Fluctuations and nonlinear oscillations in complex natural systems

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Abstract. Resonance propagation of radiation in the ionosphere, solar activity, magnetic dynamos, lightning discharges, fracture processes, plastic deformations, seismicity, turbulence and hydrochemical variability are considered as examples of complex dynamical systems in which similar fluctuation and nonlinear oscillation regimes arise. Collective effects in the systems behavior and chaotic oscillations in individual subsystems, the ratio of random and deterministic, the analysis of variability factors and the change of dynamic regimes, the scaling relation between the elements of the system and the interaction of scales are discussed. It is shown that consolidation and branching in disruptions or thunderstorm activity is the transfer of disturbances to up and down of cascades as in turbulence, and the alpha-omega effects of the magnetic dynamo are the same cascade processes, but in the presence of an external magnetic field or rotation that removes the degeneracy in the system by directions. Particular attention is paid to natural generators and oscillation amplifiers, in which the Lorentz triplet plays the role of a universal model of a nonlinear oscillator.

1 Introduction

The certain period in the development of oscillation theory was began with the investigation of non linear dissipative systems, the more famous example of which is the Lorentz's system. The triplet of Lorentz's equations began to appear in various applications so often that it can be considered as an example of a fundamental nonlinear oscillator, the typical regimes of which are used for the description of very different systems (convection, laser, magnetic dynamo and solar activity, resonant propagation and scattering, acoustic and electromagnetic emission, lightning and discharge, fracture and seismicity, chemical reactions (type of Belousov-Zhabotinsky), biogeochemical variability and so on).

The image of the fundamental nonlinear oscillator is the motion on the sphere of unit radius (spinor). The typical oscillation regimes are defined by the Lorentz or Maxwell-Bloch equations:

$$\dot{x} = \sigma(y - z), \quad \dot{y} = rx - y - z, \quad \dot{z} = -bz + xy. \tag{1}$$

For the Lorentz equations x and y are a speed and a temperature structural modes and z is a deviation of temperature gradient from the equilibrium value. For the Maxwell-Bloch equations x and y are an electric field and a polarization and z is a deviation of the two level atomic system from the equilibrium state.

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The inversion z (overturn of oscillator) plays an impotent role for nonlinear systems. When inversion is changed slowly ($\dot{z} \approx 0$) then $z \approx xy/b$ in (1) gives an analog of Van der Pol oscillator. Integration of the third equation (1) and substitution of the result in the second equation (1) give the analog of Van der Pol oscillator with a memory which may be easily generalized on a fractal case.

Near the equilibrium point $z \approx -1$ the equations (1) give the linear oscillator near the rest point, $z \approx 0$ – near the equator and $z \approx 1$ – near the nonequilibrium point.

All these regimes of oscillations are useful for understanding of a behavior of complex systems. We will consider the magnetic dynamo generated by convection where the fundamental nonlinear oscillator appears twice as the Lorentz and Maxwell-Bloch triplets. We will show how much the oscillation modes are contained in more simple case. We will discuss the selection rules. Then we will show how the nonlinear oscillator is used for the description of acoustic and electromagnetic emission. Finally, we will discuss the biochemical variability which is important for understanding living systems.

2 Magnetic dynamo as atom in the resonator

The source of the Dynamo wave is a helicity. In thick spherical rotating convective layer, the role of the atom is played the doublet of complex amplitudes of structural velocity modes ${}^{P}V_4^2$ and ${}^{T}V_4^2$ with $W = \left|{}^{P}V_4^2\right|^2 + \left|{}^{T}V_4^2\right|^2$, $C = \left|{}^{P}V_4^2\right|^2 + \left|{}^{T}V_4^2\right|^2$ and $S = {}^{P}V_4^2\left({}^{T}V_4^2\right)^*$ – the energy, inversion and helicity. Here the indices P and T denote poloidal and toroidal modes, respectively. The upper and lower indices to the right of the mode designations - are the standart spherical indices. The role of the field in resonator is played the same doublet of magnetic modes ${}^{P}B_4^2$ and ${}^{T}B_4^2$. The magnetic helicity is the polarization of the field and the velocity helicity is like the atomic polarization.

The choice of a pair of spherical indices (4,2) is due to the fact that the corresponding convective cells in a thick spherical layer have the same vertical and horizontal dimensions. Therefore, such convection is structurally stable.

We defined the ${}^{P}V_4^2$ mode as nonequilibrium. Symmetry breaking in the doublet ${}^{P,T}V_4^2$ is caused by rotation. In the ${}^{P,T}B_4^2$ doublet, the ${}^{P}B_4^2$ mode is equilibrium. We have the oscillations in the coupling pair of doublets of ${}^{P,T}V_4^2$ and of ${}^{P,T}B_4^2$.

The doublets ${}^{P,T}V_4^2$ and of ${}^{P,T}B_4^2$ are the source of the Dynamo wave (spin wave) which transfers the oscillations to other doublets including the doublet of the magnetic dipole ${}^{P,T}B_1^0$. This is the mechanism of α- dynamo which is realized through the α-doublets.

The Dynamo wave gives the phase shift in oscillations of ${}^{P,T}V_4^2$ and of ${}^{P,T}B_1^0$. This is one disk dynamo [1]. The set of ${}^{P,T}V_4^2$, ${}^{P,T}V_4^3$ and ${}^{P,T}B_1^0$ is two disk dynamo [2] and so on. In thin convective layer as on the Sun, number of modes in the Dynamo wave is large and WKB approximation may be used [3]. Fist the solar activity was considered as the Van der Pol oscillator [4]. The chaos in solar activity was discussed in [5] and equations (1) were used.

The pair of doublets of ${}^{P,T}V_4^2$ and ${}^{P,T}V_4^3$ (main and additional velocity doublets) gives the differential rotation and structural inversion. This is the mechanism of the ω-dynamo. The pair ${}^{P,T}V_4^2$ and ${}^{P,T}V_4^3$ describes the instability of Benard cells which are the main discrete elements of convections and dynamo. On the Sun, the convective zone has some layers and there are some Dynamo waves with different periods.

Nonstationarity of convection is represented by the oscillations of the temperature modes T_4^2 and T_4^3 with temperature inversion ${}_1T_0$ which will be like the atomic inversion if it will be normalized per the thickness of the convection layer and the equilibrium temperature gradient. The temperature modes T_4^2, T_4^3, ${}_1T_0$ and the velocity modes ${}^{P,T}V_4^2$ and ${}^{P,T}V_4^3$ are the sources of Convection wave which creates the Dynamo wave. The stop of the layer rotation gives the degeneration of toroidal velocity modes and the disappearance of Dynamo wave sources. The mode selection rules are presented at the fig. 1. The main influence area of (4,2) and (4,3) modes gives the modes number in the Dynamo-Convection wave. In more simple case, we can account only two diagonals. The mode equations for Convection-Dynamo waves are obtained by Galerkin method [6, 7].

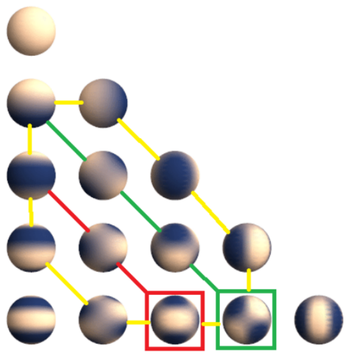

Figure 1. Red lines are diagonal of (4,2) mode, Green lines are diagonal of (4,3) mode, Yellow lines are the main influence area of (4,2) and (4,3) modes.

Note, the pair of (4,2) and (4,3) modes (ω-pair unlike α-doublet discussed above) is formed by a pair of eigenfunctions of the orbital moment with different parity. It's possible to introduce the inversion for this pair. Fig. 1 shows that this inversion is responsible for the structural changes of the magnetic field. Thus the instability of Benard cells is responsible for the changes of the magnetic field structure while the helicity is responsible for the generation of the magnetic field. Accounting the oscillation energy in the modes, we can say that the omega pair and alpha doublet splitting are the thin and super thin splitting like in atomic physics.

3 Resonance in acoustic and electromagnetic emissions

The sources of acoustic and electromagnetic emissions in the stressed medium can be entered as the non linear inverted oscillators, which are distributed on energy in the dependence on the stress and fractal properties of the medium. It's like in multilevel atom.

The role of the non linear oscillators is played the dislocations. The role of the Bloch vector is played the Burgers vector normalized per the size of the dislocation. This dimensionless vector is the characteristic of relative deformations. Using screw dislocations, the spirality of deformations may be introduced for the description of the acoustic and electromagnetic signals generation. In the result, the non linear oscillation effect in acoustic and electromagnetic emissions may be considered as it was made above for Dynamo.

The block structure of the medium creates the acoustic resonator system. The ionosphere and skin layer of the earth create the electromagnetic resonator system.

In this presentation, the resonant effects of acoustic and electromagnetic emissions may be researched by the quantum optics methods [8]. The electrical discharges and earthquake precursors may be researched too.

4 The complex natural systems

Let consider the biogeochemical variability [9] which is described by nonlinear dissipative systems [10]. As a rule, these systems are high-dimensional that makes the difficulty for their researches. However, the reducing of the high-dimensionality is possible using different relations so as the chemical parameters ratios [11], fractality of hydrochemical variability [12-14], allometry [15], flicker effects [16], the spectrum of turbulence or tides and so on.

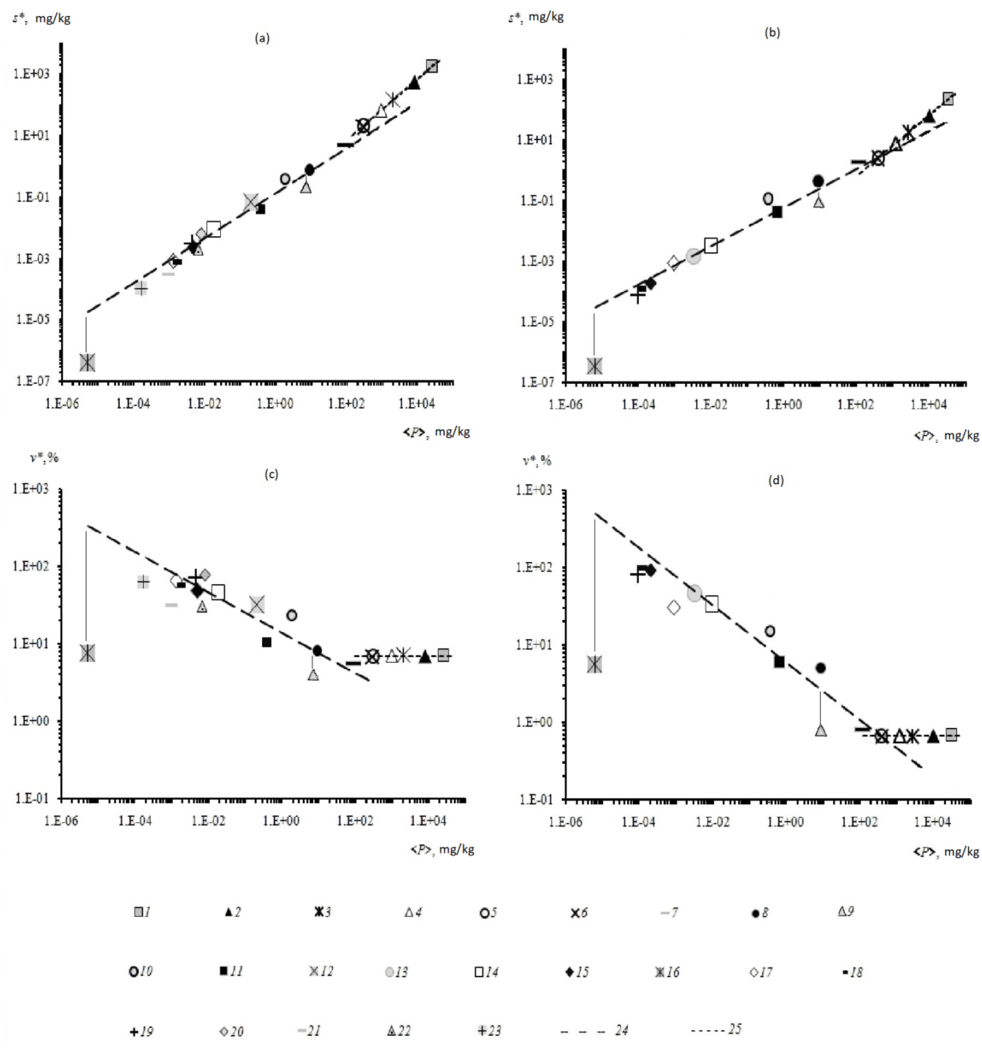

Figure 2. The relationship between the average concentration $< P >$ and s^* the standard v^* (a, b) and the average concentration and the variation coefficient (c, d) for a complex of oceanological parameters at 2 daily stations (the depth of 0 m) in various parts of Amur Bay of the Sea of Japan (see initial data in [12]). 1 – 16 – salinity, Na^+, SO_4^{2-}, Mg^{2+}, K^+, Ca^{2+}, HCO_3^-, CO_3^{2-}, O_2, SiO_3, CO_2, NH_4^+, NO_3^-, PO_4^{3-}, NO_2^-, a_{H^+}, chlorophylls a, b, c, Fe, Mn, Zn, Cu, respectively; 24, 25 – power law regression for the non-conserved and conserved parameters.

Well-known cycles of C, N, O, P and other can be used for subsystem allocation. The oscillations in the subsystems may characterize the system as a whole. The comparison of fluctuations in the biogeochemical parameters is presented at fig. 2.

We see the deviations from the power law regression which is the average characteristic of system. The positive deviation may be considered as the enhancement of the activity in subsystem. The negative deviation may be considered as the dumping. The deviations are saw especially in the variation coefficient, fig. 2, c and d. The regression diagrams at fig. 2 present the complex relation of variability factors. The regression is formed by the sum of the additive and multiplicative factors that gives the fractal relationship. The great positive deviations may be considered as a resonant in the subsystem.

It is obvious that cell membranes play the role of resonator. At the same time, the sell may be the active element in the external resonator which may be created by the environment formations. This structure hierarchy may be discussed for better understanding of the variability.

5 Conclusion

We considered the examples of the nonlinear dissipative systems in the comparison which allows to understand better the features of random fluctuations and nonlinear oscillations which was investigated earlier in convection and quantum systems. In this report, it is shown how the results obtained earlier in other arias may be used for the investigation of geophysical and astronomy objects.

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