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Analysis of reliability of systems with component-wise storages

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Abstract. Time redundancy is a method of increasing the reliability and efficiency of the operation of systems for various purposes, in particular, energy systems. A system with time redundancy is given additional time (a time reserve) for restoring characteristics. In this paper, based on the theory of semi-Markov processes with a common phase space of states, a semi-Markov model of a two-component system with a component-wise instantly replenished time reserve is constructed. The stationary reliability characteristics of the system under consideration are determined.

1 Introduction

Most modern large systems have high reliability, but they are characterized by a significant impact of operator errors (the human factor) on the functioning and efficiency of the system. Time redundancy reduces the impact of such errors by providing additional time (a time reserve) to eliminate them without affecting the operation of the system.

Time redundancy [1-9] is a method of improving the reliability and efficiency of systems, in which the system in the process of functioning is given the opportunity to spend some additional time (time reserve) for restoring characteristics. For systems with time redundancy, a malfunction of the system is not necessarily accompanied by a system failure, since it is possible to restore the system's operability for a standby time.

A time reserve is called instantly replenished if the same time is allocated for the restoration of operability after the failure of any element, regardless of the number of previous failures and the time spent on their elimination. At the end of the recovery, such a time reserve is immediately replenished to the original level [1-3].

Time reserve in the systems of a power engineering [10-12] can be created by increase in power (efficiency, a channel capacity) the generating inventory extracting inventories, subsystems of transport of energy resources, electricity transmissions and other constituents of systems of a power engineering, by creation of internal stocks of the made or transported production, introduction of parallel devices for increase in total capacity, use of the functional inertance of systems and restricted speed of development of the processes caused by adverse effects of various physical nature.

Time redundancy is used in gas transmission systems in which underground gas storage is the source of the time reserve; in the electric power industry, the time reserve is realized at the expense of high-capacity energy storage devices [10, 11].

In connection with this, problems arise in determining the capacities of storage devices and their locations.

In this paper, based on the theory of semi-Markov processes with a common phase space of states [13-18], a semi-Markov model of a two-component system with a component-wise instantly replenished time reserve is constructed. Stationary reliability characteristics of the system are found, the effect of the time reserve on the characteristics obtained is analysed.

2 Description of the system functioning

The system S, consisting of two components, time to failure of which are random variables (RVs) α_i with the distribution functions (DFs) $F_i(x)$, a restoration times are RVs β_i with DFs $G_i(x)$, $i=\overline{1,2}$. Each component of the system has a random instantly replenished time reserve τ_i with DF $R_i(x)$. RVs α_i, β_i, τ_i are assumed to be independent in aggregate, having finite mathematical expectations; DFs $F_i(x)$, $G_i(x)$, $R_i(x)$ have distribution densities $f_i(x)$, $g_i(x)$, $r_i(x)$.

The time reserve starts to be used at the time the component begins to recover. The failure of system S occurs when both elements are restored and the time reserve for each element is completely spent. It continues until the restoration of one of the failed elements, while the time reserve of the restored element is instantly replenished to the level τ_i (instantly replenished time reserve).

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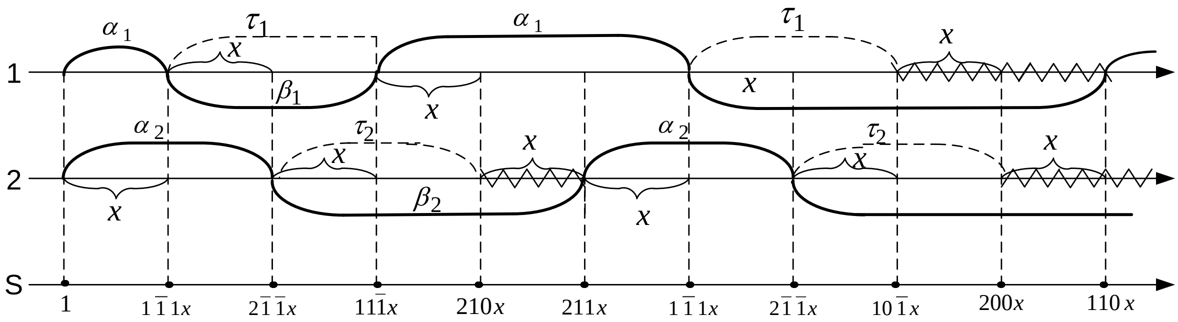

Fig. 1. Time diagram of the functioning of the system S .

3 Semi-Markov model building

To describe the functioning of the system S , we use the semi-Markov process $\xi(t)$ [7-10]. We introduce the space of states of the form:

$$E=\{1,\ i\bar{d}x:\bar{d}=(d_1,d_2),\ x>0\}, \tag{1}$$

where $i=\overline{1,2}$ is the number of the component in which the state change occurred. Component d_k of the vector $\bar{d}$ describes the physical state of the element with the number k :

$$d_k=\begin{cases}0, \text{ if k}-th\text{ component is in failure,}\\ 1, \text{ if k}-th\text{ component is operational,}\\ \bar{1}, \text{ if k}-th\text{ component is restored and}\\ \quad\text{functions due to the time reserve.}\end{cases}$$

The continuous component x indicates the elapsed time since the last change in the system state. The time diagram of the functioning of the described system is shown in Figure 1.

We construct the Markov renewal process (MRP) $\{\xi_n,\theta_n:\text{n}\geq 0\}$ that describes the functioning of the system S .

We define the transition probabilities of the embedded Markov chain (EMC) $\{\xi_n:\text{n}\geq 0\}$. Introduce the notation:

$$\bar{F}(t)=1-F(t)\ ,\ \bar{i}=\begin{cases}2,\ i=1,\\ 1,\ i=2.\end{cases}.$$

1. Let us consider the case of states id_1d_2x , $d_i=\bar{1}$, $d_{\bar{i}}=1$.

In this case, the following transitions are possible:

a) $id_1d_2x\rightarrow id_1'd_2'y$ in conditions: $d_k'=d_k$ at $k\neq i$, $d_i'=1$, $y>x$. The transition probability density in this case is calculated from the formula:

$$p_{id_1d_2x}^{id_1'd_2'y}=\frac{g_i(y-x)\bar{R}_i(y-x)\bar{F}_{\bar{i}}(y)}{\bar{F}_{\bar{i}}(x)}, \tag{2}$$

b) $id_1d_2x\rightarrow id_1'd_2'y$ in conditions: $d_k'=d_k$ at $k\neq i$, $d_i'=0$, $y>x$. The transition probability density in this case has the form:

$$p_{id_1d_2x}^{id_1'd_2'y}=\frac{r_i(y-x)\bar{G}_i(y-x)\bar{F}_{\bar{i}}(y)}{\bar{F}_{\bar{i}}(x)}, \tag{3}$$

c) $id_1d_2x\rightarrow \bar{i}d_1'd_2'y$ in conditions: $d_k'=d_k$ at $k\neq i$, $d_{\bar{i}}'=\bar{1}$, $y>0$. In this case, the probability density of the transition is

$$p_{id_1d_2x}^{\bar{i}d_1'd_2'y}=\frac{f_{\bar{i}}(y+x)\bar{G}_i(y)\bar{R}_i(y)}{\bar{F}_{\bar{i}}(x)}. \tag{4}$$

2. Let us consider the case of states id_1d_2x , $d_1=d_2=0$.

In this case, the following transitions are possible:

a) $id_1d_2x\rightarrow id_1'd_2'y$ in conditions: $d_k'=d_k$ at $k\neq i$, $d_i'=1$, $y>x$. The transition probability density is calculated by the formula:

$$p_{id_1d_2x}^{id_1'd_2'y}=\frac{\int_0^\infty r_i(t)g_i(y-x+t)dt\int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(y+t)dt}{P(\beta_i>\tau_i)\int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(x+t)dt}, \tag{5}$$

b) $id_1d_2x\rightarrow \bar{i}d_1'd_2'y$ in conditions: $d_k'=d_k$ at $k\neq i$, $d_{\bar{i}}'=1$, $y>0$. The transition probability density will be:

$$p_{id_1d_2x}^{\bar{i}d_1'd_2'y}=\frac{\int_0^\infty r_{\bar{i}}(t)g_{\bar{i}}(y-x+t)dt\int_0^\infty r_i(t)\bar{G}_i(y+t)dt}{P(\beta_i>\tau_i)\int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(x+t)dt}. \tag{6}$$

3. Consider the case of states id_1d_2x , $d_1=d_2=\bar{1}$.

a) $id_1d_2x \to id_1'd_2'y$ in conditions: $d_k' = d_k$ at $k \neq i$, $d_i' = 1$, $y > x$. The transition probability density in this case is calculated from the formula:

$$p_{id_1d_2x}^{id_1'd_2'y} = \frac{g_i(y-x)\bar{R}_i(y-x)\bar{G}_{\bar{i}}(y)\bar{R}_{\bar{i}}(y)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)}, \quad (7)$$

b) $id_1d_2x \to id_1'd_2'y$ in conditions: $d_k' = d_k$ at $k \neq i$, $d_i' = 0$, $y > x$. In this case, the transition probability density is calculated by the formula:

$$p_{id_1d_2x}^{id_1'd_2'y} = \frac{r_i(y-x)\bar{G}_i(y-x)\bar{G}_{\bar{i}}(y)\bar{R}_{\bar{i}}(y)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)}, \quad (8)$$

c) $id_1d_2x \to \bar{i}d_1'd_2'y$ in conditions: $d_k' = d_k$ at $k \neq i$, $d_{\bar{i}}' = 1$, $y > 0$. The transition probability density is

$$p_{id_1d_2x}^{\bar{i}d_1'd_2'y} = \frac{g_{\bar{i}}(y+x)\bar{R}_{\bar{i}}(y+x)\bar{G}_i(y)\bar{R}_i(y)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)}, \quad (9)$$

d) $id_1d_2x \to \bar{i}d_1'd_2'y$ in conditions: $d_k' = d_k$ at $k \neq i$, $d_{\bar{i}}' = 0$, $y > 0$. The transition probability density is found from formula:

$$p_{id_1d_2x}^{\bar{i}d_1'd_2'y} = \frac{r_{\bar{i}}(y+x)\bar{G}_{\bar{i}}(y+x)\bar{G}_i(y)\bar{R}_i(y)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)}. \quad (10)$$

For the remaining states of the system, the transition probabilities are determined in a similar manner.

4 Finding the characteristics of the system

To determine the stationary reliability characteristics of the system, we find the stationary distribution of the EMC $\{\xi_n : \mathrm{n} \geq 0\}$.

Suppose that for a stationary distribution of EMC $\{\xi_n : \mathrm{n} \geq 0\}$ exist densities $\rho(id_1d_2x)$.

We introduce the following substitutions:

$$\tilde{\rho}(id_1d_2x) = \frac{\rho(id_1d_2x)}{\bar{F}_{\bar{i}}(x)}, \quad d_{\bar{i}} = 1,$$

$$\tilde{\rho}(id_1d_2x) = \frac{\rho(id_1d_2x)}{\bar{R}_{\bar{i}}(x)\bar{G}_{\bar{i}}(x)}, \quad d_{\bar{i}} = \bar{1},$$

$$\tilde{\rho}(id_1d_2x) = \frac{\rho(id_1d_2x)}{\dfrac{\int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(x+t)dt}{P(\beta_{\bar{i}} > \tau_{\bar{i}})}}, \quad d_{\bar{i}} = 0.$$

The system of integral equations for the stationary distribution of the EMC $\{\xi_n : \mathrm{n} \geq 0\}$ has the following form:

1. If $d_i' = 1$, $d_{\bar{i}}' = d_{\bar{i}}$, then

$$\tilde{\rho}(id_1d_2x) = \int_0^x \tilde{\rho}(id_1'd_2'y)\, f_i(x-y)dy + \int_0^\infty \tilde{\rho}(id_1'd_2'y)\, f_i(x+y)dy, \quad (11)$$

2. If $d_i' = \bar{1}$, $d_{\bar{i}}' = d_{\bar{i}}$, then

$$\tilde{\rho}(id_1d_2x) = \int_0^x \tilde{\rho}(id_1'd_2'y) r_i(x-y)\bar{G}_i(x-y)dy + \int_0^\infty \tilde{\rho}(id_1'd_2'y) r_i(x+y)\bar{G}_i(x+y)dy, \quad (12)$$

3. If $d_i' = \bar{1}$, $d_{\bar{i}}' = d_{\bar{i}} = 0$, then

$$\tilde{\rho}(id_1d_2x) = \int_0^x \tilde{\rho}(id_1'd_2'y) g_i(x-y)\bar{R}_i(x-y)dy + \int_0^\infty \tilde{\rho}(id_1'd_2'y) g_i(x+y)\bar{R}_i(x+y)dy, \quad (13)$$

4. If $d_i = 1$, $d_i' = \bar{1}$, $d_i'' = 0$, $d_{\bar{i}}' = d_{\bar{i}}'' = d_{\bar{i}} \neq 0$, then

$$\tilde{\rho}(id_1d_2x) = \int_0^x \tilde{\rho}(id_1'd_2'y) g_i(x-y)\bar{R}_i(x-y)dy + \int_0^\infty \tilde{\rho}(id_1'd_2'y) g_i(x+y)\bar{R}_i(x+y)dy + \int_0^x \tilde{\rho}(id_1''d_2''y)dy \frac{\int_0^\infty r_i(t) g_i(x-y+t)dt}{P(\beta_i > \tau_i)} + \int_0^\infty \tilde{\rho}(id_1''d_2''y)dy \frac{\int_0^\infty r_i(t) g_i(x+y+t)dt}{P(\beta_i > \tau_i)}, \quad (14)$$

5. $\int_E \rho(i\bar{d}x)dx = 1.$ (normalization condition). (15)

By substitution, one can verify that the solution of the system of equations for the stationary distribution of the EMC has the form:

$$\rho(id_1d_2x) = c\rho_i\rho_{\bar{i}}\bar{F}_{\bar{i}}(x), \quad d_{\bar{i}} = 1, \quad (16)$$

$$\rho(id_1d_2x) = c\rho_i\rho_{\bar{i}}\, p_i \bar{R}_{\bar{i}}(x)\bar{G}_{\bar{i}}(x), \quad d_{\bar{i}} = \bar{1}, \quad (17)$$

$$\rho(id_1d_2x) = \frac{c\rho_i\rho_{\bar{i}}\, p_{\bar{i}} \int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(x+t)dt}{P(\beta_{\bar{i}} > \tau_{\bar{i}})}, \quad d_{\bar{i}} = 0, \quad (18)$$

$$\rho(id_1d_2x) = \frac{c\rho_i\rho_{\bar{i}}\, p_i p_{\bar{i}} \int_0^\infty r_{\bar{i}}(t)\bar{G}_{\bar{i}}(x+t)dt}{P(\beta_{\bar{i}} > \tau_{\bar{i}})}, d_i = d_{\bar{i}} = 0, \quad (19)$$

where $p_i = P(\beta_i > \tau_i) = \int_0^\infty \bar{G}_i(t) r_i(t) dt$, $\rho_i = const$, which is equal to $\rho_i = \frac{1}{2+p_i}$, the constant c is found from the normalization condition.

We calculate the mean residence times in the states of the EMC.

1. When $d_i = d_{\bar{i}} = 1$,

$$\theta_{id_1d_2x} = \alpha_i \wedge [\alpha_{\bar{i}} - x]^+, \; E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{F}_i(t)\bar{F}_{\bar{i}}(x+t)}{\bar{F}_{\bar{i}}(x)} dt .$$

2. In case of $d_i = \bar{1}$, $d_{\bar{i}} = 1$,

$$\theta_{id_1d_2x} = \beta_i \wedge \tau_i \wedge [\alpha_{\bar{i}} - x]^+ ,$$

$$E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{G}_i(t)\bar{R}_i(t)\bar{F}_{\bar{i}}(x+t)}{\bar{F}_{\bar{i}}(x)} dt .$$

3. If $d_i = d_{\bar{i}} = \bar{1}$, then

$$\theta_{id_1d_2x} = \beta_i \wedge \tau_i \wedge [\beta_{\bar{i}} - x]^+ \wedge [\tau_{\bar{i}} - x]^+ ,$$

$$E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{G}_i(t)\bar{R}_i(t)\bar{G}_{\bar{i}}(x+t)\bar{R}_{\bar{i}}(x+t)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)} dt .$$

4. When $d_i = 0$, $d_{\bar{i}} = 1$,

$$\theta_{id_1d_2x} = [\beta_i - \tau_i]^+ \wedge [\alpha_{\bar{i}} - x]^+ ,$$

$$E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{F}_{\bar{i}}(x+t)}{\bar{F}_{\bar{i}}(x) P(\beta_i > \tau_i)} dt \int_0^\infty r_i(z)\bar{G}_i(t+z)dz .$$

5. If $d_i = 1$, $d_{\bar{i}} = \bar{1}$, then

$$\theta_{id_1d_2x} = \alpha_i \wedge [\beta_{\bar{i}} - x]^+ \wedge [\tau_{\bar{i}} - x]^+ ,$$

$$E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{F}_i(t)\bar{G}_{\bar{i}}(x+t)\bar{R}_{\bar{i}}(x+t)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x)} dt .$$

6. In case of $d_i = 0$, $d_{\bar{i}} = \bar{1}$,

$$\theta_{id_1d_2x} = [\beta_i - \tau_i]^+ \wedge [\beta_{\bar{i}} - x]^+ \wedge [\tau_{\bar{i}} - x]^+ ,$$

$$E\theta_{id_1d_2x} = \int_0^\infty \frac{\bar{G}_{\bar{i}}(x+t)\bar{R}_{\bar{i}}(x+t)}{\bar{G}_{\bar{i}}(x)\bar{R}_{\bar{i}}(x) P(\beta_i > \tau_i)} dt \int_0^\infty r_i(z)\bar{G}_i(t+z)dz.$$

7. When $d_i = \bar{1}$, $d_{\bar{i}} = 0$,

$$\theta_{id_1d_2x} = \tau_i \wedge \beta_i \wedge \left[[\beta_{\bar{i}} - \tau_{\bar{i}}]^+ - x \right]^+ ,$$

$$E\theta_{id_1d_2x} = \frac{\int_0^\infty \bar{G}_i(t)\bar{R}_i(t)dt \int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+t+z)dz}{\int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+z)dz} .$$

8. If $d_i = 1$, $d_{\bar{i}} = 0$, then

$$\theta_{id_1d_2x} = \alpha_i \wedge \left[[\beta_{\bar{i}} - \tau_{\bar{i}}]^+ - x \right]^+ ,$$

$$E\theta_{id_1d_2x} = \frac{\int_0^\infty \bar{F}_i(t)dt \int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+t+z)dz}{\int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+z)dz} .$$

9. In case of $d_i = d_{\bar{i}} = 0$,

$$\theta_{id_1d_2x} = [\beta_i - \tau_i]^+ \wedge \left[[\beta_{\bar{i}} - \tau_{\bar{i}}]^+ - x \right]^+ ,$$

$$E\theta_{id_1d_2x} = \frac{\int_0^\infty dt \int_0^\infty r_i(y)\bar{G}_{\bar{i}}(t+y)dy \int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+t+z)dz}{P(\beta_i > \tau_i)\int_0^\infty r_{\bar{i}}(z)\bar{G}_{\bar{i}}(x+z)dz},$$

where $\wedge$ is the minimum sign.

RV $[\alpha - x]^+$ is the residual residence time of the semi-Markov process $\xi(t)$ in the state, provided that the residence time in this state exceeded the value x.

Then $P\{[\alpha - x]^+ > t\} = \frac{\bar{F}(x+t)}{\bar{F}(x)}$ and

$$P\{[\alpha - x]^+ \in dt\} = \frac{f(x+t)}{\bar{F}(x)} .$$

RV $[\beta - \tau]^+$ is given by the following probabilities:

$$P\{[\beta - \tau]^+ > t\} = \frac{\int_0^\infty r(z)\bar{G}(t+z)dz}{P(\beta > \tau)} ,$$

$$P\{[\beta - \tau]^+ \in dt\} = \frac{\int_0^\infty r(z)g(t+z)dz}{P(\beta > \tau)} .$$

Define a random variable $\left[[\beta_{\bar{i}} - \tau_{\bar{i}}]^+ - x \right]^+$ by

$$P\{\left[[\beta - \tau]^+ - x \right]^+ > t\} = \frac{\int_0^\infty r(z)\bar{G}(t+x+z)dz}{\int_0^\infty r(z)\bar{G}(x+z)dz},$$

$$P\{\left[[\beta-\tau]^{+}-x\right]^{+}\in dt\}=\frac{\int_0^\infty r(z)g(t+x+z)dz}{\int_0^\infty r(z)\bar{G}(x+z)dz}.$$

Let us turn to the determination of the stationary reliability characteristics of the system S: mean stationary operating time of the system to failure T_+, mean stationary restoration time T_- and stationary availability factor K_a of the system.

To find the characteristics, we use the following formulas presented in [18]:

$$T_+=\frac{\int_{E_+} m(e)\rho(de)}{\int_{E_+} P(e,E_-)\rho(de)},\quad T_-=\frac{\int_{E_-} m(e)\rho(de)}{\int_{E_+} P(e,E_-)\rho(de)},$$

$$K_a=\frac{T_+}{T_+ + T_-}, \tag{20}$$

where $\rho(de)$ is stationary distribution of EMC $\{\xi_n; n\ge 0\}$, $P(e,E_-)$ – transition probabilities of the EMC $\{\xi_n; n\ge 0\}$ to a subset of fault states, $m(e)$ is mean residence time of the semi-Markov process in the state $e\in E$.

Consider the parallel connection of system components. In this case

$$E_-=\{100x,\ 200x\},$$

$$E_+=\{i\bar{d}x:\bar{d}=(d_1,d_2),\ \bar{d}\ne(0,0),\ x>0\}.$$

We calculate the components of formulas (20) using solutions of the system of integral equations (16) - (19), the transition probabilities of the embedded Markov chain (2) – (10) and average residence times in the states obtained above.

In the transformations, we will use the following formula, proved in [19]:

$$\sum_{j=1}^{n}\int_0^\infty\!\!\dots\!\int_0^\infty \bar{F}_j(t)\left(\prod_{\substack{k=1,\\ k\ne j}}^{n}\bar{F}_k(t+y_k)dy_k\right)dt=\prod_{j=1}^{n}E\alpha_j.$$

where $F_k(t)$ are distribution functions of independent nonnegative random variables α_k with mathematical expectations $E\alpha_k$.

We write out expressions for the numerator and denominator for mean stationary operating time of the system to failure and mean stationary restoration time.

$$\int_{E_-} m(e)\rho(de)=$$
$$=c\rho_1\rho_2\int_0^\infty dx\int_0^\infty dt\int_0^\infty r_2(y)\bar{G}_2(y+t)dy\int_0^\infty r_1(z)\bar{G}_1(y+x+z)dz+$$
$$+c\rho_1\rho_2\int_0^\infty dx\int_0^\infty dt\int_0^\infty r_1(y)\bar{G}_1(y+t)dy\int_0^\infty r_2(z)\bar{G}_2(y+x+z)dz=$$
$$=c\rho_1\rho_2 p_1 p_2 E\left([\beta_1-\tau_1]^{+}\right)E\left([\beta_2-\tau_2]^{+}\right),$$

$$\int_{E_+} P(e,E_-)\rho(de)=$$
$$=c\rho_1\rho_2\int_0^\infty dx\int_0^\infty r_2(x+y)\bar{G}_2(x+y)dy\int_0^\infty r_1(t)\bar{G}_1(y+t)dt+$$
$$+c\rho_1\rho_2\int_0^\infty dx\int_0^\infty r_1(x+y)\bar{G}_1(x+y)dy\int_0^\infty r_2(t)\bar{G}_2(y+t)dt+$$
$$+c\rho_1\rho_2\int_0^\infty dx\int_0^\infty r_2(y)\bar{G}_2(y)dy\int_0^\infty r_1(t)\bar{G}_1(y+x+t)dt+$$
$$+c\rho_1\rho_2\int_0^\infty dx\int_0^\infty r_1(y)\bar{G}_1(y)dy\int_0^\infty r_2(t)\bar{G}_2(y+x+t)dt=$$
$$=c\rho_1\rho_2\left[p_1\int_0^\infty \bar{G}_2(x)R_2(x)dx+p_2\int_0^\infty \bar{G}_1(x)R_1(x)dx\right],$$

$$\int_{E_+} m(e)\rho(de)=$$
$$=c\rho_1\rho_2\left[p_1E\left([\beta_1-\tau_1]^{+}\right)\left(E\alpha_2+E(\beta_2\wedge\tau_2)\right)+\right.$$
$$+p_2E\left([\beta_2-\tau_2]^{+}\right)\left(E\alpha_1+E(\beta_1\wedge\tau_1)\right)+$$
$$\left.+\left(E\alpha_1+E(\beta_1\wedge\tau_1)\right)\left(E\alpha_2+E(\beta_2\wedge\tau_2)\right)\right].$$

Let's write down formulas for finding of stationary characteristics of reliability:

$$T_-=\frac{c\rho_1\rho_2 p_1 p_2 E\left([\beta_1-\tau_1]^{+}\right)E\left([\beta_2-\tau_2]^{+}\right)}{c\rho_1\rho_2\left[p_1\int_0^\infty \bar{G}_2(x)R_2(x)dx+p_2\int_0^\infty \bar{G}_1(x)R_1(x)dx\right]}= \tag{21}$$
$$=\frac{E\left([\beta_1-\tau_1]^{+}\right)E\left([\beta_2-\tau_2]^{+}\right)}{E\left([\beta_1-\tau_1]^{+}\right)+E\left([\beta_2-\tau_2]^{+}\right)},$$

$$T_+=\frac{p_1E\left([\beta_1-\tau_1]^{+}\right)\left(E\alpha_2+E(\beta_2\wedge\tau_2)\right)}{p_1p_2\left(E\left([\beta_1-\tau_1]^{+}\right)+E\left([\beta_2-\tau_2]^{+}\right)\right)}+$$
$$+\frac{p_2E\left([\beta_2-\tau_2]^{+}\right)\left(E\alpha_1+E(\beta_1\wedge\tau_1)\right)}{p_1p_2\left(E\left([\beta_1-\tau_1]^{+}\right)+E\left([\beta_2-\tau_2]^{+}\right)\right)}+ \tag{22}$$
$$+\frac{\left(E\alpha_1+E(\beta_1\wedge\tau_1)\right)\left(E\alpha_2+E(\beta_2\wedge\tau_2)\right)}{p_1p_2\left(E\left([\beta_1-\tau_1]^{+}\right)+E\left([\beta_2-\tau_2]^{+}\right)\right)}.$$

As an example of the use of formulas (21), (22), consider a system in which K_1 operating time $E\alpha_1$ = 8 h, K_2 operating time $E\alpha_2$ = 6 h, K_1 recovery time $E\beta_1$ = 0.71 h, K_2 recovery time $E\beta_2$ = 0.83 h, RV α_1, α_2,

β_1, β_2 have 5th order Erlang distribution. Each component has a non-random time reserve ($R_i(t)=1(t-h_i)$), which varies from 0 to 0.7 hours in 0.1 increments. The corresponding values of mean stationary operating time of the system to failure $T_+(h_1,h_2)$, mean stationary restoration time $T_-(h_1,h_2)$ and stationary availability factor $K_a(h_1,h_2)$ of the system for the specified distribution were calculated. The results are presented in Table 1, calculated on the condition that $h_1+h_2=0.7$.

Table 1. The influence of the time reserve on the system reliability characteristics.

h_1	h_2	$T_-(h_1,h_2)$	$T_+(h_1,h_2)$	$K_c(h_1,h_2)$
0	0	0.385	49.551	0.9923
0	0.7	0.241	102.274	0.99765
0.1	0.6	0.241	91.035	0.99736
0.2	0.5	0.238	84.365	0.99719
0.3	0.4	0.233	82.291	0.99717
0.4	0.3	0.228	85.172	0.99733
0.5	0.2	0.224	93.617	0.99762
0.6	0.1	0.219	108.69	0.99799
0.7	0	0.214	132.3	0.99839

Analysis of the data in the table shows the significant effect of the time reserve on reliability characteristics.

5 Conclusion

In this paper we construct a semi-Markov model of a two-component system with a component-wise random instantly replenished time reserve. On a concrete example, influence of capacities of component-wise stores on stationary characteristics of reliability of system is shown. The effect of the time reserve on the reliability characteristics obtained is analysed.

In the future, it is planned to build semi-Markov models of multicomponent systems with a component-wise time reserve.

The results of this work can be used to construct semi-Markov models of systems with different types and strategies for using the time reserve, engineering calculations and solving optimization problems associated with the use of a time reserve.

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