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Reversals of the Magnetic Field in one low-dimensional $\alpha\Omega$-dynamo model

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Abstract. Described low-dimensional model, that implements the various known regimes of cosmic dynamo systems, in particular, the various regimes of reversals. The source of regular reversals in the model is its internal dynamics. Violation of this cyclicality, failures in the a dynamo, the occurrence regime irregularity reversals caused by chaotic fluctuations in the parameters. This fluctuations are interpreted as the result of spontaneous coherent addition of the higher modes. The simplicity of the model allows you to easily modify it for various celestial bodies.

1 Introduction

For systems of cosmic dynamo (planets, stars, galaxies) are usually considered three mechanisms $\alpha\Omega$, α^2 and $\alpha^2\Omega$, of which the third is the most general [1, 2]. The first two are used when the efficiency of α- and Ω-generator differ sharply. In particular, a strong differential rotation generates a magnetic field described by a $\alpha\Omega$-dynamo.

Known properties of dynamo systems is the presence of reversals - abrupt change in the magnetic field polarity, without significant restructuring of the motion of a conducting medium. Real cosmic dynamo system demonstrate as a regular character reversals (like Sun), and very chaotic (like Earth). The length of the polarity intervals in the geomagnetic field differs by several orders of [3].

The simulation of reversals is intensively developing part of the dynamo theory. Their research is use both direct numerical simulation and simplified models. Direct numerical simulation of the magnetohydrodynamic equations reproduce multiple type of reversals, but does not answer the question of their reason. In addition, the full equations contain a lot of parameters, which estimations differ by several orders, or unknow. Therefore, using simple low-dimensional dynamical systems are trying to explain the physical cause, features, the most important properties of this phenomenon, for example [4–8].

In this paper, we simulate the inversion of the field in a simple model of $\alpha\Omega$-dynamo. This three-mode model with fluctuating of intensities α- and Ω-generators. The source of these fluctuations can be interpreted as a result of synchronization discarded modes of velocity and magnetic field. Such spontaneous formation and destruction of the coherent structures is well known in the theory of turbulence [9].

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2 Model equations

Consider a spherical shell of a viscous conducting liquid in a rotating coordinate system. The origin coincides with the center of the shell, and the Oz – axis of rotation. We will also use the spherical coordinates (r, θ, φ). On the inner r_1 and the outer r_2 boundaries the velocity is zero. This formulation of the boundary conditions is typical for geodynamo and is set for definiteness. For a magnetic field are set vacuum boundary conditions on the r_2.

The physical parameters of the shell is constant. We also believe that the turbulence in the shell is isotropic, and α-effect antisymmetric with respect to the equatorial plane. Therefore we accept scalar parametrization of α-effect in the form $\alpha(r,\theta) = \alpha_0 a(r)\cos\theta$, where $\max|a(r)| \sim 1$ and the coefficient $\alpha_0 > 0$ determines the intensity of α-effect. Now we will consider the problem of kinematic dynamo, but the further we introduce algebraic quenching of α-effect by large-scale magnetic field.

The dimensionless dynamo equation can be written as

$$\begin{aligned} &\frac{\partial \mathbf{B}}{\partial t} = \mathrm{R_m}\nabla\times(\mathbf{v}\times\mathbf{B}) + \mathrm{R}_\alpha\nabla\times(a(r)\ \cos\theta\,\mathbf{B}) + \Delta\mathbf{B}, \\ &\nabla\cdot\mathbf{B} = 0, \end{aligned} \tag{1}$$

where $\mathrm{R_m}$ - magnetic Reynolds number, and R_α- amplitude of α-effect. Velocity $\mathbf{v}$ is considered a given constant. This type of dimensionless equations corresponds to the choice of radius r_2 as length scale L, magnetic diffusion time L^2/ν_m as the time scale (ν_m - magnetic diffusion coefficient) and some velocity value U and the some magnetic field value B_0.

In the simplest form $\alpha\Omega$-dynamo considered that medium motion is a differential rotation, ie, field $\mathbf{v}$ – toroidal. But we know that in reality this movement is convection and $\mathbf{v}$ includes poloidal component. Moreover, spherically uniform heat flux is excited by convection only poloidal component and toroidal arise only as a result of the Coriolis drift.

Therefore, we set the velocity field as a linear combination of several modes of the free dissipation of the fluid in the shell. All these modes have the form $\mathbf{v}^T_{k,n,m} = \nabla\times\left[R^T_{kn}(r)Y^m_n(\theta,\varphi)\mathbf{r}\right]$ (toroidal) and $\mathbf{v}^P_{k,n,m} = \nabla\times\nabla\times\left[R^P_{kn}(r)Y^m_n(\theta,\varphi)\mathbf{r}\right]$ (poloidal), where index k determines the number of convection layers in the radial direction. The axially symmetric case $m = 0$. Parameters $R^T_{kn}(r)$ and $R^P_{kn}(r)$ depend on the aspect ratio r_1/r_2. We used the terrestrial value 0.35.

Differential rotation correspond to the modes $\mathbf{v}^T_{k,1,0}$. They are consist in the linear capsule of the set $\left\{\mathbf{v}^T_{k_1,1,0}, \mathbf{v}^P_{k_2,2,0}, \mathbf{v}^T_{k_3,3,0}, \mathbf{v}^P_{k_4,4,0}, \dots\right\}$, which is invariant under the Coriolis drift. Because any such mode are driven rest by rotation. Then, in the simplest case, the poloidal velocity component – a $\mathbf{v}^P_+ + 0,2,0$. It turns out that the distribution of this mode Coriolis drift direction in the shell volume is well approximated by a combination of four toroidal modes: $\mathbf{v}^T_{0,1,0}$, $\mathbf{v}^T_{1,1,0}$, $\mathbf{v}^T_{0,3,0}$, $\mathbf{v}^T_{1,3,0}$. Therefore, we set the velocity as a combination of these five modes. The coefficients are chosen so that the combination approximates one of the eigenmodes of the Poincare operator . The scheme of calculation of such approximations is described in [10].

For representation of the magnetic field, we will use some of the of Ohmic decay modes $\mathbf{B}^T_{k,n,m}$ and $\mathbf{B}^P_{k,n,m}$. Their structure is similar to the previously described modes of free dissipation of velocity.

We selected of magnetic modes by the scheme proposed in [11].

Let the magnetic field is represented by a linear combination of several modes. Substituting this expansion in the induction equation (1) and apply the Galerkin method. We obtain a system

$$\frac{dg_k}{dt} = \mathrm{Re}_m\sum_i W_{ki}g_i + \mathrm{R}_\alpha\sum_i A_{ki}g_i - \eta_k g_k, \tag{2}$$

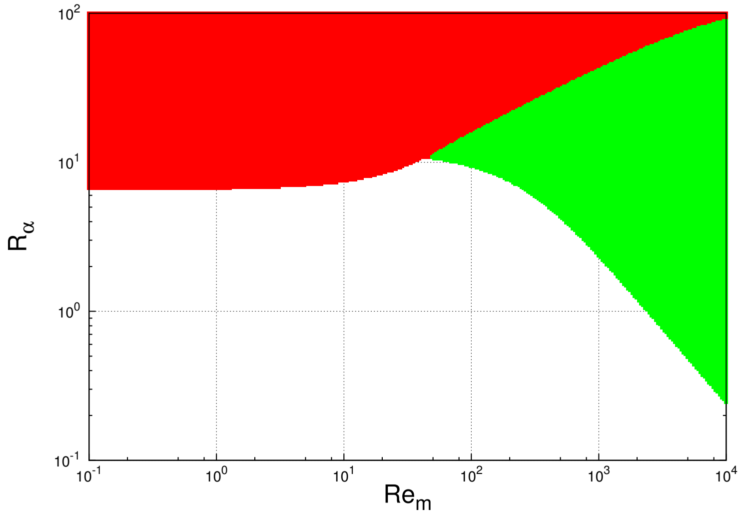

Figure 1. The areas of field generation. Red dots – non-oscillatory dynamo; green dots – oscillatory dynamo.

where $g_k(t)$ – magnitudes of the modes, and η_k – eigenvalues. Matrix W and A are formed by Galerkin coefficients .

Consider the eigenvalues of matrix of the system (2). Let us call the leading and denoted as λ it of them, which has the largest real part. Clearly, the dynamo works if and only if $\Re\mathfrak{e}\lambda > 0$. Decisions will be oscillating if the number λ imaginary.

In accordance with the approaches of work [11] magnetic field is represented to minimum number of the modes, sufficients to produce oscillatory dynamo.

By varying the parameters Re_m and R_α, we found that oscillating solutions arise, if we use the the following three lower modes: $\mathbf{B}^P_{0,1,0}$ (dipole), $\mathbf{B}^T_{0,2,0}$ and $\mathbf{B}^P_{0,3,0}$. Next, we denote their $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$, respectively.

Of course, the eigenvalues of the system (2) depend on the type of the radial part of the α-effect $a(r)$. We used three ways: $a(r) = 1$, $a(r) = r$, $a(r) = 1/r$.

Note that in the [11] for oscillating solutions authors took 5 magnetic modes for dynamo in the star with a thin convective shell, but they used a toroidal velocity like differential rotation. Most likely, a smaller number of modes in our work can be explained by a more complex structure of the velocity field.

On Fig. 1 shows the areas of oscillatory and non-oscillatory dynamo for $a(r) = 1/r$. Other variants $a(r)$ are given a very similar area, so we continue to limit ourselves this dependence.

Now we introduce in (2) algebraic quenching of α -effect and fuctuations:

$$\frac{dg_k}{dt} = \mathrm{Re}_m(1 + \zeta(t)) \sum_i W_{ki} g_i + \frac{\mathrm{R}_\alpha(1 + \xi(t))}{1 + \sum_j g_j^2} \sum_i A_{ki} g_i - \eta_k g_k, \tag{3}$$

where $\zeta(t)$ and $\xi(t)$ – stochastic processes whit zero means.

As mentioned above, these processes are simulated spontaneously emerging and spontaneously destroying the coherence impact the discarded modes velocity and magnetic field. The structure of the processes is following.

We take on the time axis of the random sequence of points $0 < \tau_1 < \theta_1 < \tau_2 < \theta_2 < \cdots < \tau_k < \theta_k < \ldots$. We assume that the k-th coherent structure is formed at the time τ_k and destroyed at the time θ_k. Then $T_k^{est} = \tau_k - \theta_{k-1}$ - waiting time of formation next structure, and $T_k = \theta_k - \tau_k$ - duration of its existence. The processes $\zeta(t)$ and $\xi(t)$ are zero during the waiting time, and $\zeta(t) = \zeta_k$ and $\xi(t) = \xi_k$ during the time of existence . Here ζ_k and ξ_k independent random variables with zero mean. The laws of the distribution of these variable, as well as T_k^{est} and T_k selected in numerical simulations.

3 Simulation results

Consider the results of computational experiments with the model (3).

From the Fig. 1 we see, that the bifurcation point in the linear approximation (2) is a boundary point of red, green and white areas, i.e. $(\mathrm{Re}_m, \mathrm{R}_\alpha) \approx (49, 11)$.

First, we carried the calculation of the solutions of (3) at $(\mathrm{Re}_m, \mathrm{R}_\alpha)$ with the vicinity of point $(49, 11)$ and zeroes $\zeta(t)$ and $\xi(t)$. In the nonlinear case, the bifurcation point has shifted to $(\mathrm{Re}_m, \mathrm{R}_\alpha) \approx (42.15, 5.45)$. It was also found that the system is highly responsive not only to changes in the parameters, but also to initial conditions.

Therefore, one might expect that the impact of even small fluctuations will switch model between different regimes of dynamo.

In the simulation, we used the exponential law of distribution for the waiting time T_k^{est} and the existence time T_k, and themselves these variables were independent. Mean values $\left\langle T_k^{est} \right\rangle = 5$ and $\langle T_k \rangle = 30$, i.e. time of the existence of coherent structures much less time of their expectations.

The values of jumps ζ_k and ξ_k are uniformly distributed in the interval $[-0.01; 0.01]$ and $[-0.1; 0.1]$, respectively.

The strong differences in the choice of variance of jumps conditioned by the following considerations. α-effect has a turbulent nature, so its response to the restructuring of the turbulence structure i.e. xi_k, can be considerable. Then, disturbance in the magnetic field through the Lorenz force changes the large-scale medium flow. We have described this change jump $zeta_k$. It therefore seems reasonable that the intensity of fluctuations in the turbulence generator should be much higher than in a large scale generator.

By selecting different values of $(\mathrm{Re}_m, \mathrm{R}_\alpha)$ in a small neighborhood of a bifurcation point and different initial conditions, we have received a variety of modes dynamo: quasi-periodic, dynamo-bursts, the disappearance of the field, followed by growth, irregular reversals.

On Fig. 2 shows the examples of the two of realizations. Dimensionless amplitude of the dipole are plotted on the vertical axis.

At the top is mainly quasi-periodic solution. There are only a few failures in the dynamo cycle. This solution is like to the solar dynamo. It is worth noting the almost complete disappearance of the field in the $340 \le t \le 370$ and several shorter intervals. There is a certain analogy with the Maunder minimum, the failure of the dynamo.

The solution at the bottom of Fig. 2 is characterized by an extremely irregular reversals, which is typical for Geodynamo.

Thus, in the proposed simple model of dynamo at close values of the parameters can reproduced different modes dynamo, which are observed in real dynamo systems. Of course, all these solutions able to get a lot of writers and earlier as a result of direct number simulation, and the simple models.

The advantage of the proposed model is that it is not associated with the structure of a particular celestial body and does not require knowledge of the distribution of its physical parameters.

The source of regular reversals in the model is its internal dynamics, but violation of this cyclicality, failures in the dynamo, the yield on the regime of chaotic reversals arises due to the restructuring of the turbulence structure.

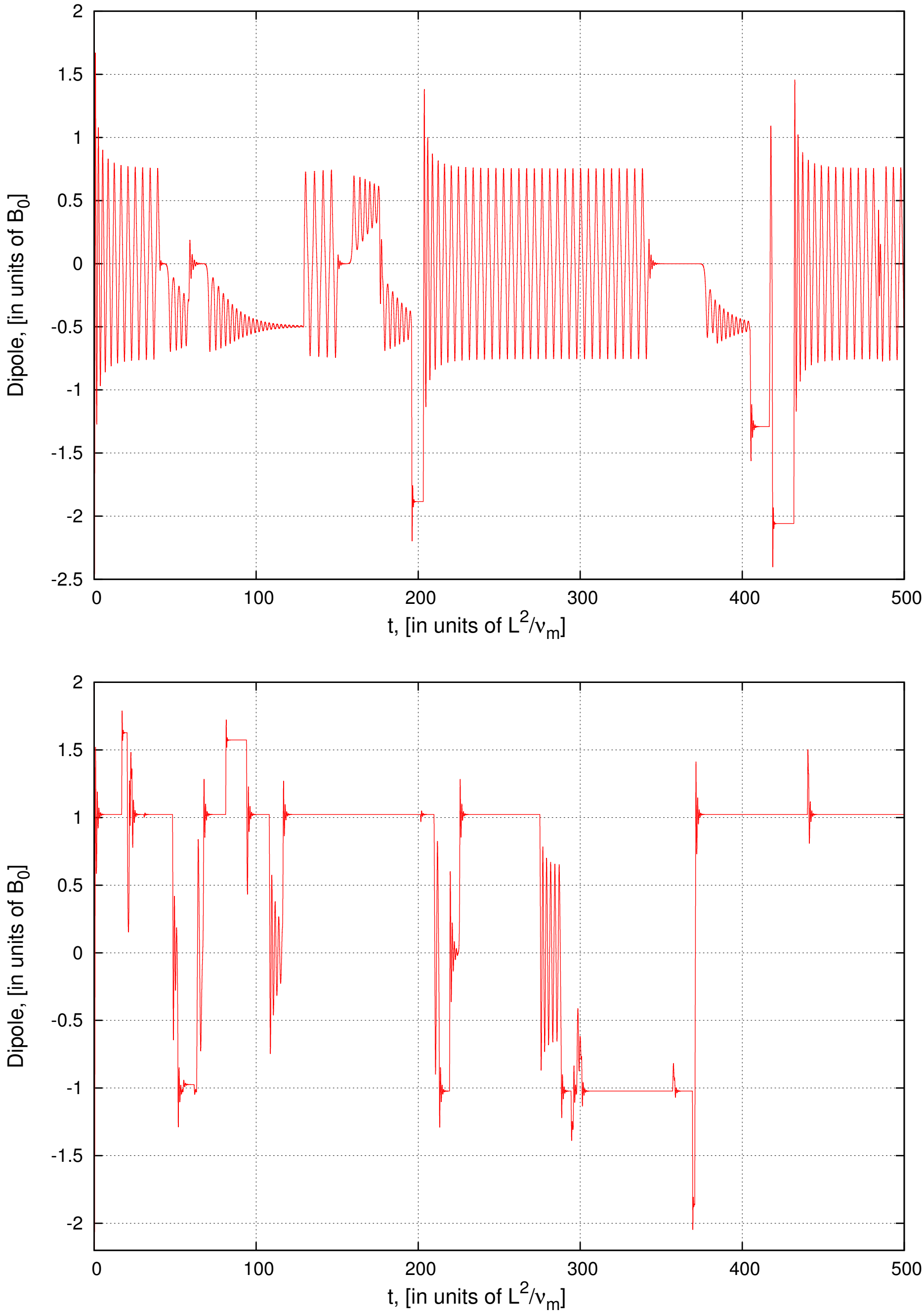

Figure 2. The different regimes of changes the amplitude of the dipole. Top: $(\mathrm{Re}_m, \mathrm{R}_\alpha) = (42.2, 5.5)$. Bottom: $(\mathrm{Re}_m, \mathrm{R}_\alpha) = (42.2, 5.7)$.

4 Conclusion

Proposed in this paper, the model allows to describe various known regimes of cosmic dynamo systems, in particular, the various regimes of reversals.

The simplicity of this model makes it easy to modify it for various celestial bodies. So, depending on the aspect ratio, you can adapt it to the terrestrial planets, giant planets, fully convective stars, stars with a thin convective shell. This will change the parameters of the velocity modes, that may change the set of magnetic modes or their number. Knowledge of the physical parameters of a particular celestial body is not required.

A wide variety of regime may be provided as different distributions of variables T_k^{est} and T_k. For simplicity, we have chosen the exponential distribution for simulation. However, it is known that for turbulence is more characteristic of the power laws. It seems that the introduction of power lows gives an even more complex statistics of reversals, for example, similar paleomagnetic polarity scale.

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