It was shown that for the considered zones of $S_1, S_2, \ldots, S_6$ and different values of the initial interval of the energy class $K_0 \pm 0.5$, for K_0 = 9.5, 10.5, 11.5 and 12.5, the probability values $P(K_0 \pm 0.5)$ differ little from each other and tend to the average value 0.6947.

On the basis of recurrence relation (3) the estimation of events which must occur in the unpresentative energy class $K_0 \pm 0.5$ was calculated for the zones of $S_1, S_2, \ldots, S_6$.

On the basis of relation (5) the estimation of time of a strong earthquake for zone S_7 was calculated. The periods of expectance of seismic events with magnitudes 8.5, 9.0 and 9.5 were 100, 300 and 960 years, respectively.

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