

Simple heat transfer correlations for turbulent tube flow

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Abstract. The paper presents three power-type correlations of a simple form, which are valid for Reynolds numbers range from $3 \cdot 10^3 \leq Re \leq 10^6$, and for three different ranges of Prandtl number: $0.1 \leq Pr \leq 1.0$, $1.0 < Pr \leq 3.0$, and $3.0 < Pr \leq 10^3$. Heat transfer correlations developed in the paper were compared with experimental results available in the literature. The comparisons performed in the paper confirm the good accuracy of the proposed correlations. They are also much simpler compared with the relationship of Gnielinski, which is also widely used in the heat transfer calculations.

1 Introduction

Heat transfer correlations for turbulent fluid flow in the tubes are commonly used in the design and performance calculations of heat exchangers [1-4]. The value of heat transfer coefficient significantly affects the value of the thermal stress [5]. Time changes in optimum fluid temperature determined from the condition of not exceeding the stress at a point lying on the inner surface of the pressure component, very strong depend on the heat transfer coefficient [6-9]. Therefore, continuous experimental research is carried out to find a straightforward and accurate heat transfer correlations. The empirical correlation of Dittus-Boelter [10-12] has gained widespread acceptance for prediction of the Nusselt number with turbulent flow in the smooth-surface tubes

$$Nu = 0.023 Re^{0.8} Pr^n \quad (1)$$

$$0.6 \leq Pr \leq 160, \quad Re \geq 10^4, \quad L / d_w \geq 60$$

where : $Nu = hd_w / k$, $Re = u_m d_w / \nu$, $Pr = c_p \mu / k$,
 $d_w = 2r_w$ - inner diameter of the tube, L - tube length,
 h - heat transfer coefficient, k - thermal conductivity, u_m - mean velocity of the fluid, c_p - specific heat at constant pressure, $\nu = \mu / \rho$ - kinematic viscosity, μ - dynamic viscosity, ρ - fluid density.

The exponent of the Prandtl number is $n = 0.4$ for heating of the fluid and $n = 0.3$ if the fluid is being cooled. The similar power-type relationship of Colburn is based on the Chilton-Colburn analogy for heat and momentum transfer [13]

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \quad (2)$$

$$0.6 \leq Pr \leq 160, \quad Re \geq 10^4, \quad L / d_w \geq 60$$

The correlations (1) and (2) are valid for moderate temperature differences $|T_w - T_m|$ under which properties may be evaluated at the mean bulk temperature \bar{T}_m . When variations of physical properties are significant, the Sieder and Tate relationship [14] can be used

$$Nu = 0.027 Re^{0.8} Pr^{1/3} \left(\frac{\mu_m}{\mu_w} \right)^{0.14} \quad (3)$$

$$0.7 \leq Pr \leq 16,700, \quad Re \geq 10^4, \quad L / d_w \geq 60$$

where all properties are evaluated at the local bulk temperature T_m except the viscosity μ_w that is estimated at the wall surface temperature T_w .

However, the power-type correlations like Dittus-Boelter, Colburn, and Colburn relationships do not provide good approximations within larger ranges of the experimental data because of its simple form. The power type correlation like that of Dittus-Boelter is not able to approximate the experimental results over a broad range of the Prandtl number. The maximum deviation between experimental data and predictions using Eq. (1) is about 20% [15-17]. When the working medium is water, the Dittus-Boelter relationship (1) overpredicts the heat transfer coefficient for smaller Reynolds numbers when $Re < 10,000$ and underpredicts for higher Reynolds numbers [18]. In the last forty years, the power-type equations (1-3) were often being replaced by the

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Gnielinski correlation [19-20]

$$Nu = \frac{\frac{\xi}{8} (Re-1000) Pr}{1 + 12.7 \sqrt{\frac{\xi}{8}} (Pr^{2/3} - 1)} \left[1 + \left(\frac{d_w}{L} \right)^{2/3} \right] \left(\frac{Pr_m}{Pr_w} \right)^{0.11} \quad (4)$$

$$4 \cdot 10^3 \leq Re \leq 10^6, \quad 0.5 \leq Pr \leq 200$$

where the symbol ξ denotes the friction factor for smooth tubes calculated from the Filonienko relationship [21]

$$\xi = (1.82 \log Re - 1.64)^{-2} \quad (5)$$

The symbols Pr_m and Pr_w designates the Prandtl number at the bulk and wall temperature, respectively.

The Gnielinski relationship (4) is a modification of the Petukhov formula [22]

$$Nu = \frac{\frac{\xi}{8} Re Pr}{1.07 + 12.7 \sqrt{\frac{\xi}{8}} (Pr^{2/3} - 1)} \left(\frac{\mu_m}{\mu_w} \right)^n \quad (6)$$

$$10^4 \leq Re \leq 10^6, \quad 0.5 \leq Pr \leq 200$$

to increase the accuracy of Eq. (6) for smaller Reynolds numbers. In Eqs. (4) and (6), the subscript m refers to evaluating the fluid properties at the mean (bulk) average temperature \bar{T}_m that is usually defined as the average of the inlet and exit bulk temperature. The subscript w refers to the evaluation of the fluid properties at the wall surface temperature T_w . The exponent n in Eq. (6) is equal to 0.11 for $T_w > \bar{T}_m$ and $n = 0.25$ for $T_w < \bar{T}_m$. The constant n is equal zero for gases. The relationship (4) of Gnielinski is very widely used, as well approximates the experimental data [17, 23-25].

In recent years, however, a new rise in popularity of power-type correlation to an approximation of the measurement data is observed [24-26].

The thermal-hydraulic performance of a dimpled enhanced tube was studied experimentally and numerically by Li et al. [24]. The power type correlation based on the experimental results was proposed. The Wilson method was used to adjust unknown constants in the searched relationship. The Prandtl number changed from 5.2 to 30.7 and the Reynolds number from 500 to 8,000.

Simple power correlations are also proposed to fit experimental data for turbulent flows of nanofluids in tubes. Li and Xuan [25] and Azmi and al. [26] also used electrically heat tubes to study hydrodynamically fully developed flow. Before starting the actual experiments with nanofluids, tests were conducted in which the working fluid was water [25] or a mixture of water and

ethylene glycol [26]. The Reynolds number varied in the study of Azmi et al. from 3,000 to 25,000. The Nusselt numbers determined experimentally showed good agreement with the Nusselt numbers obtained from the Dittus-Boelter equation.

Li and Xuan [25] also found a heat transfer correlation for internal flow of Cu-water nanofluid that for water reduces to

$$Nu = 0.0059 Re^{0.9238} Pr^{0.4} \quad (7)$$

$$2.5 \cdot 10^3 \leq Re \leq 2.5 \cdot 10^4$$

Previous studies indicate that power-type correlations are still used successfully to approximate the experimental results when the Reynolds and Prandtl numbers vary in a narrow range.

Unknown constants occurring in the power-type relationships can be readily determined by standard [27] or modified [28] Wilson method while maintaining reasonable effort.

Easy determination of searched constants based on the experimental results is also the reason for the use of power type functions to find heat transfer correlations for the turbulent flow of nanofluids or molten salts in tubes.

2 Nusselt numbers for turbulent tube flow

Energy conservation equation for turbulent tube flow averaged by Reynolds has the following form [23, 29]

$$\rho c_p \bar{u} \frac{\partial \bar{T}}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} (r q) \quad (8)$$

where: q – heat flux density, x – Cartesian coordinate, r – radius, \bar{u} - time averaged velocity.

The heat flux q consists of the molecular q_m and turbulent q_t component

$$q = q_m + q_t \quad (9)$$

where

$$q_m = k \frac{\partial \bar{T}}{\partial r}, \quad q_t = \rho c_p \varepsilon_q \frac{\partial \bar{T}}{\partial r} = k \frac{Pr}{Pr_t} \frac{\varepsilon_\tau}{\nu} \frac{\partial \bar{T}}{\partial r} \quad (10)$$

The symbols in Eq. (10) stand for: \bar{T} - time averaged temperature, ε_q - eddy diffusivity for heat transfer, ε_τ eddy diffusivity for momentum transfer (turbulent kinematic viscosity), $Pr_t = \varepsilon_\tau / \varepsilon_q$ - turbulent Prandtl number. Equation (8) is subject to the following boundary conditions

$$k \frac{\partial \bar{T}}{\partial r} \Big|_{r=r_w} = q_w \quad (11)$$

$$\left[\frac{\partial \bar{T}}{\partial r} \right]_{r=0} = 0 \quad (12)$$

$$\bar{T}|_{x=0} = T_m|_{x=0} \quad (13)$$

where q_w is the wall heat flux. The mass averaged (bulk) temperature $T_m(x)$ is defined as follows

$$T_m(x) = \frac{2}{r_w^2 u_m} \int_0^{r_w} \bar{u}(r) \bar{T}(x,r) r dr \quad (14)$$

The time averaged velocity profile $\bar{u}(r)$ in tube cross-section is necessary to determine the temperature distribution in the fluid. The eddy diffusivity for momentum transfer ε_τ and time averaged fluid velocity $\bar{u}(r)$ were calculated using Reichardt's [30-31] empirical relationships, which are straightforward and accurate.

Equation (8) with the boundary conditions (11)-(13) was solved using the finite difference method [29]. The Nusselt number was determined based on the temperature distribution determined numerically using the following formula

$$Nu = \frac{2hr_w}{k} = \frac{2q_w r_w}{k(T|_{r=r_w} - T_m)} \quad (15)$$

The Nusselt number was evaluated for various Reynolds and Prandtl numbers, and the results are listed in Table 1 [29].

Table 1. Nusselt Nu number as a function of the Reynolds and Prandtl numbers obtained from the solution of the energy conservation equation for fully developed turbulent flow in tubes with constant wall heat flux [29].

Re \ Pr	3·10 ³	5·10 ³	7.5·10 ³	10 ⁴
0.1	7.86	9.42	11.13	12.69
0.2	9.41	11.86	14.58	17.08
0.5	12.65	16.96	21.81	26.31
0.71	14.32	19.60	25.57	31.12
1	16.22	22.61	29.86	36.61
3	24.43	35.57	48.39	60.47
5	29.56	43.64	59.94	75.36
7.5	34.34	51.15	70.68	89.21
10	38.16	57.14	79.24	100.25
12.5	41.40	62.20	86.47	109.56
15	44.23	66.62	92.78	117.70
30	56.73	86.09	120.53	153.40
50	67.98	103.56	145.36	185.32
100	86.64	132.44	186.35	237.96
200	110.11	168.69	237.71	303.83
1000	190.70	292.75	413.17	528.61

Re \ Pr	3·10 ⁴	5·10 ⁴	7.5·10 ⁴
0.1	22.76	31.02	40.20
0.2	33.53	47.23	62.59
0.5	56.55	82.31	111.63
0.71	68.78	101.16	138.18
1	82.91	123.04	169.15
3	145.31	220.65	308.45
5	184.68	282.64	397.43
7.5	221.38	340.53	480.68
10	250.64	386.73	547.14
12.5	275.31	425.71	603.25
15	296.85	459.72	652.22
30	391.25	608.73	866.75
50	475.39	741.46	1,057.70
100	613.70	959.44	1,371.11
200	786.27	1,231.01	1,761.37
1000	1,373.01	2,153.24	3,085.03

Re \ Pr	10 ⁵	3·10 ⁵	10 ⁶
0.1	48.62	104.68	256.61
0.2	76.78	172.57	437.11
0.5	138.97	327.20	860.91
0.71	172.84	413.35	1,102.29
1	212.50	515.41	1,391.88
3	391.85	987.68	2,764.63
5	506.89	1,295.71	3,677.45
7.5	614.66	1,586.18	4,544.73
10	700.78	1,818.99	5,242.89
12.5	773.51	2,015.88	5,834.61
15	836.98	2,187.81	6,352.24
30	1,115.04	2,941.80	8,624.88
50	1,362.53	3,612.61	10,647.89
100	1,768.51	4,711.28	13,959.35
200	2,273.65	6,076.39	18,067.10
1000	3,986.35	10,692.06	31,968.41

The Nusselt numbers shown in Table 1 with Prandtl numbers ranging from 0.1 to 1,000 and Reynolds numbers ranging from 3,000 to 10⁶ were approximated by a product of two power functions of the Reynolds and Prandtl numbers

$$Nu = c_1 Re^{c_2} Pr^{c_3} \quad (16)$$

Unknown constants $c_1, c_2,$ and c_3 occurring in the approximating function (16) were determined using the least squares method

$$S(c_1, c_2, c_3) = \sum_{i=1}^{n_{Re}} \sum_{j=1}^{n_{Pr}} (Nu_{ij}^m - Nu_{ij}^c)^2 = \min \quad (17)$$

where: $Nu_{ij}^m = Nu^m(Re_i, Pr_j)$ and

$Nu_{ij}^c = Nu(Re_i, Pr_j), i = 1, \dots, n_{Re}, j = 1, \dots, n_{Pr}$ - given and approximated values of the Nusselt number, respectively, n_{Re} and n_{Pr} - the number of the Reynolds and Prandtl numbers which are considered in the sum (17). As Nu_{ij}^m the Nusselt numbers shown in Table 1 were taken. The values of Nu_{ij}^c were evaluated using Eq. (17).

Substituting the constants c_1, c_2 , and c_3 determined from the condition (17) into the correlation (16), the following power-type formulas for turbulent flow in round tubes were obtained

$$Nu = 0.02155 Re^{0.8018} Pr^{0.7095} \quad (18)$$

$$3 \cdot 10^3 \leq Re \leq 10^6, 0.1 \leq Pr \leq 1$$

$$Nu = 0.01253 Re^{0.8413} Pr^{0.6179} \quad (19)$$

$$3 \cdot 10^3 \leq Re \leq 10^6, 1 < Pr \leq 3$$

$$Nu = 0.00881 Re^{0.8991} Pr^{0.3911} \quad (20)$$

$$3 \cdot 10^3 \leq Re \leq 10^6, 3 < Pr \leq 1000$$

The analysis of equations (18)-(20) shows that power exponents at the Reynolds and Prandtl numbers strongly depend on the Prandtl number. If the Prandtl number is smaller than one, power exponents with Reynolds and Prandtl numbers are almost equal to each other, as in the case of liquid metals when $Pr < 0.1$ [1, 17, 23]. The exponent with Reynolds number increases with the Prandtl number, while the exponent of Prandtl number behaves conversely. As the Prandtl number increases, the power with Prandtl number decreases.

Equation (16) can be generalized to a tube of finite length by introducing the same multiplier as in the Gnielinski formula (4)

$$Nu = c_1 Re^{c_2} Pr^{c_3} \left[1 + \left(\frac{d_w}{L} \right)^{2/3} \right] \quad (21)$$

The heat transfer correlations (18) - (20) were compared with the experimental data available in the literature taking into account the correction factor $\left[1 + (d_w / L)^{2/3} \right]$.

3 Comparison of the proposed correlations with experimental data

The proposed correlation (18) was compared with the experimental results of turbulent air flow in the pipes and the correlation (20) with experimental data for water, which are available in the literature. Comparison

of the results of experimental studies of doctoral dissertations [32-35] for the air flowing inside the heating pipe is shown in Fig. 1. Abraham [36] was the first who presented the experimental data for smooth tubes with uniform wall heat flux reported in these theses and used by him to verify the CFD results. Equation (18) gives lower values of Nusselt number as compared to experimental results, whereas the correlation of Dittus-Boelter (1) overpredicts them. The proposed relationship (18) was also compared in Fig. 2 with the experimental data of Eiamsa-ard and Promvonge [37] for turbulent air flow in a tube with constant wall heat flux.

Measurements were performed in an electrically heated tube. The outer and inner diameter of the copper tube were 47 mm and 50 mm, respectively. The tube was heated uniformly over the length of 1,250 mm. An unheated calming section preceded heated pipe section.

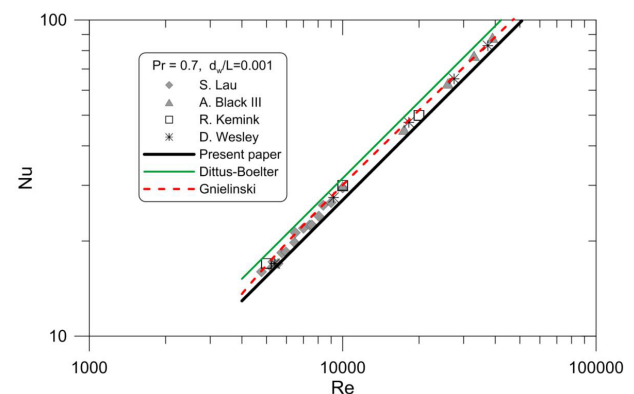


Fig. 1. Comparison of the Nusselt number for circular tubes obtained from the correlation (18) proposed in the paper with the experimental results of Lau, Black, Kemink, and Wesley [32-35] and the Gnielinski relationship (4) [19-20].

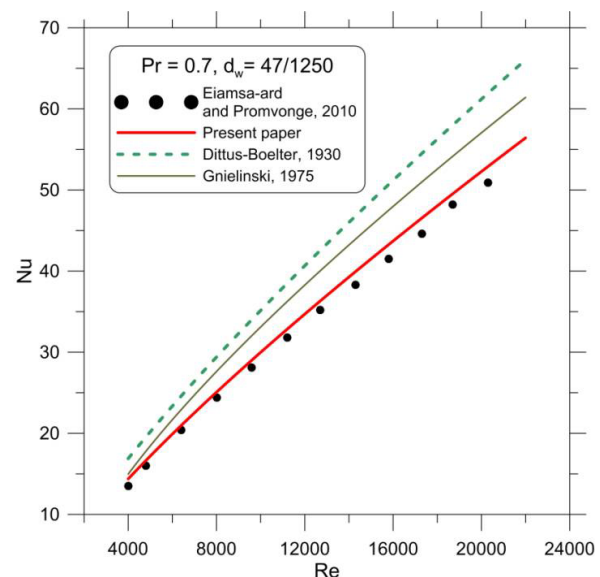


Fig. 2. Comparison of the correlation (18) proposed in the present paper and the correlation of Dittus-Boelter [10] and Gnielinski [19-20] with the experimental data of Eiamsa-ard and Promvonge [37] obtained for turbulent air flow in a tube with constant wall heat flux.

The hydrodynamically developed turbulent flow of water enters a uniformly heated tube. The Nusselt number determined experimentally was 10 to 20% higher than values predicted by the Dittus-Boelter equation. The discrepancy between the results of measurements and values of Nusselt number obtained from the Dittus-Boelter formula is greater for higher Reynolds numbers.

Then, Eq. (20) was compared with experimental results available of Allen and Eckert [16]. The smooth-tube heat transfer results reported by Allen and Eckert [16] were obtained for developed turbulent flow of water under the uniform wall heat flux boundary condition at $Pr=7$ and $Pr=8$, and $1.3 \cdot 10^4 \leq Re \leq 1.11 \cdot 10^5$ [16]. Steel pipe with an inner diameter of 19.05 mm and a wall thickness of 1.5875 mm was heated electrically so that the heat flux at the internal surface of the tube was constant.

Allen and Eckert [16] found friction and heat transfer correlations for turbulent pipe flow of water at uniform wall heat flux for $Pr=8$ experimentally. The friction factor and Nusselt number determined experimentally were approximated by the following functions [16]

$$\xi = 0.00556 + \frac{0.432}{Re^{0.308}} \quad (22)$$

$$1.3 \cdot 10^4 \leq Re \leq 1.11 \cdot 10^5, Pr = 8$$

$$Nu = 0.1576 \frac{\xi}{8} Re^{1.0685} Pr \quad (23)$$

$$1.3 \cdot 10^4 \leq Re \leq 1.11 \cdot 10^5, Pr = 8$$

The relationship (20) proposed in this paper is close to the correlation (23) of Allen and Eckert [16](Fig.3). The best agreement is observed between Eq. (4) of Gnielinski [19-20]. The correlations of Dittus-Boelter (1) and Li and Xuan (7) give Nusselt number smaller than the correlation of Allen and Eckert (23). Evaluating the relative difference from the relation

$$e = \frac{Nu_{A-E} - Nu}{Nu_{A-E}} 100 \quad (24)$$

the correlations (20) and (1) were compared with (23) for the following Reynolds numbers: 10,000, 50,000, and 110,000. The symbol Nu_{A-E} designates the Nusselt number obtained from the Allen and Eckert relation (23) while Nu is the Nusselt number calculated using Eq. (20) or (1). The relative differences for the Eq. (20) developed in the paper are: 14.2% , 3.9% , and 0.08% , respectively, whereas for the Dittus-Boelter equation (1) the differences e are: 8.4%, 12.5%, and 15.9% .

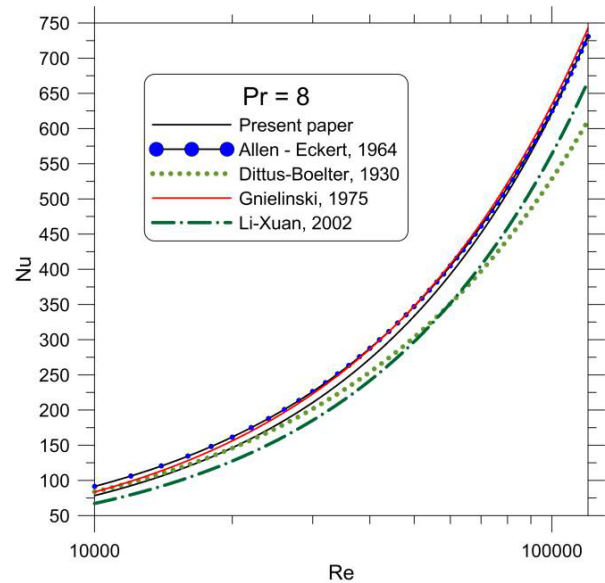


Fig. 3. Comparison of experimental correlation (23) of Allen and Eckert [16] with the correlation (20) proposed in the present paper, the correlation of Dittus-Boelter (1) [10], the Gnielinski relationship (4) [19-20], and the correlation (7) developed by Li and Xuan [25].

The correlation (20) found in the paper closely approximates the experimental equation (23) of Allen and Eckert for Reynolds numbers greater than 20,000 as compared to the Dittus-Boelter equation (1).

Comparisons of the correlations (8) and (20) to experimental data obtained by different researchers confirm quite well accuracy of the proposed relationships, especially when taking into account a variety of test conditions and different ways of experimental data processing.

4 Conclusions

The paper presents three power-type correlations of a simple form, which are valid for Reynolds numbers range from $3 \cdot 10^3 \leq Re \leq 10^6$, and for three different ranges of Prandtl number: $0.1 \leq Pr \leq 1.0$, $1.0 < Pr \leq 3.0$, and $3.0 < Pr \leq 10^3$.

Formulas proposed in the paper have good theoretical basis, as they have been obtained by approximating Nusselt numbers obtained from the solution of the energy conservation equation for turbulent flow in a pipe by the method of least squares. Heat transfer correlations developed in the paper were compared with experimental results available in the literature. The performed comparisons confirm the good accuracy of the proposed correlations, better than the accuracy of the Dittus-Boelter correlation. Heat transfer correlations proposed in the paper can be used in a broader ranges of Reynolds and Prandtl numbers compared with a widely used correlation of Dittus-Boelter. They are also much simpler in comparison to the relationship of Gnielinski, which is also widely used in the heat transfer calculations.

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