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Determination of rational transformation coefficients of transformers distribution networks

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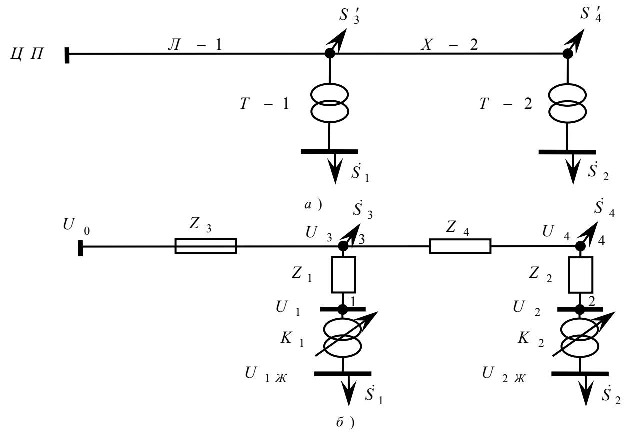
Abstract. The thesis proposed a systematic approach to the determination of rational coefficients transformation of transformers distribution networks. Analytic expressions are obtained for transformation coefficient of transformer from the point of view of synthesis of distribution networks in the field of permissible voltage regimes. The Newton-Raphson method was used to solve nonlinear equations for transformation coefficients. This eliminates the need for calculation of steady distribution network modes.

1 Synthesis formalization for the simplest networks

For the sake of clarity and simplicity of presentation, we consider the stationary mode of the distribution network shown in Pic. 1, a.



а – principle scheme; б - replacement scheme
Fig. 1. Electrical network scheme

- Z_1, Z_2 - complex resistance of transformers
- Z_3, Z_4 - line resistance of power transmission;
- $\dot{\underline{S}}_1, \dot{\underline{S}}_2, \dot{\underline{S}}_3, \dot{\underline{S}}_4$ - calculating power of nodal loads;
- K_1, K_2 - transformation coefficients of ideal transformers

Assume that providing the required voltage quality changes achieved transformation ratio transformers with tap changer on the basis of a counter voltage regulation. Adjusting range taps of transformers is considered sufficient to provide the required voltage levels. Consequently, the desired value of the stress on the tires reducing substation considered as given quantities.

The resulted voltages of the secondary windings of transformers with adjustable transformation coefficients, in accordance with the scheme of Fig. 6.3b, are determined by the expressions:

$$\left.\begin{aligned} \dot U_1 = U_0 &- \frac{\sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{\hat U_1} - \frac{\sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{\hat U_2} - \\ &- \frac{\sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{\hat U_3} - \frac{\sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{\hat U_4} \\ \dot U_2 = U_0 &- \frac{\sum_{j=1}^{4} \underline{C}^t_{2j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{\hat U_1} - \frac{\sum_{j=1}^{4} \underline{C}^t_{2j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{\hat U_2} - \\ &- \frac{\sum_{j=1}^{4} \underline{C}^t_{2j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{\hat U_3} - \frac{\sum_{j=1}^{4} \underline{C}^t_{2j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{\hat U_4} \end{aligned}\right\} \quad (1)$$

where, $\sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j1}, \sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j2}$, ... $\sum_{j=1}^{4} \underline{C}^t_{2j}\underline{Z}_j\underline{C}_{j1}, \sum_{j=1}^{4} \underline{C}^t_{1j}\underline{Z}_j\underline{C}_{j2}$- partial systemic functions of resistance relatively to the first and second nodes.

Without taking into account the influence of the phases of the node voltages on the magnitude of the driving currents, the system (1) is written in the form:

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$$\left.\begin{aligned}
\dot U_1 &= U_0 - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{U_1} - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{U_2} - \\
&- \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{U_3} - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{U_4} \\
\dot U_2 &= U_0 - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{U_1} - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{U_2} - \\
&- \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{U_3} - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{U_4}
\end{aligned}\right\} \quad (2)$$

Known values of node voltages are such that the system (2) describes the stationary condition of the electrical network within the allowable regimes by voltage.

Desired voltages on tires 6-10 kV of reducing substations are provided by voltage regulation by changing the coefficient transformation of transformers. It is known that the selection of the regulating branches of the transformers is carried out individually for each substation according to the results of calculation of the stationary regime. In this case, the mutual influence of the regulating branches can not be taken into account when choosing the most suitable modes within the range of admissible regimes. The most suitable mode can be found in the system approach to the determination of the regulating branches of all transformers of the lowering substations.

Let the necessary expedient regime of the voltage of the network in question be ensured by changing the transformation ratios of the transformers.

Then the system of equations (2), taking into account the transformation coefficients of ideal transformers, can be written as follows:

$$\left.\begin{aligned}
\dot U_1 &= U_0 - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{K_1 U_{1Ж}} - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{K_2 U_{2Ж}} - \\
&- \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{U_3} - \frac{\sum_{j=1}^{4} \underline{C}'_{1j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{U_4} \\
\dot U_2 &= U_0 - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j1}\hat S_1}{K_1 U_{1Ж}} - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j2}\hat S_2}{K_2 U_{2Ж}} - \\
&- \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j3}\hat S_3}{U_3} - \frac{\sum_{j=1}^{4} \underline{C}'_{2j}\underline{Z}_j\underline{C}_{j4}\hat S_4}{U_4}
\end{aligned}\right\} \quad (3)$$

Taking into account $U_1 = U_{1Ж} / K_1; U_2 = U_{2Ж} / K_2$ and the values of the distribution coefficients, the system (3) can be written in the form:

$$\left.\begin{aligned}
U_{\Delta 11}K_1^2 + U_{\Delta 12}K_2K_1 - \left(U_0 - U_{\Delta 13} - U_{\Delta 14}\right)K_1 + U_{1Ж} = 0 \\
U_{\Delta 21}K_1K_2 + U_{\Delta 12}K_2^2 - \left(U_0 - U_{\Delta 23} - U_{\Delta 24}\right)K_2 + U_{2Ж} = 0
\end{aligned}\right\} \quad (4)$$

where the coefficients of the unknowns for the scheme under consideration are equal to:

$$U_{\Delta 11} = \sqrt{\left(\frac{P_1(R_1+R_3)+Q_1(X_1+X_3)}{U_{1Ж}}\right)^2 + \left(\frac{P_1(X_1+X_3)-Q_1(R_1+R_3)}{U_{1Ж}}\right)^2}$$

$$U_{\Delta 12} = \sqrt{\left(\frac{P_2R_3+Q_2X_3}{U_{2Ж}}\right)^2 + \left(\frac{P_2X_3-Q_2R_3}{U_{2Ж}}\right)^2}$$

$$U_{\Delta 13} = \sqrt{\left(\frac{P_3R_3+Q_3X_3}{U_3}\right)^2 + \left(\frac{P_3X_3-Q_3R_3}{U_3}\right)^2}$$

$$U_{\Delta 14} = \sqrt{\left(\frac{P_4R_3+Q_4X_3}{U_4}\right)^2 + \left(\frac{P_4X_3-Q_4R_3}{U_4}\right)^2}$$

$$U_{\Delta 21} = \sqrt{\left(\frac{P_1R_3+Q_1X_3}{U_{1Ж}}\right)^2 + \left(\frac{P_1X_3-Q_1R_3}{U_{1Ж}}\right)^2}$$

$$U_{\Delta 22} = \sqrt{\left(\frac{P_2(R_2+R_3+R_4)+Q_2(X_2+X_3+X_4)}{U_{2Ж}}\right)^2 + \left(\frac{P_2(X_2+X_3+X_4)-Q_2(R_2+R_3+R_4)}{U_{2Ж}}\right)^2};$$

$$U_{\Delta 23} = \sqrt{\left(\frac{P_3R_3+Q_3X_3}{U_3}\right)^2 + \left(\frac{P_3X_3-Q_3R_3}{U_3}\right)^2}$$

$$U_{\Delta 24} = \sqrt{\left(\frac{P_4(R_3+R_4)+Q_4(X_3+X_4)}{U_4}\right)^2 + \left(\frac{P_4(X_3+X_4)-Q_4(R_3+R_4)}{U_4}\right)^2}$$

As an illustration, calculations are made by the proposed and conventional methods for the circuit in Fig. 1 with the specified parameters of the transformers, line and load:

$\square P_к = 45{,}5 кВА, \square P_х = 9{,}4 кВт, U_к = 7{,}5\%, I_х = 0{,}9\%,$
$\pm 6 \times 1{,}5\%, U_Н = 11 кВ$

Л1-2ХАС-70, Л2-2ХАС-35, L_1=L_2=18км., S_1= S_2= 3,5+ j 2,5 МВА.

Design parameters: transformers Z_1= Z_2=1,6+j16,1 Ом,

line -Z_3=4,05+j3,78 Ом, Z_4=8,19+j4 Ом.

Calculations of non-linear equations were carried out by the Newton-Raphson method in the MATLAB environment, and the working mode of the network by the two-stage method according to the data of the beginning. The results are shown in Table 1.

Table 1. Results of calculations

Direct method	Classical method
Mode of maximum loads	
K_2=0,3217 K_1=0,3123 U_2=34,19 kV U_1=35,22 kV	$U_{жел}$=10,5 kV $U_{отв}$= $U_н$-nx1,5% $U_{отв2}$=34,47 kV $U_{отв1}$=35,52 kV
Minimum mode	
K_2=0,279 K_1=0,277 U_2=39,42 kV U_1=39,71 kV	$U_{жел}$=10,0 kV $U_{отв2}$=39,4 kV $U_{отв1}$=39,78 kV

As can be seen from the table, the results obtained by different methods give good coincidences, which allows us to conclude that the proposed method can be used to solve both operational and design tasks.

2 Synthesis of distribution networks of complex structure

Assume that the desired voltage busbar transformer substations included first n complex electrical circuit nodes network provided by the transformation ratio transformers changes. Then a nonlinear matrix equation (8.8), after simple transformation can be written in the form of voltage balance, for example i-th node:

$$\dot{\underline{U}}_{i\text{ж}} = \dot{\underline{U}}_{\Delta i1} K_i K_1 + \dot{\underline{U}}_{\Delta i2} K_i K_2 + \ldots + \dot{\underline{U}}_{\Delta in} K_i K_n + (\sum_{j=n+1}^{N} \dot{\underline{U}}_{\Delta ij} + \dot{\underline{U}}_0) K_i \quad (5)$$

where $\dot{\underline{\mathbf{U}}}_{\Delta i1} = \sum_{j=1}^{m} \underline{\mathbf{C}}_{ij}^{t} \underline{\mathbf{Z}}_j \underline{\mathbf{C}}_{j1} \dot{\underline{\mathbf{U}}}_{1\text{ж}}^{-1} \dot{\underline{\mathbf{S}}}_1$, $\dot{\underline{\mathbf{U}}}_{\Delta in} = \sum_{j=1}^{m} \underline{\mathbf{C}}_{ij}^{t} \underline{\mathbf{Z}}_j \underline{\mathbf{C}}_{jn} \dot{\underline{\mathbf{U}}}_{n\text{ж}}^{-1} \dot{\underline{\mathbf{S}}}_n$ - partial nodal voltage reduces of the i-th node.

$\dot{\underline{U}}_{i\text{ж}}$ - complex desired voltage of the i-th node;
$\dot{\underline{U}}_0$ - the voltage of the basic node;
$K_1, K_2, ..., K_n$ -transformation coefficients of transformers;
N - number of independent nodes.

The dependent variables in equation (4) are the node voltage phases (δ) and the transformation coefficients transformer (K).

The system of nonlinear equations for the balance of the stress of the node under consideration concerning the variables (δ, K) can be obtained from (5) by selecting the real and imaginary parts in the form:

$$\left.\begin{aligned}
&\omega_{\underline{U}_i'}(\delta,K) = -U_{i\text{ж}}\cos\delta_i + \sum_{j=1}^{n} U_{\Delta ij} K_i K_j \cos(\delta_j - \varphi_j + \psi_{ij}) + \\
&+(\sum_{j=n+1}^{N} U_{\Delta ij}\cos(\delta_j - \varphi_j + \psi_{ij}) + U_0) K_i \\
&\omega_{\underline{U}_i''}(\delta,K) = -U_{i\text{ж}}\sin\delta_i + \sum_{j=1}^{n} U_{\Delta ij} K_i K_j \sin(\delta_j - \varphi_j + \psi_{ij}) + \\
&+(\sum_{j=n+1}^{N} U_{\Delta ij}\sin(\delta_j - \varphi_j + \psi_{ij})) K_i
\end{aligned}\right\} \quad (6)$$

Where:

U_i', U_i'' are the real and imaginary parts of the voltage;

$U_{\Delta ij}$ - module of partial nodal voltage drop;

φ_j - phase shift of the load power of the j-th node;

ψ_{ij} - angle of mutual complex nodal resistance.

It is seen from (6) that a system of 2n equations with 2n unknowns is compiled, the solution of which can be obtained by the Newton method. The Jacobi matrix is not degenerate and can be written in block form in the form:

$$\frac{\partial \mathbf{W}_U(x)}{\partial x} = \begin{Vmatrix} \dfrac{\partial \omega_{\upsilon'}(\delta,K)}{\partial \delta} & \dfrac{\partial \omega_{\upsilon'}(\delta,K)}{\partial K} \\ \dfrac{\partial \omega_{\upsilon''}(\delta,K)}{\partial \delta} & \dfrac{\partial \omega_{\upsilon''}(\delta,K)}{\partial K} \end{Vmatrix}$$

The partial derivatives of the i-th row of each block matrix can be calculated as follows:

$$\frac{\partial \omega_{\upsilon' i}(\delta,K)}{\partial \delta} = U_{i\text{ж}}\sin\delta_i - U_{\Delta ii} K_i^2 \sin(\delta_i - \varphi_i + \psi_{ii}) - \sum_{\substack{j=1\\ j\neq i}}^{n} U_{\Delta ij} K_i K_j \sin(\delta_j - \varphi_j + \psi_{ij});$$

$$\frac{\partial \omega_{\upsilon' i}(\delta,K)}{\partial K} = 2U_{\Delta ii} K_i \cos(\delta_i - \varphi_i + \psi_{ii}) + \sum_{j=n+1}^{n} U_{\Delta ij}\cos(\delta_j - \varphi_j + \psi_{ij}) + U_0 +$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{N} U_{\Delta ij} K_i \cos(\delta_j - \varphi_j + \psi_{ij});$$

$$\frac{\partial \omega_{\upsilon'' i}(\delta,K)}{\partial \delta} = -U_{i\text{ж}}\cos\delta_i - U_{\Delta ii} K_i^2 \cos(\delta_i - \varphi_i + \psi_{ii}) + \sum_{\substack{j=1\\ j\neq i}}^{n} U_{\Delta ij} K_i K_j \cos(\delta_j - \varphi_j + \psi_{ij});$$

$$\frac{\partial \omega_{\upsilon'' i}(\delta,K)}{\partial K} = 2U_{\Delta ii} K_i + \sum_{j=n+1}^{N} U_{\Delta ij}\sin(\delta_j - \varphi_j + \psi_{ij}) + \sum_{\substack{j=1\\ j\neq i}}^{n} U_{\Delta ij} K_i \sin(\delta_j - \varphi_j + \psi_{ij}).$$

If we neglect the influence of the phases of the node voltages on the values of the driving currents, then the system (6.32) can be written in the form:

$$\left.\begin{aligned}
&\omega_{\underline{\dot{U}}_i'}(\delta,K) = -U_{i\text{ж}}\cos\delta_i + \sum_{j=1}^{n} U_{\Delta ij} K_i K_j \cos(-\varphi_j + \psi_{ij}) + \\
&+(\sum_{j=n+1}^{N} U_{\Delta ij}\cos(-\varphi_j + \psi_{ij}) + U_0) K_i \\
&\omega_{\underline{\dot{U}}_i''}(\delta,K) = -U_{i\text{ж}}\sin\delta_i + \sum_{j=1}^{n} U_{\Delta ij} K_i K_j \sin(-\varphi_j + \psi_{ij}) + \\
&+(\sum_{j=n+1}^{N} U_{\Delta ij}\sin(-\varphi_j + \psi_{ij})) K_i
\end{aligned}\right\} \quad (7)$$

Then the partial derivatives of the matrix are defined as:

$$\frac{\partial \omega_{\upsilon' i}(\delta,K)}{\partial \delta} = U_{i\text{ж}}\sin\delta_i;$$

$$\frac{\partial \omega_{\upsilon' i}(\delta,K)}{\partial K} = 2U_{\Delta ii} K_i \cos(-\varphi_i + \psi_{ii}) + \sum_{j=n+1}^{N} U_{\Delta ij}\cos(-\varphi_j + \psi_{ij}) + U_0 +$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n} U_{\Delta ij} K_i \cos(-\varphi_j + \psi_{ij});$$

$$\frac{\partial \omega_{\upsilon'' i}(\delta,K)}{\partial \delta} = -U_{i\text{ж}}\cos\delta_i;$$

$$\frac{\partial \omega_{\upsilon''i}(\delta, K)}{\partial K} = 2U_{\Delta ii} K_i \sin(-\varphi_i + \psi_{ii}) + \sum_{j=n+1}^{N} U_{\Delta ij} \sin(-\varphi_j + \psi_{ij}) +$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} U_{\Delta ij} K_i \sin(-\varphi_j + \psi_{ij}).$$

In distribution networks calculation formula for determining the elements of the Jacobian matrix, greatly simplified and reduced amount of calculations performed since the voltage balance equation I - th node can be reduced to the form:

$$\omega_{\underline{\dot{U}}_i}(K) = -U_{i\text{ж}} + \sum_{j=1}^{n} U_{\Delta ij} K_i K_j \cos(-\varphi_j + \psi_{ij}) + \Big(\sum_{j=n+1}^{N} U_{\Delta ij} \cos(-\varphi_j + \psi_{ij}) + U_0\Big) K_i \tag{8}$$

which can easily be written in the matrix form as follows:

$$\mathbf{K}_{Д} \mathbf{U}_{\Delta} \mathbf{K} + (\mathbf{U}_{\Delta\Sigma} + \mathbf{U}_0 \mathbf{E}) \mathbf{K} - \mathbf{U}_{ж} = 0 \tag{9}$$

$\mathbf{U}_{\Delta}$ - square matrix of nodal voltage drops from load currents, powered from transformer buses;

$\mathbf{U}_{\Delta\Sigma}$ - column matrix of nodal voltage drops from load currents without transformer links;

$\mathbf{U}_{ж}$ - column matrix of the desired stresses;

$\mathbf{K}$ - column matrix of transformation coefficients of transformers;

$\mathbf{E}$ - the identity matrix.

Given the initial approximations of the transformation coefficients of transformers and expanding in a Taylor series, we can obtain a linear approximation of each nonlinear equation of the system (6.34), for example, the nonlinear approximation of the i-th node has the following form:

$$-U_{i\text{ж}} + U_{\Delta i1} \cos(-\varphi_1 + \psi_{i1}) K_i^0 K_1^0 + \ldots + U_{\Delta ii} \cos(-\varphi_i + \psi_{ii}) (K_i^0)^2 + \ldots +$$

$$+ U_{\Delta in} \cos(-\varphi_n + \psi_{in}) K_i^0 K_n^0 + \Big(\sum_{j=n+1}^{N} U_{\Delta ij} \cos(-\varphi_j + \psi_{ij}) + U_0\Big) K_i^0 +$$

$$+ U_{\Delta i1} \cos(-\varphi_1 + \psi_{i1}) K_i^0 K_1^{(1)} + \ldots + (2U_{\Delta ii} \cos(-\varphi_i + \psi_{ii}) K_i^0 +$$

$$+ \sum_{j=n+1}^{N} U_{\Delta ij} \cos(-\varphi_j + \psi_{ij}) + U_0) \Delta K_i^{(1)} + \ldots + U_{\Delta in} \cos(-\varphi_n + \psi_{in}) K_i^0 \Delta K_n^{(1)} = 0 \tag{10}$$

We write the system of linearized equations in the matrix form as follows:

$$W(K^0) + \frac{\partial W(K^0)}{\partial K} \Delta K^{(1)} = 0 \tag{11}$$

Where $\Delta K^{(1)}$ is the matrix of corrections of the first iteration of Newton. Each step of the iterative process involves solving the linear system (11) with the subsequent determination of the approximation:

$$K^{(i+1)} = K^i + \Delta K^{(i+1)} \tag{12}$$

The Newtonian iteration process, in general, can be written in matrix form:

$$K^{(i+1)} = K^{(i)} - \left[\frac{\partial W(K^i)}{\partial K} \right]^{-1} W(K^i) \tag{13}$$

Control of convergence is carried out on the vector of residuals:

$$W_{\upsilon}(K) \leq \varepsilon$$

Where ε - is a predetermined small value.

From the calculated values of the transformation coefficients, it is easy to find the tap voltage and the regulating stage of each transformer.

3 Conclusions

1. A systematic method for calculating the transformer ratios of distribution network transformers has been developed.
2. Nonlinear equations are obtained with respect to transformer transformation coefficients.
3. To solve the problem, the Newton-Raphson method is proposed.

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