Quantitative assessment of the identifiability of pipeline systems*

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Abstract. The article is devoted to the issues of quantitative assessment of the identifiability of the pipeline systems (heat, water, gas supply systems etc.). Identifiability is first considered as a complex property, including such particular properties as observability and parametric identifiability. A brief description of the topic relevance and a review of available development in this sphere allow giving the structuring of identifiability analysis problems. The technique of differentiate quantitative analysis of this property is disclosed. It based on the use of analytical expressions for covariance matrices of parameters. New concepts of experimental matrices, parametric identifiability and observability of pipeline systems are introduced. Analytic expressions for these matrices are given. The substantiation of the integral indicators of the pipeline systems identifiability is presented, including the covariance matrix determinant for the estimated parameters and the relative variance of the prediction for non-measurable state parameters. The analytical interrelation of these indicators is opened. These indicators can be accepted in a role of criteria at decision of synthesis problems for optimal measurements composition.

1 Introduction

The urgency of the article theme is due to increasing rates of technological transformation, renovation and modernization of pipeline systems (PLS) due to the integration of new equipment, devices of control, monitoring and measuring, computer technology, mathematical modelling methods, computer systems for measuring and processing measurements and making control decisions. The main purpose of these transformations has recently been increasingly associated with the intelligent automation of PLS. First of all, the intelligence of PLS is the intelligent automation of control. In this regard, there are questions of quantitative estimation of the intellectual property of the PLS from the point of view of their satisfaction with the basic cybernetic properties – controllability and identifiability. As [1,2] were shown the main attention is paid to the identification of PLS as a complex property, including such particular properties as observability and parametric identifiability.

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2 The problem of pipeline systems identifiability

Identifiability is understood as the possibility of restoring the mathematical model of PLS with accuracy to parameters from the results of measurements and observations. The relevance of independent study of this property in respect to PLS is determined by the following main circumstances: 1) the characteristics of PLS equipment undergo significant changes during operation, and the coefficients of these characteristics are inaccessible to direct measurement or observation; 2) lack of information on the actual characteristics of PLS reduces the effectiveness and validity of decisions on control, maintenance and repair, reconstruction and development of PLS; 3) the existing practice of PLS equipping with measuring devices at the main facilities (at sources, pumping stations, etc.) leads to the fact that the flow distribution processes remain unobservable, and the extent of achievement of control objectives remains uncontrolled; 4) the lack of adequate PLS models invalidates practically the entire arsenal of mathematical modelling and optimization methods for PLS control.

Recently, in connection with the emergence of advanced information and measurement systems, more attention is being paid to the development and application of identification (calibration) methods of PLS [3], water [4,5], heat [6,7], gas [8], oil supply systems [9], etc., as well as problems of optimizing of measurements composition for the possibility of using these methods [10].

In ESI SB RAS, the issues of PLS identifiability have been being systematically begun to be investigated since the early 1990th [11 - 13, etc.]. This investigation included the development of a new approach to the identification of PLS [14, 15]. In the framework of this approach, practically all methodological limitations were removed. These limitations were related to the type of model, possible combinations of measured or sought-for parameters, and other factors. Accordingly, it became possible to objectively study of the identifiability as a property of PLS itself, rather than the methods for identifying of them.

The structuring of the problems for identifiability analysis can be performed depending on [1,2]: 1) identification purposes (states, parameters, models) – analysis of observability, parametric or structural identifiability; 2) factors determining the possibility of achievement these goals – *qualitative and quantitative analysis*; 3) the conditions for studying of these factors – *a priori or a posteriori analysis* (before the experiment or after processing its results); 4) the purposes of further use of the analysis results – *differential or integral analysis*.

Thus, identifiability is a complex property that requires independent study, development of its own methodical apparatus. Below, problems and methods of a priori quantitative assessment of identifiability from the point of view of information and objective aspects of this property are considered.

3 Differential assessments of identifiability

Qualitative analysis of identifiability makes it possible to establish the principle solvability of the identification problem depending on the composition of the measured parameters [11, 12], but does not answer the question of the identification accuracy under conditions of measurement errors presence. In [14], the technique of differential quantitative analysis of identifiability was proposed. It based on the use of analytic expressions for covariance matrices (CM) of an arbitrary subset of PLS parameters.

The following main results were obtained here. Suppose that the system of equations is used as a model of PLS, which we represent in the form

$$\mathrm{U}(\mathbf{Z}) = 0, \tag{1}$$

where: $\mathbf{Z}$ – vector of model parameters.

We introduce the following decomposition methods for $\mathbf{Z}$:

1) $\mathbf{Z}=\{\mathbf{R},\boldsymbol{\alpha}\}$, where $\mathbf{R}$ – vector of state parameters (pressure, flow rates, etc.), $\boldsymbol{\alpha}$ – vector of the parameters of the elements (the model coefficients) specifying the internal technical condition of the PLS (the control vector u is considered here as known, deterministic given);

2) $\mathbf{Z}=\{\mathbf{Z}_1,\mathbf{Z}_2\}$, where $\mathbf{Z}_1,\mathbf{Z}_2$ – vectors of measured and non-measurable parameters;

3) $\mathbf{Z}=\{\mathbf{X},\mathbf{Y}\}$, where $\mathbf{X},\mathbf{Y}$ – vectors of independent and dependent parameters, and $\mathbf{X}$ always must provide a single-valued computation $\mathbf{Y}$ from (1);

4) $\mathbf{X}=\{\mathbf{X}_R,\boldsymbol{\alpha}\}$. That is, we assume that $\mathbf{X}$ always includes the vector of elements parameters $\boldsymbol{\alpha}$, and $\mathbf{X}_R$ is the vector of the independent state parameters. Wherein

$$\mathbf{Y}=\mathbf{Y}(\mathbf{X})=\mathbf{Y}(\mathbf{X}_R,\boldsymbol{\alpha}) \tag{2}$$

If we involve the measurement results for N conditions, we assume that $\mathbf{Z}=\{\mathbf{R},\boldsymbol{\alpha}\}=\{\mathbf{R}^{(1)},\mathbf{R}^{(2)},\ldots,\mathbf{R}^{(N)},\boldsymbol{\alpha}\},\quad \mathbf{R}^{(t)}=\{\mathbf{X}_R^{(t)},\mathbf{Y}^{(t)}\},\ u=\overline{1,N}$,
$\mathbf{X}=\{\mathbf{X}_R^{(1)},\mathbf{X}_R^{(2)},\cdots,\mathbf{X}_R^{(N)},\boldsymbol{\alpha}\},\quad \mathbf{Y}=\{\mathbf{Y}^{(1)},\mathbf{Y}^{(2)},\cdots,\mathbf{Y}^{(N)}\}$.

The error in the definition $\mathbf{Z}$ is determined as $\boldsymbol{\varepsilon}_Z=\hat{\mathbf{Z}}-\overline{\mathbf{Z}}$, where $\hat{\mathbf{Z}}$ is some estimate of the true value $\overline{\mathbf{Z}}$ of the model parameters vector $\mathbf{Z}=\{\mathbf{R}^{(1)},\mathbf{R}^{(2)},...,\mathbf{R}^{(N)},\boldsymbol{\alpha}\}$. As the value $\overline{\mathbf{Z}}$ is unknown, the error level can be determined statistically in the form of a CM $\mathbf{C}_Z\equiv\mathbf{E}(\boldsymbol{\varepsilon}_Z\boldsymbol{\varepsilon}_Z^T)$, on the diagonal of which there are component variances $\boldsymbol{\varepsilon}_Z$, and off-diagonal elements correspond to their covariance. As shown in [14], *the CM of model parameters* have a structure $\mathbf{C}_Z=\left[\begin{array}{c|c}\mathbf{C}_X & \mathbf{C}_{XY}\\ \hline \mathbf{C}_{XY}^T & \mathbf{C}_Y\end{array}\right]=\left[\begin{array}{c|c}\mathbf{C}_X & \mathbf{C}_X(\mathbf{Y}_X')^T\\ \hline \mathbf{Y}_X'\mathbf{C}_X^T & \mathbf{Y}_X'\mathbf{C}_X(\mathbf{Y}_X')^T\end{array}\right]$.
Here $\mathbf{Y}_X'=\partial\mathbf{Y}/\partial\mathbf{X}$ – a matrix of partial derivatives, $\mathbf{C}_X=(\mathbf{J}^T\mathbf{C}_{Z1}^{-1}\mathbf{J})^{-1}$ – a CM of estimation errors for $\mathbf{X}$, $\mathbf{C}_{Z1}$ – a CM of measurement errors of the vector $\mathbf{Z}_1$, $\mathbf{J}=\partial\mathbf{Z}_1/\partial\mathbf{X}=\mathbf{I}_{Z1}\begin{pmatrix}\mathbf{E}\\ \mathbf{Y}_X'\end{pmatrix}=\mathbf{J}(\mathbf{Y}_X')$ – a matrix of the partial derivatives of the measured parameters as function of the independent, $\mathbf{E}$ – the identity matrix, $\mathbf{I}_{Z1}$ – a matrix of the correspondence of components of the vectors $\mathbf{Z}_1$ and $\mathbf{Z}$, consisting of zeros and ones.

Thus, $\mathbf{C}_Z$ is completely defined if $\mathbf{C}_X$ and $\mathbf{Y}_X'$ are given, and $\mathbf{C}_X$ if the matrices $\mathbf{I}_{Z1}$, $\mathbf{C}_{Z1}$ and $\mathbf{Y}_X'$ are given. Accordingly, the accuracy of the estimates $\hat{\mathbf{Z}}$ (and hence the identifiability) is determined by three factors are the composition of measurements, their accuracy and sensitivity $\mathbf{X}$ to $\mathbf{Z}_1$, determined by the implicit function $\mathbf{Z}_1(\mathbf{X})$ (PLS model) in the conditions for which these measurements are made. I.e, $\mathbf{C}_X=\mathbf{C}_X(\mathbf{I}_{Z1},\mathbf{C}_{Z1},\mathbf{X})$.

4 Private information and covariance matrices

We call the matrix $\mathbf{F}_X=\mathbf{J}^T\mathbf{C}_{Z1}^{-1}\mathbf{J}$ *the information matrix of the experiment*, as it includes all the information extracted from this experiment and is necessary for estimate the residual degree of uncertainty about the unknown parameters, regardless of the purpose of the experiment. Then $\mathbf{C}_X=\mathbf{F}_X^{-1}$ is a *CM of experiment*. Taking into account the introduced decomposition $\mathbf{X}=\{\mathbf{X}_R,\boldsymbol{\alpha}\}$, these matrices have the structure

$$\mathbf{F}_X = \mathbf{J}^T \mathbf{C}_{Z1}^{-1} \mathbf{J} = \left[\begin{array}{c|c} \mathbf{J}_R^T \mathbf{C}_{R1}^{-1} \mathbf{J}_R & \mathbf{J}_R^T \mathbf{C}_{R1}^{-1} \mathbf{J}_\alpha \\ \hline \mathbf{J}_\alpha^T \mathbf{C}_{R1}^{-1} \mathbf{J}_R & \mathbf{J}_\alpha^T \mathbf{C}_{R1}^{-1} \mathbf{J}_\alpha + \mathbf{I}_{\alpha 1}^T \mathbf{C}_{\alpha 1}^{-1} \mathbf{I}_{\alpha 1} \end{array} \right], \tag{3}$$

$$\mathbf{C}_X = \mathbf{F}_X^{-1} = \left[\begin{array}{c|c} \mathbf{C}_{XR} & \mathbf{C}_{XR\alpha} \\ \hline \mathbf{C}_{XR\alpha}^T & \mathbf{C}_\alpha \end{array} \right], \tag{4}$$

where the following notation is used: $\mathbf{J}_R = \partial \mathbf{R}_1 / \partial \mathbf{X}_R$; $\mathbf{J}_\alpha = \partial \mathbf{R}_1 / \partial \boldsymbol{\alpha}$; $\mathbf{I}_{\alpha 1}$ – a matrix, similar in meaning $\mathbf{I}_{Z1}$, but with respect to given pseudo-measurements $\boldsymbol{\alpha}_1$; vectors of measurement of the state parameters $\mathbf{R}_1$ and elements parameters $\boldsymbol{\alpha}_1$ are assumed to be uncorrelated. Wherein $\mathbf{J} = \begin{bmatrix} \mathbf{J}_R & \mathbf{J}_\alpha \\ \mathbf{0} & \mathbf{I}_{\alpha 1} \end{bmatrix}$, $\mathbf{C}_{Z1} = \begin{bmatrix} \mathbf{C}_{R1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\alpha 1} \end{bmatrix}$, $\mathbf{I}_{Z1} = \begin{bmatrix} \mathbf{I}_{R1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\alpha 1} \end{bmatrix}$.

We apply to $\mathbf{F}_X$ the inversion rule of the block matrix [16], as a result of which we obtain finite expressions for the CM of the independent state parameters $\mathbf{C}_{XR} = \mathbf{F}_{XR}^{-1}$ and the CM of elements parameters, where

$$\mathbf{F}_{XR} = \mathbf{J}_R^T \mathbf{H}_\alpha \mathbf{J}_R, \tag{5}$$

$$\mathbf{F}_\alpha = \mathbf{J}_\alpha^T \mathbf{H}_{R1} \mathbf{J}_\alpha + \mathbf{I}_{\alpha 1}^T \mathbf{C}_{\alpha 1}^{-1} \mathbf{I}_{\alpha 1}, \tag{6}$$

and

$$\mathbf{H}_\alpha = \mathbf{C}_{R1}^{-1} - \mathbf{C}_{R1}^{-1} \mathbf{J}_\alpha (\mathbf{J}_\alpha^T \mathbf{C}_{R1}^{-1} \mathbf{J}_\alpha + \mathbf{I}_\alpha^T \mathbf{C}_{\alpha 1}^{-1} \mathbf{I}_\alpha)^{-1} \mathbf{J}_\alpha^T \mathbf{C}_{R1}^{-1}, \tag{7}$$

$$\mathbf{H}_{R1} = \mathbf{C}_{R1}^{-1} - \mathbf{C}_{R1}^{-1} \mathbf{J}_R (\mathbf{J}_R^T \mathbf{C}_{R1}^{-1} \mathbf{J}_R)^{-1} \mathbf{J}_R^T \mathbf{C}_{R1}^{-1}. \tag{8}$$

Accordingly, we call: $\mathbf{F}_{XR}$ – *a matrix of observability*, $\mathbf{F}_\alpha$ – *a matrix of identifiability*.

Note that if the vector $\boldsymbol{\alpha}$ is known and defined deterministically, then $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}$, $\mathbf{C}_{\alpha 1} = 0$, $\mathbf{H}_\alpha = 0$ and the observability matrix takes the traditional form

$$\mathbf{F}_{XR} = \mathbf{J}_R^T \mathbf{C}_{R1}^{-1} \mathbf{J}_R. \tag{9}$$

Therefore, (5) gives a formula for the observability matrix, taking into account the uncertainty in the values of the vector α taken into account by $\mathbf{H}_\alpha$.

Relation (6) reveals the contribution to the resulting identifiability of a priori information about the parameters α (the second term on the right side), taking into account the uncertainty about the states parameters taken into account through the matrix $\mathbf{H}_{R1}$.

An important consequence of the introduced concepts and formulas is that: 1) if the system is identifiable, then it is observable, but not vice versa, therefore observability is a particular property of identifiability; 2) the obtained expressions for CM allow to perform a differentiate analysis of accuracy for any model parameter or a set of them, including by building confidential intervals or region with a given probability covering the true values; 3) such analysis can be performed depending on the purpose of the experiment (observation or parametric identification); 4) this technique can be used both for a posteriori and for a priori differential quantitative analysis of identifiability. The only difference is that the derivatives $\mathbf{Y}_X'$ are taken either at the point of the estimation problem solution, or at the point of some assumed values of the vector $\mathbf{X}$.

5 Integral indicators of identifiability

Differentiate analysis does not allow unambiguous comparison of different variants of experimental conditions, as each variant may be better than the other in its group of estimated parameters. For this comparison, as well as for a purposeful impact on these conditions (the identifiability synthesis), it is proposed to use integral quantitative indicators of the identifiability of PLSs as a whole [1,2,13,15]. In this case, the use of the *D*-criterion (the determinant of the CM of the estimated parameters) is expedient in the role of a basic indicator of identifiability. This criterion has the following important properties:

1) the reduction in the degree of uncertainty of the estimated PLS parameters can be related to the requirement to reduce the volume of the confidence ellipsoid, that is equivalent to the requirement of *D*-criterion minimizing [16];

2) the useful property of the *D*-criterion is that if the accuracy of a subset is of particular interest from the whole set of parameters, then the determinant of the submatrix can be applied. This submatrix is obtained by deleting rows and columns corresponding to parameters that are not of interest [16];

3) the most important property of this criterion in the context under consideration follows from the well-known result of the theory of experimental design [16,17], according to which the optimal value of the *D*-criterion corresponds to the minimum value of the maximum variance of the response (the dependent model variables);

4) it is also possible to interpret this criterion from the standpoint of Shannon information theory [1,18], as the minimum of entropy coincides with the minimum of the *D*-criterion.

Thus, for the integral estimation of observability and parametric identifiability of PLS, it is proposed using indicators:

$$D_{XR} = \det(\mathbf{C}_{XR}), \quad D_\alpha = \det(\mathbf{C}_\alpha). \tag{10}$$

The smaller values of these indicators, the less uncertainty in the corresponding groups of model parameters, and, accordingly, the higher the degree of observability and parametric identifiability of PLSs. These indicators already allow complex comparison of the experimental conditions. However their numerical values depend on the dimensionality of the variables and do not allow them to be given a physical meaning. Therefore, in the role of an auxiliary indicator, it is proposed to use the criterion of relative variance of the prediction

$$d = \max_{i \in I_2} \left(\frac{\hat{\sigma}_i^2}{\sigma_i^2} \right), \tag{11}$$

where I_2 – a set of unmeasurable state parameters, σ_i^2 – the a priori dispersion of the direct measurement of this parameter, determined by the metrological characteristics of the corresponding measuring devices, $\hat{\sigma}_i^2$ – the a posteriori (predicted) dispersion of this parameter by the model. An a posteriori estimate of the CM of non-measurable parameters is defined as

$$\mathbf{C}_{Z2} = \mathbf{J}_{Z2}\mathbf{C}_X\mathbf{J}_{Z2}^T, \text{ where } \mathbf{J}_{Z2} = \partial\mathbf{Z}_2 / \partial\mathbf{X} = \mathbf{I}_{Z2}\begin{pmatrix} \mathbf{E} \\ \mathbf{Y}'_X \end{pmatrix}, \tag{12}$$

where $\mathbf{I}_{Z2}$ – a matrix of correspondence of the components of the vectors $\mathbf{Z}_2$ and $\mathbf{Z}$, consisting of zeros and ones. From here

$$\hat{\sigma}_i^2 = (\mathbf{J}_{Z2})_i \mathbf{C}_X (\mathbf{J}_{Z2})_i^T, \ i \in I_2, \tag{13}$$

where $(\mathbf{J}_{Z2})_i$ – *i*-th row of the matrix $\mathbf{J}_{Z2}$.

Thus, the criterion (11) already makes sense of the relative accuracy of the non-measurable parameters determination in comparison with their direct measurement. If, $d \gg 1$ the identifiability is poor, and at $d \approx 1$ – high.

We will reveal the relationship between the criteria *D* and *d* for the simplest case of the analysis of observability under deterministic given α and diagonal matrix $\mathbf{C}_{R1} = diag\{\sigma_1^2, \sigma_2^2, ..., \sigma_l^2\}$, where $l = |I_1|$, I_1 – the set of measured state parameters. Here, the observability matrix (9) can be represented in the form

$$\mathbf{\Phi}_{XR} = \mathbf{J}_R^T \mathbf{C}_{R1}^{-1} \mathbf{J}_R = \sum_{i \in I_1} (\mathbf{J}_R)_i^T \sigma_i^{-2} (\mathbf{J}_R)_i \,. \tag{14}$$

If we add some measure, for example, with an index $i = l+1$ (moving it from the set I_2 to I_1), instead of the matrix (14), which we denote by $F_{XR}(l)$, we get the observability matrix $\mathbf{F}_{XR}(l+1) = \mathbf{F}_{XR}(l) + (\mathbf{J}_R)_{l+1}^T \sigma_{l+1}^{-2} (\mathbf{J}_R)_{l+1}$.

It is known [19, 20] that for some square block matrix $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$, when $\det(\mathbf{A}_{11}) \neq 0$ and $\det(\mathbf{A}_{22}) \neq 0$, we have the relation $\det(\mathbf{A}) = \det(\mathbf{A}_{11}) \det(\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}) = \det(\mathbf{A}_{22}) \det(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21})$.

We denote $\mathbf{A}_{11} = \mathbf{F}_{XR}(l)$, $\mathbf{A}_{22} = -\sigma_{l+1}^2$, $\mathbf{A}_{12} = (\mathbf{J}_R)_{l+1}^T$, $\mathbf{A}_{21} = (\mathbf{J}_R)_{l+1}$. Then $\det[\mathbf{F}_{XR}(l)] \det[\sigma_{l+1}^{-2} + (\mathbf{J}_R)_{l+1} \mathbf{F}_{XR}^{-1}(l) (\mathbf{J}_R)_{l+1}^T] = \det(\sigma_l^{-2}) \det[\mathbf{F}_{XR}(l) + (\mathbf{J}_R)_{l+1} \sigma_{l+1}^2 (\mathbf{J}_R)_{l+1}^T]$. As $(\mathbf{J}_R)_l \mathbf{F}_{XR}^{-1}(l) (\mathbf{J}_R)_l^T = (\mathbf{J}_R)_l \mathbf{C}_{XR} (\mathbf{J}_R)_l^T = \hat{\sigma}_l^2$, $\det(\sigma_l^2) = \sigma_l^2$, $\det[\mathbf{F}_{XR}(l) + (\mathbf{J}_R)_{l+1} \sigma_{l+1}^2 (\mathbf{J}_R)_{l+1}^T] = \det[\mathbf{F}_{XR}(l+1)]$ и $\det(\sigma_{l+1}^2 + \hat{\sigma}_{l+1}^2) = \sigma_{l+1}^2 + \hat{\sigma}_{l+1}^2$, then

$$\det[\mathbf{F}_{XR}(l+1)] = \det[\mathbf{F}_{XR}(l)] \left(1 + \frac{\hat{\sigma}_{l+1}^2}{\sigma_{l+1}^2} \right). \tag{15}$$

Thus, the determinant of the observability matrix increases by an amount $1 + d_{l+1}$ with adding a measurement $l+1$. As $\det(\mathbf{F}_{XR}) = 1/\det(\mathbf{F}_{XR}^{-1}) = 1/\det(\mathbf{C}_{XR}) = 1/D_{XR}$, then the desired interrelation of the exponents d and D has the form

$$D_{XR}(l+1) = D_{XR}(l) / \left(1 + d_{l+1}\right). \tag{16}$$

It is seen that when adding a measurement, the indicator D_{XR} decreases inversely proportional to the value $1 + d_{l+1}$.

On these principles, algorithms for measurements placement [13, 15, etc.] have been developed. These algorithms provide the possibility of successively improving of the measurements composition by the *D*-criterion, being guided by the current values of the d-criterion. Possible formulation of problems for optimization of the measurements composition are minimization of the measurements number, taking into account limitations on the accuracy of identification, or maximization of accuracy with a limitation on the measurements number. The corresponding algorithms allow finding the optimal solution for the final steps; take into account the permissible locations of new measuring devices and the existence of existing ones.

6 Conclusion

1. Due to the commonality of development and functioning problems for various types and purposes PLSs, it is justified to develop complex intellectuality indicators of PLS from the point of view of their satisfaction with the basic cybernetic properties – controllability and identifiability.
2. The relationship between the parametric identifiability and the observability of PLS is revealed. It is shown that observability is a particular property of identifiability under conditions of uncertainty of actual PLS characteristics.
3. Integral quantitative indices of PLS identifiability and methods of their calculation are proposed. Also it is proposed *D*-criterion, which can give an interpretation of information amount by Shannon and d-criterion that reflects the predictive properties of models obtained as a result of identification. The interrelation of these criteria is revealed.

4. The proposed criteria can potentially be used to optimize information and measurement systems, as well as to develop standards for the identifiability of PLSs.

References

1. N.N. Novitsky, *Pipeline Energy Systems: Methodological and Applied Problems of Mathematical Modelling*, 159 (Science, Novosibirsk, 2015) [in Russian]
2. N.N. Novitsky, *System research in power engineering: a retrospective of scientific trends*, 279 (Science, Novosibirsk, 2010) [in Russian]
3. K. Menberg, Y. Heo, R. Choudhary, Proceed. of the 15th IBPSA Conf., 1212 (2017)
4. Y. Zheng, T. Walski, R. Mankowski, G. Herrin, R. Gurrieri, M. Tryby. AWWA IMTech Conference, 1 (2002)
5. S. Lingireddy, L. E. Ormsbee, Civil. Eng. and Env. Syst., **19(1)**, 13 (2000)
6. J. Wang, Z. Zhou, J. Zhao, Energy Conversion and Management, **120**, 294 (2016)
7. V.I. Panferov. Bulletin of SUSU, **13**, 87 (2005)) [in Russian]
8. M.G. Sukharev, R.V. Samoylov, *Analysis and control of stationary and non-stationary conditions of gas transport* (Russian State University of Oil and Gas, Moscow, 2016) [in Russian]
9. A.F. Barkhatov, D.A. Terre, Oil and gas business, **3**, 282 (2015) [in Russian]
10. L. Cozzolino, R. Morte, A. Palumbo, D. Pianese, Civil Engineering and Environmental Systems, **28:1**, 75 (2011)
11. N.N. Novitsky, *Methods of analysis and optimal synthesis of pipeline systems*, 142 (ESI SB RAS, Irkutsk, 1991) [in Russian]
12. N.N. Novitsky, *Mathematical modeling and optimization of heat, water, oil and gas supply systems*, 215 (Science, Novosibirsk, 1992) [in Russian]
13. N.N. Novitsky, O.A. Grebneva, *Pipeline energy systems: models, applications, information technologies*, 90 (Oil and gas, Moscow, 2000) [in Russian]
14. N.N. Novitsky, *Estimation of hydraulic circuit parameters* (Science, Novosibirsk, 1998) [in Russian]
15. N.N. Novitsky, Proceedings of the Russian Academy of Sciences. Power Engineering, **6**, 87 (2000) [in Russian]
16. V.V. Fedorov, *Theory of Optimal Experiment* (Science, Moscow, 1971) [in Russian]
17. J. Kiefer, J. Wolfowitz, The Equivalence to Two Extremum Problems. Canad. J. Math., **12**, 363 (1960)
18. E. Shannon, W. Weaver, *The Mathematical Theory of Communication* (Univ of Illinois Press, 1963)
19. F.R. Gantmakher, *Theory of matrices* (Science, Moscow, 1988) [in Russian]
20. R. A. Horn, C. R. Johnson, *Matrix Analysis* (Cambridge University Press, Cambridge, 2013)