Model of adequacy optimization of electric power systems under market conditions

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Abstract. We present a new model of adequacy optimization of electric power systems under market conditions in the article. Optimization is realized by a criteria of maximum of social welfare. Social welfare includes profits of generating companies according penalties of unreliable electricity supply, consumer surplus, costs for development and servicing of electricity grids. Adequacy analysis of variants of development of electric power system is based on multiple estimation of electricity shortage in a random hour of system work. We analyze system work in each of a random hour in two stages. For the first stage we define equilibrium electricity demand in each system node and equilibrium price of electricity according to Cournot model. For the next stage failures of power generating equipment and transmission lines are simulated. We also estimate of electricity shortage in a current hour on the second stage. Reliability indexes and profits of generating companies are formed after a whole cycle of computations. Values of a profit are depended on reliability of electricity supply. Simulation of random values is carried out by the Monte Carlo method.

1 Introduction

Problems of development of electric power systems were traditionally solved on a base of minimization of reduced costs [1, 2]. We should make a note of follow reliability minimization problems such as: a problem of attaining of a given values of reliability indexes with minimal costs [1]; a problem of minimization of reduced costs and the mathematical expectation of financial damage of electricity shortage [2]; a problem of maximization of reliability index under a given level of costs [2]. For solving these problems, they use information about aggregate electricity demand, a structure of generating equipment and configuration of a network of electric power system.

Modern electric power systems are liberalized in many countries. Liberalized energy markets have different features [3] which we must take into account under planning of development of electric power systems. [4]. A type of market interaction is a reason of a choice of behavior criteria of electricity market agents. They are: profit maximization of generating and network companies; maximization of social welfare; maximization of consumer utilities. A result of interaction of agents is market equilibrium with a price and a quantity of electricity supply and demand. This equilibrium is not perfect competitive. It is often oligopolistic. Herewith networks of electric power systems are a monopoly segment of electricity market.

Let's consider review of some adequacy optimization methods for power electric systems in market conditions. A mechanism of forming of payment for generating power was researched in [5]. The payment is a motivation for investment in development of electric power system under conditions of competitive electricity market. The given approach is based on using of reliability index LOLP (Loss of Load Probability) [6]. Cournot oligopoly model [7] is applied as an instrument of adequacy optimization in [8]. According to the model a profit of generating company depends on choosing of a volume of power reserve. But random nature of generation, consumption and transmission capacity of electricity lines are not taken into account in these models. Authors of articles [8, 9] consider scenarios of development of electric power systems and estimate of adequacy level for each scenario.

An aim of our research is to develop of a model of adequacy optimization of electric power system based on imitation of market interaction for each hour of system work and simulation of random values such as electricity demand and failures of electric equipment. We offer sequential search of the best of development variant of electric power system from a set of given variants. Search of the best variants is realized by a criteria of maximum of social welfare. Social welfare includes profits of generating companies according penalties of unreliable electricity supply, consumer surplus, costs for development and servicing of electricity grids. Adequacy analysis of variants of development of electric power system is based on multiple estimation of electricity shortage in a random hour of system work. We analyze system work in each of a random hour in two stages. For the first stage we define equilibrium electricity demand in each system node and equilibrium price of electricity according to

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Cournot model. For the next stage failures of power generating equipment and transmission lines are simulated. We also estimate of electricity shortage in a current hour on the second stage. Reliability indexes and profits of generating companies are formed after a whole cycle of computations.

2 Analysis and optimization of adequacy of electric power system

According to [10] adequacy analysis of electric power system is realized in the next order. First of all, set of random hours of system work is formed. After that electricity shortage is estimated for all simulated hours of system work. Finally, reliability indexes are computed.

Taking account of market conditions under adequacy analysis we have to modificate the given approach. Let’s consider that a simulated hour of electric power system work characterizes by set of random functions of electricity demand in system nodes. We assume a low elasticity of electricity demand. It is vindicable because we discuss a long-term period of developing of electric power system. Consequently, we can present a demand function as a linear.

Generating companies maximize their profit with orientation on function of total electricity demand and power capacity of their plants. We suggest to use Cournot model for simulation of described interaction on the electricity market for every hour [7, 11]. A price and quantity of electricity are found as a result of searching of equilibrium on electricity market.

Further we simulate failures of generating and network equipment which determine quantity of generated electricity and capacity of network lines. On this stage it is necessary to estimate of electricity shortage for every hour taking into account equilibrium values of electricity demand, generated electricity in system nodes and real capacity of transmission power lines. Difference between equilibrium and computed values of electricity demand is electricity shortage for the considered hour.

After all computations we define follow reliability indexes such as the probability of electricity shortage, the mathematical expectation of electricity deficit and the mathematical expectation of electricity price. The reliability fee is established on a base on these reliability indexes. We also compute profits of generating companies taking into account possible penalties of low reliability electricity supply.

Optimization of adequacy of electric power system is realized on a base of enumeration and adequacy analyzing of variants of development of electric power system. A variant of system development with highest level of social welfare is optimal. Social welfare is sum of profits of generating companies, consumer surpluses minus network development costs. We suggest that government is social planner which takes himself of network exploitation costs and costs of building of electric power lines.

3 Price setting in the model

Let’s consider a scheme of electric power system with n nodes and some set of system links. According to method of adequacy analysis of electric power system it is necessary multiple forming of computed hours of system work. Let N is a given number of modelled hours.

Inverse function of electricity demand is given for k-th hour, $k=1,\ldots,N$, in node i, $i=1,\ldots,n$,

$$p_i^k = \beta_i^k - \alpha_i y_i^k .$$

Here y_i^k is quantity of consumed electricity in a node i for k-th hour, kWh, p_i^k is electricity price in i-th node for one kWh, α_i is a positive coefficient of i-th demand function, β_i^k is meaning of random value β_i with given distribution law, for example, truncated normal law for interval $[\underline{\beta}_i; \overline{\beta}_i]$, $i=1,\ldots,n$, $k=1,\ldots,N$.

Let’s suggest that equilibrium price is equal for all system nodes $p^k = p(y^k)$ where y^k is total amount of consumed electricity for k-th hour

$$y^k = \sum_{i=1}^{n} y_i^k , \; k=1,\ldots,N . \tag{1}$$

We assume that S is amount of generating companies with generating capacities located in different nodes. All generating capacities are selected on competitive market of capacity. Formed price in model includes payment for provided capacity and consumed electricity. Generating company s produces x_{si}^k quantity of electricity in i-th node and x_s^k total amount of electricity along all nodes

$$x_s^k = \sum_{i=1}^{n} x_{si}^k , \; s=1,\ldots,S , \; k=1,\ldots,N .$$

Installed power $\overline{x}_{si}$ is given for every node i. Cost function for s-th generating company depends on volume of generated power $C_s(x_s^k)$, $s=1,\ldots,S$. Profit function of generating company s, $s=1,\ldots,S$, is written as

$$\pi_s(x_s^k, y^k) = p(y^k)x_s^k - C_s(x_s^k) . \tag{2}$$

For searching of equilibrium of Cournot-Nash it is necessary to maximize function (2) for every hour k, $k=1,\ldots,N$, subject to

$$\sum_{i=1}^{n} y_i^k = \sum_{s=1}^{S}\sum_{i=1}^{n} x_{si}^k , \tag{3}$$

$$0 \le x_{si}^k \le \overline{x}_{si}, \; i=1,\ldots,n. \tag{4}$$

Standard method for solving the problem (2) – (4) is Cournot tatonnement method [12]. The main idea of the method consists in sequential maximization of profit of every generating company with fixed values of generated electricity by others companies. Computational process converges if demand functions are linear and cost functions are quadratic [13].

Let’s denote by means of $\tilde{x}_{si}^k$, $\tilde{y}_i^k$ equilibrium in the model (2) – (4) in a k-th hour, $s=1,\ldots,S$,

$i = 1, \ldots, n$, $k = 1, \ldots, N$. From here according to (1) it is defined $\tilde{y}^k$ then price $p(\tilde{y}^k)$ and profits of generating companies $\pi_s(\tilde{x}_s^k, \tilde{y}^k)$ are computed, $s = 1, \ldots, S$, $k = 1, \ldots, N$.

It is important to note that we don't take into account network constraints in problem (2) – (4). These constraints essentially increase complexity of computation process. Electric power system is considered as a combined node on this stage of computations. Network constraints are taken into account on the second stage of developed approach under searching of electricity shortage in electric power system.

4 The model of electricity shortage estimation

After finding equilibrium in the model (2) – (4) it is necessary to compute optimal distribution of electricity consumption in system nodes for *k*-th hour, $k = 1, \ldots, N$. Herewith we have to simulate meanings of random values of maximal generated electricity $\bar{x}_i^k$ and random values of transmission capacities of power lines $\bar{z}_{ij}^k$ between nodes *i* and *j*, $i = 1, \ldots, n$, $j = 1, \ldots, n$, $i \neq j$.

For estimation of electricity shortage in computed hours we use the follow problem [11]. Generated electricity x_i in node *i*, kWh, consumed electricity y_i in node *i*, kWh, transmitted electricity z_{ij} from node *i* to node *j*, kWh, are variables in the problem, $i = 1, \ldots, n$, $j = 1, \ldots, n$. Problem of estimation of electricity shortage in *k*-th hour, $k = 1, \ldots, N$, is written as

$$\sum_{i=1}^{n} y_i \to \max, \quad (5)$$

$$x_i - y_i + \sum_{j=1}^{n} (1 - \gamma_{ji} z_{ji}) z_{ji} - \sum_{j=1}^{n} z_{ij} = 0, \; i = 1, \ldots, n, \quad (6)$$

$$0 \le y_i \le \tilde{y}_i^k, \; i = 1, \ldots, n, \quad (7)$$

$$0 \le x_i \le \bar{x}_i^k, \; i = 1, \ldots, n, \quad (8)$$

$$0 \le z_{ij} \le \bar{z}_{ij}^k, \; i = 1, \ldots, n, \; j = 1, \ldots, n, \; i \neq j. \quad (9)$$

Here γ_{ij} are positive coefficients of electricity losses under transmission from node *i* to node j, $i \neq j$.

As a rule, the adequacy analysis of electric power system is realized for year. Every hour of work of electric power system is modelled. The failures of generators and power lines are used as random parameters. Time of repair of equipment and fluctuations of load in the year are taken into account under simulating. The rules of simulations of random values such as available capacity of generator, transmission capacities of power lines or load value are discussed in [14].

Let set of $\hat{x}_i^k$, $\hat{y}_i^k$, $\hat{z}_{ij}^k$ is optimal solution of the problem (5) – (9), $k = 1, \ldots, N$, $i = 1, \ldots, n$, $j = 1, \ldots, n$, $i \neq j$. Value of electricity shortage in node *i*, $i = 1, \ldots, n$, according to optimal solution of the problem (5) – (9) is defined by the formula

$$w_i^k = \tilde{y}_i^k - \hat{y}_i^k, \; k = 1, \ldots, N.$$

An hour with number *k*, $k = 1, \ldots, N$, is deficit if value

$$w^k = \sum_{i=1}^{n} w_i^k$$

is different from zero. Let's numerate all deficit hours and denote their total amount by means of *H*. Then the probability of non-shortage work of electric power system is computed by the next formula

$$P = 1 - \frac{H}{N}.$$

The mathematical expectation of electricity shortage in nodes of electric power system is computed by the next rule

$$MW_i = \sum_{j=1}^{H} \frac{w_i^j}{N}, \; i = 1, \ldots, n.$$

That's way the mathematical expectation of electricity shortage in electric power systems is calculated by

$$MW = \sum_{i=1}^{n} MW_i .$$

The mathematical expectation of price is computed in the same way

$$MP = \frac{1}{N} \sum_{k=1}^{N} p(\tilde{y}^k).$$

The mathematical expectation of maximal price in every system node *i*, $i = 1, \ldots, n$, is found by $M\beta_i = \frac{1}{N} \sum_{k=1}^{N} \beta_i^k$.

5 Modelling of behavior of economic agents taking into account reliability level

As a result of the reliability estimation of every variant of equipment setting in electric power system we find a set of meanings of reliability indices. Willingness of consumers to pay for electricity depends on meanings of reliability indices. Let R is a some reliability index which characterizes fault rate, their duration and depth. In the general case, we can use n-dimensional vector instead of *R*. Components of the vector are meanings of some reliability index obtained for system nodes. We assume that values of the index *R* can not be below a certain level $\underline{R}$ and can not be above a level $\overline{R}$. Simulation and analysis of N computed hours for every scenario of development of electric power system allow to estimate volume of satisfied demand $\dot{y}_i$, volume of produced electricity $\dot{x}_i$ in every node and volume of electricity flows $\dot{z}_{ij}$ in the system

$$\dot{y}_i = \frac{1}{N}\sum_{k=1}^{N}\hat{y}_i^k\,,\ \dot{x}_i = \frac{1}{N}\sum_{k=1}^{N}\hat{x}_i^k\,,\ \dot{z}_{ij} = \frac{1}{N}\sum_{k=1}^{N}\hat{z}_{ij}^k\,,$$

$k = 1,\,...\,,\,N$, $i = 1,\,...\,,\,n$, $j = 1,\,...\,,\,n$, $i \neq j$.

Using these indexes, we evaluate of behavior of agents interacting in the electricity market.

Payment for reliability is price which consumer agrees to pay. The payment can be presented in the form of an increasing function $g(R)$. View of function $g(R)$ depends on choice of the reliability index.

5.1 Modelling of electricity demand

The reliability payment is defined either for every type of consumer or every system node. Computation of reliability indexes R_i, $i = 1,\,...\,,\,n$, is necessary for every system node in the last case. Price of high reliability electricity supply in node i is presented as function

$$g(R_i) = (R_i)^m MP. \qquad (10)$$

Here MP is the mathematical expectation of price of electricity supply without electricity shortages, m is parameter defining of importance of reliability factor for consumer.

5.2 Modelling of generating company behavior

Generating company with number s, $s = 1,\,...\,,S$, obtains profit depending of reliability indexes R_i, $i = 1,\,...\,,\,n$,

$$\pi_s(R) = \sum_{i=1}^{n} g(R_i)\dot{x}_{si} - \sum_{i=1}^{n} C_s\left(R_i,\,\dot{x}_{si}\right). \qquad (11)$$

Here R is n-dimensional vector with R_i components, $\dot{x}_{si}$ is the mathematical expectation of generated electricity of s-th generating company in node i, $C_s\left(R_i,\,\dot{x}_{si}\right)$ are operating and investment costs of generating company s needed for providing a certain level of reliability R_i and some volume of generated electricity $\dot{x}_{si}$ in a node i, $i = 1,...,n$, $s = 1,...,S$.

Generating company s, $s = 1,...,S$, solves problem of maximization of function (11) subject to constraints

$$\underline{R}_i \le R_i\,,\ i = 1,...,n\,, \qquad (12)$$

or

$$R_i \le \overline{R}_i\,,\ i = 1,...,n\,, \qquad (13)$$

depending on view of reliability index R_i.

5.3 Modelling of network company behavior

Network component seriously impacts on electricity price. Costs of building of new power lines are given for every variant of development of electric power system. We take into account these costs under computation of social welfare. This fact is explained of usual market structure where network companies are a natural monopoly segment regulated by government. Profit of a such company can be considered zero or normal. Electricity price can be more expensive due to network additive in condition of free market pricing.

5.4 Choice of the best development variant for electric power system

Development variant of electric power system is confessed the best if social welfare is maximal for this one. Therefore, problem of adequacy optimization is written as

$$SW = \sum_{s=1}^{S}\pi_s(R) + CS(\dot{y},\,R) - NC \to \max_R\,. \qquad (14)$$

Here NC is cost of network development, $CS(\dot{y},\,R)$ is consumer surplus which depends on reliability level of electricity supply and satisfied electricity demand in nodes. Consumer surplus is defined as difference between utility of electricity using and the mathematical expectation of electricity price

$$CS(R) = \sum_{i=1}^{n}\int_0^{\dot{y}_i}(M\beta_i - \alpha l_i)dl_i - \sum_{i=1}^{n}\dot{y}_i \cdot g\left(R_i\right).$$

The problem (14) is solved by enumeration of all possible development variants.

6 Conclusion

We suggest a new approach to adequacy optimization of electric power systems under market conditions supposing of competition of different agents of electricity market. This particularity determines impropriety of implementation of traditional methods for solving the problem.

Adequacy analysis of development variants of electric power system is based on method with using Monte Carlo method for simulation of random values. Analysis of adequacy is realized in two stages. On the first stage of computational process equilibrium values of electricity demand and electricity price are determined according to Cournot model and with taking into account random nature of demand functions. Failures of generating and network equipment are simulated on the second stage. Electricity shortage is also estimated for a current hour on this step. We compute reliability indexes, profits of generating companies taking into account payments for high reliability electricity supply at the end of computational process.

We apply the model with quadratic losses of electricity for estimation of electricity shortage in a computed hour. The given model guaranties of uniqueness of distribution of electricity shortage by system nodes and single-valuedness of reliability indexes.

Further it is necessary to concretize network company's role, realize representative computing experiment and compare different models of consumer behavior.

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