Innovative numerical modelling of technogenic rock arrays structure

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Abstract. Numerical simulation of bulk technogenic rock arrays, unlike the full-scale, has a number of advantages. They are: the large amount of low cost measurements; operative selection of parameters necessary for the environment with the given properties; calculation of stress-strain state and assessment of the array sustainability; calculation of rock arrays filtration properties that are used as filters for mining enterprises sewage, etc. The article describes the method of numerical simulation of structure of bulk disjointed rock arrays, describes the basic initial and boundary conditions of the model; represents the numerical model visualization of the array, created with the use of peripheral dumping (as an array with more textured and visual segregation of granulometric composition adjustment). The basic applications of numerical modeling of technogenic rock arrays in the mining processes are described. Comparative analysis of the real and simulated array structure shows the adequacy of the numerical model and the developed for its creation algorithm, which in turn allows judging the quality of the study. Also, the adequacy of building models is supported by a high convergence of the experimental and calculated figures depending on the filtration rate values of the particle size distribution of technogenic rock arrays.

1 Introduction

The relevance of research lies in the fact that mathematical models of technogenic rock masses can predict and evaluate the operational structural and mechanical properties, which are very difficult or impossible to study instrumentally [1, 2]. Simulation of man-made structures rock masses is convenient to carry out by the method of discrete elements, where each particle in the array structure is described separately. The study of the internal geometry (structure and texture) of man-made rock masses on mathematical models [3, 4] is largely a solution for determining their physical and strength properties that may be of practical use to calculate the filtration characteristics of the dams, the stability of dumps and bulk structures.

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2 Material and Method

In general, when an artificial rock array has initially a stable, static state, its numerical simulation means stable packaging of each of its particles in a given half-plane or hypothetic "container". This "container" dimensions correspond to the dimensions of simulated array, and the choice of the initial position of the packed particles reminds the dumping process. The particle shape in the plane case is taken as a circle because it reduces the total number of coordinates describing the place of particles in the space (the center coordinates x, z and radius r). Physical parameters (particle's density, friction rate in the contacts between the particles) are given separately on the basis of the field studies.

The research methods include photomapping, statistical analysis, field observations (used to set the initial conditions), the method of discrete elements (implemented in the algorithm of particles' packing).

3 Results and Discussion

Simulation algorithm consists of the following steps.

Step 1. Choosing the particle size.

The choice of particle size (r_0 radius) is based on granulometric distribution of simulated rock array. For overburden rock arrays of Kuzbass open pits the granulometric distribution for upper part of the array is determined by sieve method (particle size is up to 0.1 m), for lower and middle parts – by oblique photomapping method [5-7] using "Split Desktop" software available on the official website of the "Split Engineering" company.

The granulometric distribution study revealed that the total content of fractions (based on the number of particles) in the artificial rock arrays is in the range from $2.99 \cdot 10^{-4}$% for the pieces of 0.7-1 m, and 91.42% for the size of the pieces less than 0.003 m. The values for the fractions of 0.5-0.7 m ($4.02 \cdot 10^{-4}$%)); 0.25-0.5 m ($1.32 \cdot 10^{-3}$%); 0.1-0.25 m ($6.39 \cdot 10^{-3}$%); 0.025-0,1 m ($8.78 \cdot 10^{-2}$%); 0.01-0,025 m (0.37%); 0.007-0.01 m (0.56%); 0.005-0.007 m (1.46%); 0.003-0.005 m (6.10%) were calculated for the highest quality modeling.

Step 2: Determining the initial coordinates of the particle.

In general, when modeling rock mass, limited only by the bottom, initial coordinates of the particle are x_0 and z_0. Here x_0 is within the width B (if only the cross-section of the array is being simulated) or the length L (if only longitudinal section of the array is being simulated) of the upper platform of piling rock array in the simulation of spread technology, i.e., it varies from $z_0 \, x \, ctg \, \beta$ to $z_0 x \, ctg \, \beta + B$ up to ($z_0 \cdot ctg \, \beta$ + L), respectively (β - natural angle of slope, degrees).

Coordinate z_0 shall be equal to the maximum of height of the bench. When z_0 is selected, the condition of non-intersection of packed particles with each already packed *i-th* particle must be fulfilled:

$$(x_i - x_0)^2 + (z_i - z_0)^2 \geq (r_i + r_0)^2, \quad (1)$$

where x_i, z_i, r_i - generalized coordinates of previously packed particles m.

If this condition is not fulfilled a number of times equal to the number of particles with z coordinate close to the height of piling array, the dumping process should be considered as finished for spread technology. For peripheral dumping simulation or for the dump piling using transportless technology, the value of initial coordinate x_0 at the beginning of the simulation is calculated as $z_0 \, x \, ctg \, \beta$ and when the simulated array height reaches the full height of a dump, x_0 is shifting by the value of Δx (which is equal to r_0 for bulldozer dumping, or to the width of n-s dumping stope for excavator dumping) up to the maximum length (width) of the upper dump's platform (Fig. 1).

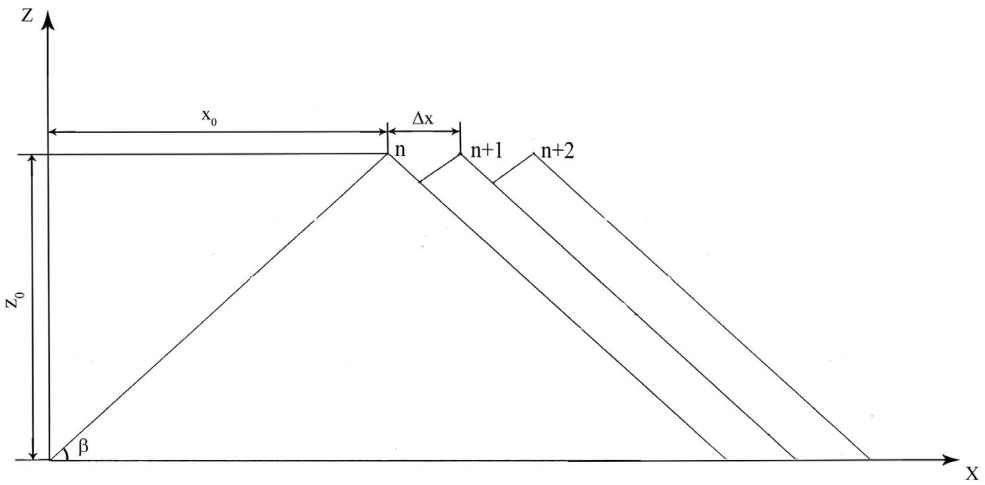

Fig. 1. The selection of initial parameters of the simulation

Step 3: Determining the conditions of contact of the particles

Simulation of particle packing process does not involve considering the dynamics of their collisions. After determining the initial coordinates of the particle (Fig. 1, point *n, n + 1, n + 2 ...*) at its moving straight down the minimum-valued coordinate *z* can be found. If the particle does not meet other particles, the coordinate z_0 value becomes equal to the particle radius r_0.

In order for the particle being packed could "meet" the previously packed *j*-th particle, the following three conditions must be observed (Fig. 2):

- the sum of the particles' radiuses must be greater than the horizontal projection of the distance between them:

$$(r_0 + r_j) > |x_0 - x_j|; \tag{2}$$

- the distance between them d_j should be minimal (it can be calculated by the search of previously packed particles):

$$d_j = \sqrt{(x_0 - x_j)^2 + (z_0 - z_j)^2} - (r_0 + r_j) \to \min; \tag{3}$$

- the coordinate z_j for previously packed particle must be less than the coordinate-ordinates z_0 of particle being packed:

$$z_j < z_0. \tag{4}$$

To find the minimum coordinate z_0, corresponding to the touch of particle being packed the previously packed particle, i.e. under the conditions (2) - (4), we consider the triangle ABC (Figure 2.). At an angle:

$$\varphi = \arccos\left(\frac{|x_0 - x_j|}{r_0 + r_j}\right) \tag{5}$$

coordinate z_0 value will be:

$$z_0 = z_j + (r_j + r_0)\cdot\sin\varphi. \tag{6}$$

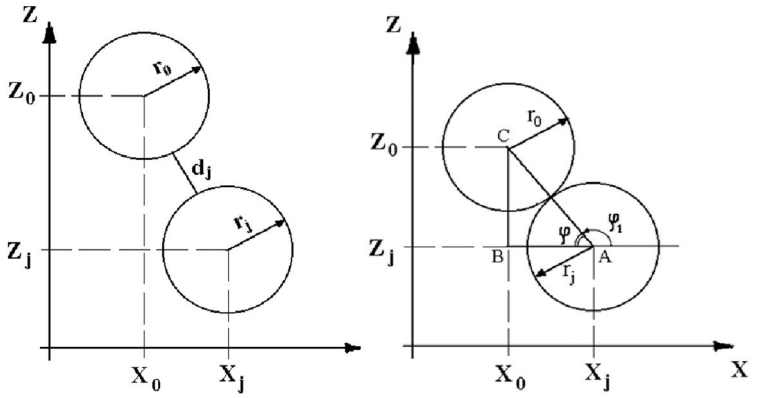

Fig. 2. The scheme of finding a sustainable position of the particle

In the particular case when $x_j = x_0$ coordinate z_0 is set to:

$$z_0 = z_j + (r_j + r_0). \tag{7}$$

Step 4: Determining the stable position of the particle.

Having calculated z_0 under the condition of contact between the particles (6) or (7), we begin to "roll" particle being packed around the previously packed one, reducing the angle φ to $d\varphi$ until the angle $\varphi = 0$. It was empirically found that for the array with particles size of the same order, optimum value $d\varphi = 0{,}1 \cdot r_{min}$ (r_{min} - the minimum value of the radius of packed particles), and $d\varphi = 0{,}0001 \cdot r_{min}$ for arrays with higher inhomogenuity of granulometric distribution.

The condition of contact of the particles being packed with i-th particle (i.e. for $i \neq j$) can be written as:

$$d_i = \sqrt{(x_0 - x_i)^2 + (z_0 - z_i)^2} - (r_0 + r_i) \leq 0, \tag{8}$$

where: d_i – the distance between the particle being packed and i-the packed particle.

This condition is checked every time the angle φ is getting changed respectively to all the particles lying near the particle being changed.

"The rolling" is stopped as soon as the condition (8) becomes equal to 0 (which means that the rolling particle contacts with the two previously packed particles), or becomes lower than 0, which means the intersection of the second from the previously packed particles.

The condition of the solid packing in this case is solving simultaneous equations:

$$\begin{cases} (x_i - x_0)^2 + (z_i - z_0)^2 - (r_i + r_0)^2 = 0 \\ (x_j - x_0)^2 + (z_j - z_0)^2 - (r_j + r_0)^2 = 0 \end{cases}, \tag{9}$$

where: x_i, z_i, r_i, x_j, z_j, r_j – coordinates and radius of packed particle that touches the one being packed, $i \neq j$; x_0, z_0, r_0 – the radius and the coordinates of the particles being packed.

The numerical solution of this system of equations simply implemented by Newton-Ruffson iterative method, with a given relative error $\varepsilon = 10^{-7}$ and the number of equations $N = 2$, the maximum number of iterations $M = 1000$, and the vector of initial approximations X_{i0} (with components $x_{10}, x_{20}, \ldots, x_{N0}$). In our case, $x_{10} = x_i + x_j$, $x_{20} = z_i + z_j$.

Decomposition of nonlinear equations $F_i(X_i)$ in a Taylor series formed Jacobian matrix $[\partial F_i / \partial X_i]$ necessary to calculate the increments of $F_i(X_i)$ at the small changes of the variables. The expanded form of the Jacobian matrix can be written as:

$$\begin{bmatrix} \partial F_1 / \partial x_1 & \partial F_1 / \partial x_2 & \dots & \partial F_1 / \partial x_N \\ \partial F_2 / \partial x_1 & \partial F_2 / \partial x_2 & \dots & \partial F_2 / \partial x_N \\ \dots & & & \\ \partial F_N / \partial x_1 & \partial F_N / \partial x_2 & \dots & \partial F_N / \partial x_N \end{bmatrix} \tag{10}$$

Analytical differentiation $F_i(X_i)$ can be avoided by replacing the partial derivatives in the matrix of their approximate values:

$$\frac{\partial F_i}{\partial X_i} \approx \frac{F_i(X_i + H_i) - F_i(X_i)}{H_i}, \tag{11}$$

where: $H_i = \varepsilon |X_i|$ – the small increment X_i.

The solving $\Delta x_1, \Delta x_2, ..., \Delta x_N$, i.e. ΔX_i can be found from the linear equation system:

$$\begin{bmatrix} \partial F_1 / \partial x_1 & \partial F_1 / \partial x_2 & \dots & \partial F_1 / \partial x_N \\ \partial F_2 / \partial x_1 & \partial F_2 / \partial x_2 & \dots & \partial F_2 / \partial x_N \\ \dots & & & \\ \partial F_N / \partial x_1 & \partial F_N / \partial x_2 & \dots & \partial F_N / \partial x_N \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \dots \\ \Delta x_N \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ \dots \\ -F_N \end{bmatrix} \tag{12}$$

Then the corrected values can be calculated:

$$\begin{aligned} X_{1(n+1)} &= x_{1(n)} + \Delta x_1, \\ X_{2(n+1)} &= x_{2(n)} + \Delta x_2, \\ &\dots \\ X_{N(n+1)} &= x_{N(n)} + \Delta x_N. \end{aligned} \tag{13}$$

For every ΔX_i one of the following conditions must be checked:

$$\begin{aligned} &|\Delta X_i| > \varepsilon, \\ &\left|\frac{\Delta X_i}{\Delta X_j}\right| > \varepsilon. \end{aligned} \tag{14}$$

With its completion another iteration must be performed, otherwise the vector $X_{i(n+1)}$ is considered to be the solution.

After accurate calculation of the coordinates the stability of the particles to be packed must be checked: if its coordinate x_0 is within the interval $[x_i - x_j]$ then the position of the particle is stable (only in the absence of collision dynamics and the impact of inertia).

Under the conditions:

$$\begin{cases} |x_0 - x_i| < |x_j - x_i| \\ |x_0 - x_j| < |x_j - x_i| \end{cases} \quad (15)$$

it is believed that the particle be packed has a steady position, otherwise it continues to "roll down" over the *i*-th particle.

When the model of technogenic rock array corresponds to its real size the simulation of its structure is considered to be completed. After modeling the structure it is possible to study density, strength and filtration properties of technogenic rock array. Fig. 3 shows a fragment of the visualized model technogenic rock array - the waste dump of 30 m height, pilled by bulldozer peripheral technology.

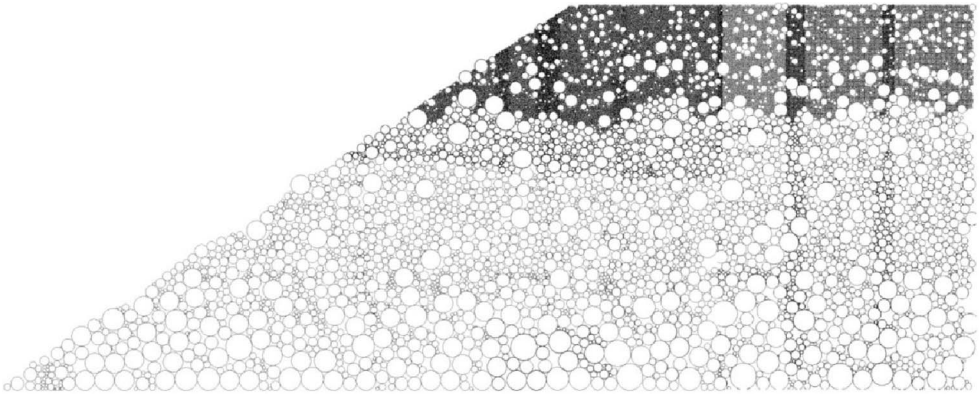

Fig. 3. The fragment of the dump array

4 Conclusion

The developed algorithm of technogenic rock array simulation has a sufficiently high reliability because reconstituted structure almost completely matches the real dumps.

The numerical modeling of granular media and technogenic rock masses which are their particular case can be used in the study of their physical and strength properties. This is due to the fact that the modeling algorithms allow flexibly change the input parameters of the simulated array. It allows selecting the parameters of dumping technology, simulating the arrays with any defined properties.

References

1. M. A. Tyulenev, T. N. Gvozdkova, S. A. Zhironkin, E.A. Garina, Geotech. Geol. Eng., **35**, 203 (2017)
2. M.Y. Blaschuk, A.A. Dronov, S.S. Ganovichev, IOP Conf. Ser. Mater. Sci. Eng., **142**, 012128 (2016)
3. A. V. Koperchuk, A. V. Murin, V. V. Filonov, IOP Conf. Ser. Mater. Sci. Eng., **127**, 012040 (2016)
4. M.Y. Blashchuk, A.A. Kazantsev, R.V. Chernukhin, App. Mech. Mat, **682**, 418 (2014)
5. J.M. Kemeny, A. Devgan et al., Journal of Geotechnical Engineering, **119**, 1144 (1993)
6. J.M. Kemeny, Mining Engineering, **46**, 1281 (1994)
7. J.C. Russ, *The Image Processing Handbook* (CRC Press, 1995)