

# Calculation of critical heat transfer in horizontal evaporator pipes in cooling systems of high-rise buildings

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**Abstract.** An exact calculation of the heat exchange of evaporative surfaces is possible only if the physical processes of hydrodynamics of two-phase flows are considered in detail. Especially this task is relevant for the design of refrigeration supply systems for high-rise buildings, where powerful refrigeration equipment and branched networks of refrigerants are used. On the basis of experimental studies and developed mathematical model of asymmetric dispersed-annular flow of steam-water flow in horizontal steam-generating pipes, a calculation formula has been obtained for determining the boundaries of the zone of improved heat transfer and the critical value of the heat flux density. A new theoretical approach to the solution of the problem of the flow structure of a two-phase flow is proposed. The applied method of dissipative characteristics of a two-phase flow in pipes and the principle of a minimum rate of entropy increase in stabilized flows made it possible to obtain formulas that directly reflect the influence of the viscous characteristics of the gas and liquid media on their distribution in the flow. The study showed a significant effect of gravitational forces on the nature of the phase distribution in the cross section of the evaporative tubes. At a mass velocity of a two-phase flow less than  $700 \text{ kg} / \text{m}^2\text{s}$ , the volume content of the liquid phase near the upper outer generating lines of the tube is almost an order of magnitude lower than the lower one. The calculation of the heat transfer crisis in horizontal evaporative tubes is obtained. The calculated dependence is in good agreement with the experimental data of the author and a number of foreign researchers. The formula generalizes the experimental data for pipes with the diameter of 6-40 mm in the pressure of 2-7 MPa.

## 1 Introduction

Design and calculation of heat exchange surfaces with their different orientations in space, including horizontal ones, is of wide use in steam generators of thermal installations in various branches of industrial heat power engineering. Today the development of reliable and economical devices with boiling heat-transfer agents is one of the most important tasks in heat and power engineering, the successful development of which largely depends on the

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knowledge of the processes of hydrodynamics and heat transfer in two-phase working media, as well as issues related to the intensification of heat exchange and the prevention of crisis phenomena. In the present article, in order to simplify the analysis, the annular structure of a two-phase flow is the design model. The solution is based on the principle of a minimum of energy dissipation in a stabilized flow associated with irreversible losses by internal friction during the movement of real media. From the thermodynamic point of view, in the flow of a viscous liquid degradation occurs (energy depreciation), as in all other real processes taking place in nature, the characteristic feature of which is the increase in the entropy of the system. Internal nonequilibrium processes always act in a direction that causes a decrease in the rate of entropy increase. In the thermodynamics of irreversible processes, this law is formulated as follows: as the system goes over to the stationary-nonequilibrium state, the value of the incremental increase in entropy decreases, and when the stationary-nonequilibrium state is reached, it is of the smallest value compatible with external constraints [1, 2].

## 2 Methods

Let us consider the annular regime of a stabilized gas-liquid flow in a pipe with radius of  $R_0$ , shown in the Fig. 1. This flow will consist of a ring layer of liquid with thickness  $\delta$  and a cylindrical gas core with radius  $R_G = R_0 - \delta$ . The mass flow of fluid  $G_l$  and gas  $G_g$  and their physical parameters (density, viscosity)  $\rho_g, \rho_l, \mu_g, \mu_l$  are assumed to be known [3,4].

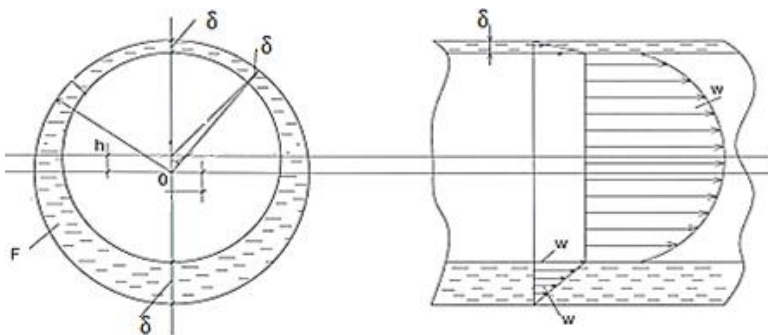
In horizontal and low-inclined pipes, the lifting force (Archimedeova) exerts an essential effect on the steam core:

$$P_A = \pi R_r^2 (\rho' - \rho'') g \Delta l \tag{1}$$

where  $R_r$  - radius of vapor nucleus, m;  $\rho'$  и  $\rho''$  - liquid and gas density, kg/m<sup>3</sup>;  $g$  - projection of the acceleration of gravity on the plane of the pipe cross-section, m/s<sup>2</sup>;  $\Delta l$  - length of the pipe element, m.

Under this force, the co-alignment of the vapor nucleus and the annular layer of the liquid is disrupted. The thickness of the liquid layer  $\delta$  becomes not uniform along the perimeter (Fig. 1). At the top of the pipe, the liquid layer becomes thinner, in the lower one, the liquid layer thickens. The vertical displacement of the flow nucleus will be accompanied by an increase in the energy dissipation intensity associated with increasing friction losses.

It is obvious that a certain amount of displacement of the steam nucleus  $h$  corresponding to a minimum of the second change in the energy of the system for given boundary conditions will correspond to the stabilized flow.



**Fig. 1.** The model of horizontal asymmetric annular flow of steam-water flow

We start from the assumption that the total specific work in each section of the stabilized vapor-liquid flow has a minimum [2]:

$$\sum e_i(h) = \min, \tag{2}$$

where  $e_i = \frac{E}{V}$ ;

$E$  - secondary work of internal friction forces,  $W$ ;  $V$  - the volume of the pipe  $\Delta l$ . Energy flows, independent of  $h$ , are not introduced here.

Let us consider the problem of finding the magnitude of the displacement of the vapor nucleus, as well as the minimum thickness of the liquid film  $\delta_{cr}^B$ , and of corresponding boundary heat flow density  $q_{cr}$  ( $W/m^2$ ), at which a sharp deterioration of heat transfer in the region of the upper generator of the evaporation tube is observed [3].

We assume that when the vapor nucleus is displaced, its cross section is deformed insignificantly, and the absolute velocity  $\omega_{oL}$  at the boundary of the vapor nucleus and the liquid layer is unchanged. In this case, the second energy loss by friction in the liquid layer  $V_L = F' \Delta l$  can be expressed by the following integral.

where  $\tau_L$  - frictional stress in a liquid layer,  $N/m^2$ ;  $\tau_L = \mu_L \cdot \frac{dw_L}{dR}$ ;  $F'$  - sectional area occupied by a liquid;  $w_L$  - local fluid velocity in the section under consideration, m/s.

$$dF = \Delta l R d\beta \tag{3}$$

Assuming the law of velocity distribution  $w_L(R)$  in the liquid layer to be linear, we can rewrite the integral (3) in the following form

$$E_\tau = 2\Delta l \int_0^\pi d\beta \int_{R_r}^{R_r+\delta} \mu_L \frac{w_{oL}^2}{\delta^2} R dR, \tag{4}$$

where  $\delta$  - thickness of the liquid layer.

Integrating over the variable  $R$ , we have:

$$E_\tau = \Delta l \int_0^\pi \frac{\mu_L w_{oL}^2}{\delta} (2R_r + \delta) d\beta. \tag{5}$$

Given that  $R_r = \sqrt{\varphi} \cdot R_o$ ,

we receive

$$E_\tau = \Delta l \mu_L w_{oL}^2 \left( \pi + 2\sqrt{\varphi} \cdot R_o \int_0^\pi \frac{d\beta}{\delta} \right). \tag{6}$$

According to the Fig. 1

$$\delta = \sqrt{R_o^2 + h^2 - 2R_o h \cos\beta} - R_r, \tag{7}$$

where  $h$  - displacement of the center of the vapor nucleus relative to the axis of the pipe. With a relatively small thickness of the liquid layer, the function  $\delta = f(\beta)$  can be represented as the following formal power series:

$$\frac{1}{\delta} = B_0 + B_1\beta + B_2\beta^2 + B_3\beta^3 + \dots \tag{8}$$

Restricting ourselves to the first four terms of the written expansion of the function  $1/\delta$  in powers of  $\beta$ , we find the coefficients of the polynomial  $B_0, B_1, B_2$  and  $B_3$  from the following boundary conditions:

- 1)  $\beta = 0 \quad 1/\delta = \frac{1}{\alpha-h}$ ;
- 2)  $\beta = \pi \quad 1/\delta = \frac{1}{\alpha+h}$ ;
- 3; 4)  $\begin{cases} \beta = 0 & \frac{\partial(\frac{1}{\delta})}{\partial\beta} = 0 \\ \beta = \pi & \frac{\partial(\frac{1}{\delta})}{\partial\beta} = 0 \end{cases}$ ;

where  $\alpha = R_0 - R_r = R_0(1 - \sqrt{\varphi})$ .

From the first boundary condition

$$B_0 = \frac{1}{\alpha - h}. \tag{9}$$

From the second boundary condition

$$\pi B_1 + \pi^2 B_2 + \pi^3 B_3 = -\frac{2h}{\alpha^2 - h^2}. \tag{10}$$

$$\text{From the third boundary condition } B_1 = 0. \tag{11}$$

From the fourth boundary condition

$$B_1 + 2\pi b_2 + 3\pi^3 B_3 = 0. \tag{12}$$

Solving jointly (10), (11), and (12), we obtain

$$B_2 = \frac{6h}{\pi^2 \cdot (\alpha^2 - h^2)}; \tag{13}$$

$$B_3 = \frac{4h}{\pi^3 \cdot (\alpha^2 - h^2)}. \tag{14}$$

Thus, the desired dependence can be represented in the following form:

$$\frac{1}{\delta} = \frac{1}{\alpha - h} - \frac{6h\beta^2}{\pi^2(\alpha^2 - h^2)} + \frac{4h\beta^3}{\pi^3(\alpha^2 - h^2)}. \tag{15}$$

Evaluate the integral (6):

$$\int_0^\pi \frac{d\beta}{\delta} = \int_0^\pi \left[ \frac{1}{\alpha - h} - \frac{6h\beta^2}{\pi^2(\alpha^2 - h^2)} + \frac{4h\beta^3}{\pi^3(\alpha^2 - h^2)} \right] d\beta = \frac{\pi}{\alpha - h} - \frac{2\pi h}{\alpha^2 - h^2} + \frac{\pi h}{\alpha^2 - h^2} = \frac{\pi\alpha}{\alpha^2 - h^2}. \tag{16}$$

Taking into account (16), the expression of the second frictional losses (6) takes the following form:

$$E_\tau = \pi \Delta l \mu_L \omega_{0L}^2 \left( 1 + \frac{2\sqrt{\varphi} R_0 \alpha}{\alpha^2 - h^2} \right). \tag{17}$$

Consider a second change in the energy of the selected element of the flow in the region  $\Delta l$ , which will be determined by the work spent against the gravitational forces on the displacement of the liquid  $M$  in the transverse direction relative to the channel:

$$E_\pi = Mg(y'_c - y''_c), \tag{18}$$

where  $g$  - projection of the acceleration of gravity on the plane of the pipe cross-section,  $m/s^2$  (for horizontal pipe  $g = 9,81 \text{ m/s}^2$ );  $y''_c$ - the ordinate of the center of gravity of the vapor nuclear.

According to the earlier assumptions  $y''_c = h$ . (19)

The ordinate  $y'_c$  of the center of gravity of the cross section of the liquid layer when the steam nucleus is displaced by an amount  $h$ , is determined from the equation of moments of static equilibrium with respect to the horizontal axis of the section of the pipe 0-0 (Fig. 1):

$$\rho' F' g y'_c = g(F\rho' \cdot 0 - hF''\rho'); \tag{20}$$

As  $F' = F(1 - \varphi)$  and  $F'' = \varphi F$ , then

$$F(1 - \varphi)\rho' g y'_c = -gh\varphi F\rho'. \tag{21}$$

$$\text{Hence } y'_c = -\frac{\varphi h}{1 - \varphi}. \tag{22}$$

Substituting (19) and (22) into (18), we obtain

$$E_\pi = -Mg \frac{h}{1 - \varphi}. \tag{23}$$

For the selected element with the length of  $\Delta l$  the second-long transverse transport of the mass  $M$  will be composed of the following flows [4]:

- 1) the mass of the liquid, changing into a vapor state in the amount of  $\Delta D$ , kg/s

$$\Delta D = \frac{2\pi R_0 \Delta l q}{r}, \tag{24}$$

where  $q$  - heat flow density,  $W/m^2$ ;  $r$  - heat of vaporization,  $J / kg$ .

- 2) the transverse displacement of the liquid during the evaporation process, which is proportional to  $\Delta D$ :

$$M_1 = C_1 \Delta D; \tag{25}$$

- 3) liquid particles in the result of their mechanical detachment from the inner surface of the liquid layer and entrainment into the vapor nucleus [5, 6, 7].

We assume that the flow density of dispersed particles from the surface of the liquid layer is proportional to the average mass flow rate of the liquid phase:

$$M_2 = C_2(\rho w) \cdot (1 - x) \cdot \Delta F. \tag{26}$$

In this case the coefficient of proportionality  $C_2$  is a function of the Weber number

$$C_2 = C_2' \cdot W_e = \frac{C_2' \rho'' (\bar{w}'' - \bar{w}')^2 2R_0}{\sigma}, \tag{27}$$

where  $\bar{w}''$  и  $\bar{w}'$  - average cross-sectional velocities of the vapor and liquid phases in the flow;  $\sigma$  - surface tension coefficient,  $N/m$ . As

$$\bar{w}'' F'' \rho'' = (\rho w) x F_0; \tag{28}$$

$$\bar{w}' F' \rho' = (\rho w) (1 - x) F_0, \tag{29}$$

where  $F'$  и  $F''$  - the cross-sectional area of the pipe, occupied by liquid and steam,  $m^2$ ;  $\rho w$  - mass flow rate,  $kg/m^2 \cdot s$ , then

$$\bar{w}'' = \frac{(\rho w) \cdot x}{\varphi \rho''}, \tag{30}$$

$$\bar{w}' = \frac{(\rho w) \cdot (1-x)}{(1-\varphi) \rho'}. \tag{31}$$

Thus, we obtain an expression for the Weber number

$$W_e = \frac{2R_0 \rho'' (\rho w)^2}{\sigma} \cdot \left[ \frac{x}{\varphi \rho''} - \frac{(1-x)}{(1-\varphi) \rho'} \right]^2. \tag{32}$$

Taking into account (24), (25) and (26) the total mass flow be:

$$M = \Delta D + C_1 \cdot \Delta D + (\rho w)(1 - x) \cdot 2\pi R_0 \Delta l \left[ \frac{Cq}{r} + C_2(\rho w)(1 - x) \right]. \tag{33}$$

where  $\Delta D$  - mass of a liquid passing into a vapor state;

$$C=C_1+1.$$

The specific work against gravitational forces is as follows:

$$e_n = \frac{E_n}{V} = - \frac{2Cgh}{R_0 \cdot (1-\varphi)} \left[ \frac{q}{r} + C_3 (\rho w)(1 - x) \right], \tag{34}$$

where  $C_3 = \frac{C_2}{c}$ .

Specific work of internal friction forces:

$$e_\tau = \frac{E_\tau}{V} - \frac{\mu w_0^2}{R_0^2} \left( 1 + \frac{2\sqrt{\varphi} R_0 \alpha}{a^2 - h^2} \right) \quad (35)$$

We find the value of  $h$  from the condition for the transition of the system to a stationary state. Moreover, the latter will be achieved when  $\sum e(h) = \min$  [8]

$$\frac{\partial}{\partial h} (e_\tau + e_n) = 0 \quad (36)$$

Substituting expression (34) and (35) into equation (36), after differentiation, we obtain:

$$\frac{2 \cdot \mu w_0^2 \sqrt{\varphi} \alpha h}{(a^2 - h^2)^2} - \frac{Cg}{1 - \varphi} \left[ \frac{q}{r} + C_3 (\rho w) - (1 - x) \right] = 0 \quad (37)$$

Then

$$\frac{t_h}{(1 - t_h^2)^2} = \frac{C \alpha^2 g \left[ \frac{q}{r} + C_3 (\rho w) - (1 - x) \right]}{2 \mu w_0^2 \sqrt{\varphi} (1 - \varphi)}, \quad (38)$$

where  $t_h = \frac{h}{a} = 1 - \frac{\delta^B}{a}$  – relative displacement of the vapor nucleus.

As

$$w_0' = 2\overline{w} = \frac{2(\rho w)(1 - x)}{(1 - \varphi)\rho}, \quad (39)$$

expression (38) can be represented in the following form:

$$\frac{t_h}{(1 - t_h^2)^2} = \frac{CgR_0^2 (1 - \sqrt{\varphi})^3 (1 + \sqrt{\varphi}) \rho^2 \left[ \frac{q}{r} + C_3 (\rho w) - (1 - x) \right]}{8\mu (1 - x)^2 (\rho w)^2 \sqrt{\varphi}}. \quad (40)$$

### 3 Results

Based on the analysis of experimental studies of the structure of the horizontal steam-water flow by the method of "electrosounding" and generalization of the author's experimental data in the range  $P = 2-7\text{MPa}$ ;  $\rho w = 100 \div 1500 \frac{\text{kg}}{\text{m}^2} \cdot \text{s}$ ;  $x > 0.2$ ;  $0.012\text{m} \leq d_0 \leq 0.019 \text{ m}$ , it was obtained an expression for the reduced thickness of the liquid film at the upper pipe generator at the boundary of the zone of deteriorated heat transfer.

$$\delta_{2p}^e = \sqrt{\frac{k(\rho w)\mu}{g(\rho' - \rho'')\rho'}}; \quad (41)$$

The validity of expression (41) is confirmed by experimental measurements of the effective thickness of the liquid film at the upper pipe generator at the boundary of the onset of thermal pulsations. Moreover, it is established that for smooth pipes  $K_\alpha = 1, 1$ .

From the expression (40) we obtain following expression for the boundary density of the heat flow:

$$\frac{q_{cr}}{r} = C_1 \left[ \frac{T}{B} - CW_e (\rho w) (1 - x) \right], \quad (42)$$

$$T = \frac{t_h}{(1-t_h^2)^2} = \frac{1-\delta_{cr}^B/\alpha}{[1-(1-\delta_{cr}^B/\alpha)^2]^2}; \tag{43}$$

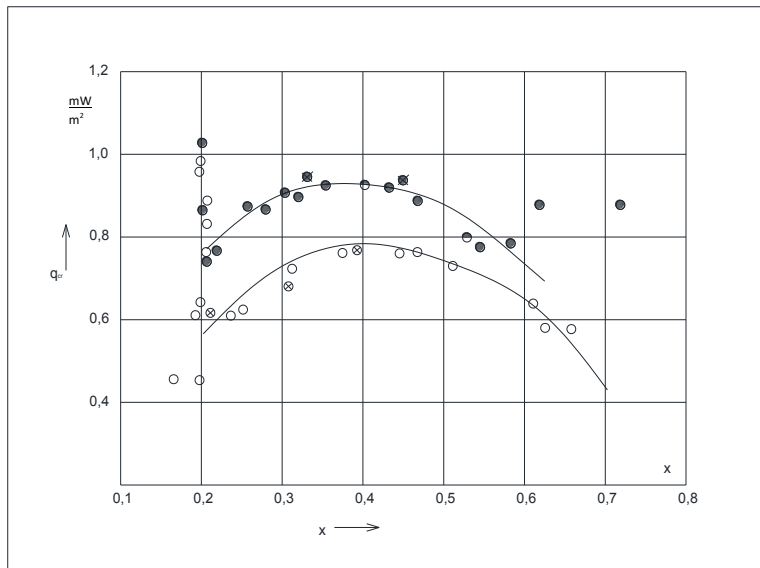
$$B = \frac{gR_0^2(1-\sqrt{\varphi})^3(1+\sqrt{\varphi})\rho^2}{8\mu \cdot (1-x)^2 \cdot (\rho w)^2 \sqrt{\varphi}}. \tag{44}$$

Substituting the values of the numerical coefficients found from the experiments, we finally obtain the critical density of the heat flow:

$$q_{2p} = r \left[ \frac{T}{B} - W_e(\rho w)(1-x) \cdot 10^{-3} \right] \cdot 1.6 \cdot 10^{-4} \tag{45}$$

where  $T = \frac{1-\delta_{ip}^B/\alpha}{[1-(1-\delta_{ip}^B/a)^2]^2}$ ;  $B = \frac{gR_0^2(1-\sqrt{\varphi})^3(1+\sqrt{\varphi})\rho^2}{8\mu \cdot (1-x)^2 \cdot (\rho w)^2 \sqrt{\varphi}}$ .

In the Fig. 2 the dependences of the boundary heat load on the mass gas content according to the formula (45) are given. In the same place, the experimental data obtained by the author and also by other researchers are plotted [9, 10, 11].



**Fig. 2.** Comparison of experimental data with calculation of  $q_{cr}$  for a horizontal evaporator pipe ( $d_0 = 0.017$  m,  $P = 5$  MPa)

### 4 Conclusion

The formula generalizes the experimental data obtained on steam-vaporizing pipes with an internal diameter  $d_o = 0,012 \div 0,019$  in the pressure range 3-7 MPa, the range of mass flow velocities  $300-1400$  kg/m<sup>2</sup> · s for relative enthalpies of the flow  $x > 0.2$  with an accuracy  $\pm 20\%$  [12].

In the range of relative enthalpies  $0.2 < x < 0.3$ , in the experimental values  $q_{cr}$  an appreciable spread takes place, and in the range  $x = 0.2$ , the effect of the heat flow on the boundary of the zone of deteriorated heat transfer practically degenerates. The character of the dependence  $q_{cr} = f(x)$  is analogous to a crisis of the second kind in vertical pipes (the experimental values obtained under these conditions in the  $q, x$  coordinates are grouped around almost vertical lines) [14, 15]. And in the normative method, it is proposed to take the degraded heat transfer zone outside the range of medium pressures for horizontal evaporative tubes at  $x = 0.3$  for  $P < 5.0$  MPa for any  $q$ .

In this case, the dispersed-annular flow regime is only formed and has an appreciable hydrodynamic instability. At low mass velocities  $\rho w < 700$  kg/m<sup>2</sup> · s, the vapor clusters moving along the upper generator periodically cause "dry spots" on the channel wall, which in turn leads to the thermal pulsations of the wall near the upper generator of the pipe.

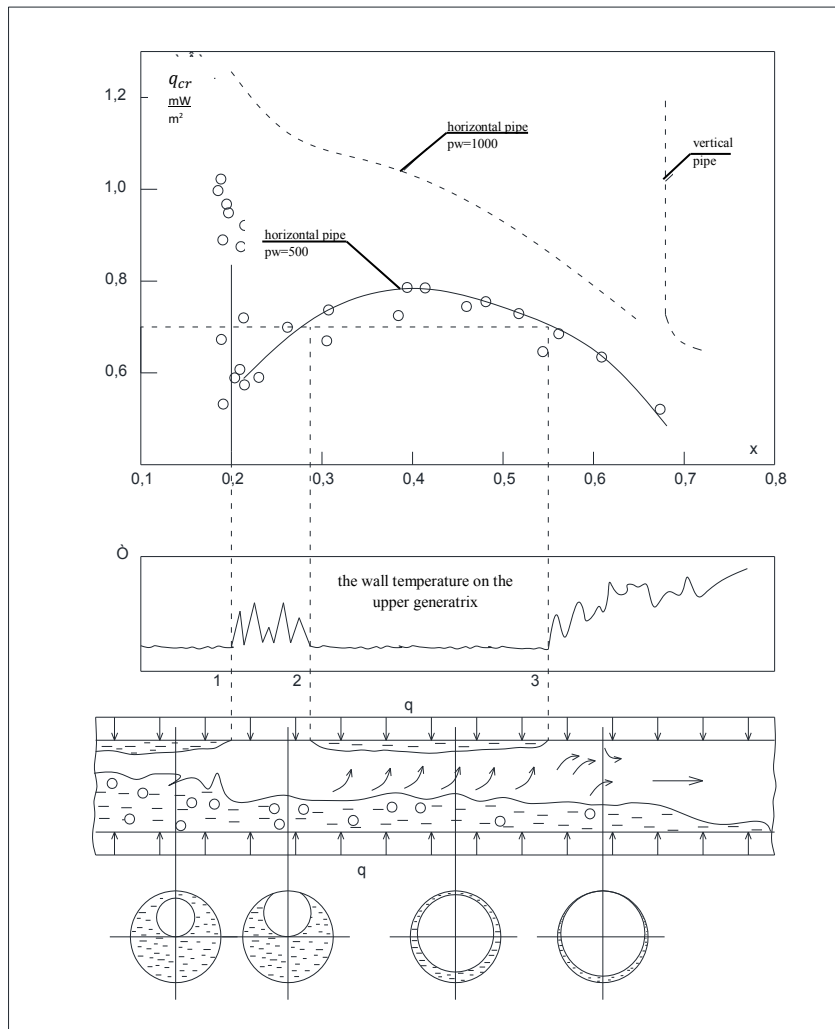
With decreasing  $q$  to a certain value  $q_{min}$ , the dependence  $q_{cr} = f(x)$  sharply deflects to the right and the boundary density of the heat flow  $q_{cr}$  even slightly increases. It is explained by the fact that the steam-water flow rate increases as the steam flow rate increases, which in turn reduces the asymmetry of the flow and increases the velocity of liquid droplets in the flow nucleus (i.e., the irrigation conditions of the channel wall improve) [13].

In the range of relative enthalpies  $0.2 < x < 0.3$ , the boundary heat flow density  $q_{cr}$  calculated by formula (45) corresponds to the minimum experimental values of  $q$  when the wall thermal impulses on the upper generator of the horizontal steam generating pipe were observed. A further increase in  $x$  greater than 0.4 is associated with a decrease in the thickness of the wall-mounted liquid film along the entire perimeter of the channel. At the same time, the liquid droplets in the nucleus of the stream decrease, which significantly reduces the irrigation intensity of the channel wall. It leads to a monotonic decrease  $q_{cr}$ .

In some regimes it was possible to observe simultaneously two separate zones of thermal pulsations along the length of the channel (Fig. 3): one in case of small values of  $x$  of 0.2 order and the other near the outlet section of the pipe. A similar phenomenon was also noted in [14, 15]. Two separate zones of thermal pulsations on the upper generator of a horizontal pipe can be explained on the basis of the combined influence of many factors on the nature of the dependence  $q_{cr} = f(x)$ , which is in complete agreement with the above physical model of the horizontal steam-water flow.

The dependence  $q_{cr}$  on the mass-velocity value for horizontal evaporative tubes is shown in the Fig. 2. It should be noted that obtained dependence has an inversion. An increase of  $\rho w$  up to 1000-1500 kg/m<sup>2</sup> · s leads to an increase in  $q_{cr}$ . In the range of 1000-1500 kg/m<sup>2</sup> · s an increase in the mass velocity value of  $q_{cr}$  decreases. Moreover, at large vapor contents of the flow, the inversion point corresponds to smaller mass velocities. The observed phenomenon confirms the previously stated position that at high velocities the hydrodynamic processes in horizontal and vertical pipes are similar.





**Fig. 3.** Zones of deteriorated heat transfer in a horizontal evaporative tube

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