Reversals in the low-mode model dynamo with $\alpha\Omega$-generators

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Abstract. In the dynamic model $\alpha\Omega$-dimensions are simulated reversions of the magnetic field with a varying intensity of the α-generator. We consider such changes in intensity as a consequence of the synchronization of the higher discarded modes of the velocity field and the magnetic field. Dynamo regimes are studied depending on the change in the intensity of the generator.

1 Introduction

The generation of the magnetic field with a strong differential rotation is described by $\alpha\Omega$-dinamo. The property of dynamo systems is the presence of a reversal without a significant rearrangement of the motion of the conducting medium [1, 2]. In this paper are described the large-scale $\alpha\Omega$-dinamo model and simulated reversions of the magnetic field with a varying intensity of the α-generator. We consider such changes in intensity as the effect of synchronization of the higher discarded modes of the velocity field and the magnetic field [4]. Dynamo regimes are studied depending on the intensity change of the a generator.

2 Equations and model parameters

We assume in the $\alpha\Omega$-dynamo model that the velocity field $\mathbf{v}$ and the magnetic field $\mathbf{B}$ are axially symmetric in a spherical shell of viscous incompressible liquid rotating around an axis $0z$ with a constant angular velocity Ω. We consider that the velocity field of a viscous fluid $\mathbf{v}$ is zero on the inner $r = r_1$ and the outer $r = r_1$ spherical envelope boundaries (boundary conditions of adhesion); the magnetic permeability of the inner and outer nuclei are the same, the medium outside the nucleus ($r > r_2$) is not conductive (vacuum boundary conditions at the outer boundary and conditions of boundedness at the center of the Earth are assumed). We assume that the mean flow $\overline{\mathbf{v}}$ has the character of differential rotation, corresponding to the modes $\mathbf{v}^T_{k,1,0}$ from the linear shell $\{\mathbf{v}^T_{k_1,1,0}, \mathbf{v}^P_{k_2,2,0}, \mathbf{v}^T_{k_3,3,0}, \mathbf{v}^P_{k_4,4,0}, \dots\}$ is invariant under the Coriolis drift. Any such mode generates the rest of the chain [5].

The velocity field of a viscous fiuid is approximated by the following combination [3, 5]:

$$\mathbf{v} = u(t)\mathbf{v_0} = u(t)(\alpha_1\mathbf{v}^T_{0,1,0} + \alpha_2\mathbf{v}^P_{0,2,0} + \alpha_3\mathbf{v}^T_{0,3,0} + \alpha_{11}\mathbf{v}^T_{1,1,0} + \alpha_{13}\mathbf{v}^T_{1,3,0}), \quad (1)$$

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where $\mathbf{v_0}$ is the Poincare mode, $|v_0| = 1, u(t)$ is the velocity amplitude, the components of the velocity field are independent of time.

The magnetic field is represented by the minimum number of lower eigenmodes $\mathbf{B}^P_{0,1,0}, \mathbf{B}^T_{0,2,0}, \mathbf{B}^P_{0,3,0}$, sufficient to obtain an oscillating dynamo [5]

$$\mathbf{B} = B^T_2(t)\mathbf{B}^T_{0,2,0}(\mathbf{r}) + B^P_1(t)\mathbf{B}^P_{0,1,0}(\mathbf{r}) + B^P_3(t)\mathbf{B}^P_{0,3,0}(\mathbf{r}), \tag{2}$$

where the components of the magnetic field are considered independent of time and the component $\mathbf{B}^P_1(\mathbf{r})$ is dipole.

The physical parameters of the fluid are assumed to be unchanged, the turbulence in the core is isotropic and we use the scalar parametrization of the α-effect as a function $\alpha(r, \theta) = \alpha(r)\, cos\theta$, where $max|\alpha(r, \theta)| \sim 1$. Two variants of the radial part of the α-effect are used: $\alpha(r) = -sin(\pi(r - r_1))$ and $\alpha(r) = r$.

Magnetohydrodynamic (MGD) equation, containing the equation of Navier-Stokes, the magnetic field $\mathbf{B}$ induction equation, continuity condition of the velocity field $\mathbf{v}$, the magnetic field $\mathbf{B}$ solenoidal condition, boundary conditions and taking into account the α-effect, in the Boussinesq approximation have the following form, respectively:

$$\begin{array}{l} \dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = \nu\Delta\mathbf{v} - \dfrac{1}{\rho_0}\nabla P - \mathbf{f}_K + \mathbf{f}_{out} + \mathbf{f}_L, \\ \dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nu_m\Delta\mathbf{B} + \nabla \times (\alpha(r, \theta)\,\mathbf{B}), \\ \nabla \cdot \mathbf{v} = 0, \\ \nabla \cdot \mathbf{B} = 0, \\ \mathbf{v}(\mathbf{r}_1) = \mathbf{v}(\mathbf{r}_2) = \mathbf{0}, \end{array} \tag{3}$$

where P is pressure, ρ_0 is density, ν is kinematic viscosity, ν_m is magnetic viscosity, $\mathbf{f}_{out}$ is mass density of the external-force field (conditions a poloidal mode of velocity), the mass density of Lorentz force is assumed equal

$$\mathbf{f}_L = \frac{1}{\rho_0\mu_0\mu}(\nabla \times \mathbf{B}) \times \mathbf{B},$$

the mass density of Coriolis force is given by

$$\mathbf{f}_K = 2\Omega \times \mathbf{v},$$

$\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vector to the inner and outer boundaries of the envelope, respectively.

We make dimensionless the Navier-Stokes equation and the magnetic induction equation in according to accepted restrictions. Let us take for the unit of measurement the characteristic time scale of the dissipation of the magnetic field $t \sim (L^2/\nu_m)$, where L – radius of the liquid core, accepted as a unit of the length. In addition, we introduce the mechanism of algebraic suppression of the α-effect and oscillations due to the function $Z = Z(t)$, which is given by the differential equation of the following form

$$\frac{\partial Z}{\partial t} = \mathbf{B}^2 - bZ \tag{4}$$

where b – scale factor, with the initial condition

$$Z(0) = 0. \tag{5}$$

As a result, the system (3) is converted to the dimensionless form with the inclusion of the conditions (4) and (5) in it:

$$\begin{aligned}
&\frac{\partial \mathbf{v}}{\partial t} + (Re_m \mathbf{v}\nabla)\mathbf{v} = P_m \Delta \mathbf{v} - \nabla P - E^{-1} P_m (e_z \times \mathbf{v}) + (1+\zeta(t))\,\mathbf{f}_{out} + (\nabla \times \mathbf{B}) \times \mathbf{B},\\
&\frac{\partial \mathbf{B}}{\partial t} = Re_m \nabla \times (\mathbf{v} \times \mathbf{B}) + \Delta \mathbf{B} + (R_\alpha - Z)\nabla \times (\alpha(r,\theta)\,\mathbf{B}),\\
&\nabla \cdot \mathbf{v} = 0,\\
&\nabla \cdot \mathbf{B} = 0,\\
&\mathbf{v}(\mathbf{r}_1) = \mathbf{v}(\mathbf{r}_2) = \mathbf{0},
\end{aligned} \tag{6}$$

where P_m is magnetic Prandtl number, Re_m is magnetic Reynolds number, R_α is range of the α-effect, $\mathbf{f}_{out}$ is mean mass density of the external-force field, the fluctuations of which are ensured by the stochastic process $\zeta(t)$ with zero mean in this case. This process simulates the spontaneously arising and vanishing coherent effect of the discarded higher modes of the velocity field. The structure of the process was determined as follows [3]: a random sequence of points $0 < \tau_1 < \theta_1 < \tau_2 < \theta_2 < \ldots < \tau_k < \theta_k < \ldots$ is given on the time axis. We assume that the k–th account of the coherent structure is formed at the moment τ_k and is resolved at the moment θ_k. Then $T_k^{est} = \tau_k - \theta_{k-1}$ is the waiting time for the formation of the next structure, and $T_k = \theta_k - \tau_k$ is the time of its existence. During the delay T_k^{est} the process $\zeta(t) = 0$, and during the existence time $\zeta(t) = \zeta_k$, where ζ_k are independent random variables with zero average, uniformly distributed on the segment $[-0.01, 0.01]$. In modeling, the exponential distribution rule for the delay T_k^{est} and the existence time T_k was used and these variables themselves were independent. The average values of $\langle T_k^{est}\rangle = 5$ and $\langle T_k\rangle = 30$, that is the characteristic time of existence of coherent structures is much shorter than their waiting time.

We apply the Galerkin method to the system (6) and get the following system's form:

$$\begin{aligned}
&\frac{\partial u}{\partial t} = -P_m u(t) \sum_k \alpha_k^2 \lambda_k + (1+\zeta(t))\, f_{out} + \sum_{i,j,k} \alpha_i L_{ijk} B_j B_k,\\
&\frac{\partial B_i}{\partial t} = Re_m u(t) \sum_{j,k} \alpha_j W_{ijk} B_k - \mu_i B_i + (R_\alpha - z) \sum_k W_{ik}^{\alpha} B_k,\\
&\frac{\partial z}{\partial t} = \sum_k B_k^2 - bz,
\end{aligned} \tag{7}$$

where f_{out} is mass density of the external–force field , μ_i is the viscous dissipation parameter, λ_i is eigenvalues of the Poincare mode, parameters $L_{ijk}, W_{ijk}, W_{ij}^{\alpha}$ are the volume integrals of the fields under consideration.

3 The results of the numerical simulation

We'll carry out a numerical investigation of the obtained system (7). The computational experiments with the model were carried out for the following boundary conditions at the initial instant of time $t = 0$:

$$u = 1,\ B_2^T = 0,\ B_1^P = 1,\ B_3^P = 0,\ Z = 0. \tag{8}$$

and the accepted values of the model parameters: magnetic Reynolds number Re_m varied in the range $(0,\ 1000]$, α-effect amplitude R_α was considered on the interval $(0,\ 100]$, average density of external force f_{out} and the scale factor b are equal to one and to ten.

We note, that the different regimes of dynamo with oscillations and reversions were obtained only for the case of specifying the scalar parametrization of the α-effect in the form of a function

$\alpha(r, \theta) = r\,cos\theta$. For the second case, only two types of dynamo regimes were obtained: the steady-state regime in the region of limited parameter values $R_\alpha \in [10,\ 100]$ and $Re_m \in [100,\ 1000]$, for the remaining values of the parameters the magnetic field attenuated.

An increase in the mass density of external forces f_{out} leads to an increase in the amplitudes of the modes under consideration, but does not change the dynamo regimes, although in this case the regions of the parameters $Re_m,\ R_\alpha$ vary for each of the regimes obtained. Variations in the scale factor b determine the frequency and amplitude of the oscillations and also do not lead to the disappearance or appearance of new dynamo regimes in the model under consideration.

As a result of the computational experiment for the model (7) the following modes of dynamo were obtained: lack of inversion, the steady-state regime (fig.1 a), evanescent field (fig.1 b), dynamo burst (fig.1 c), the steady regime (fig.1 d), in contrast to the work [6], where the algebraic suppression of oscillations was also realized by using of energy, but another function we used. In the proposed model, it is possible to reproduce more regimes of dynamo observed in real dynamo-systems, including geodynamo. The regimes obtained are of a periodic or quasi-periodic nature, which to a certain extent depends on the choice of the function $Z(t)$ for algebraic suppression of oscillations. The regions for each of the dynamo regimes obtained are marked (fig.2) on the phase plane of the parameters (Re_m, R_α).

4 Discussion and conclusions

In the framework of large-scale $\alpha\Omega$-dynamo model with a varying intensity of the α-generator it is possible to reproduce various dynamo regimes that are observed in real dynamo systems. The advantage of the proposed model is that the source of regular and quasiregular reversions is its internal dynamics.

Reversals were obtained in the region bounded by the values of the parameters $Re_m \in [10; 1000]$, $R_\alpha \in [10, 60]$. The model is stable to changes in the parameters f_{out} and b, that is, their variations do not lead to the appearance of new dynamo regimes, but it affects the change in the frequency and amplitude of the oscillations.

Two variants of specifying the radial part of the scalar parametrization of the α-effect are used. When the radial part was assigned a linear function in the range of the parameters $Re_m,\ R_\alpha$, the dynamo regimes as with inversions were obtained: evanescent field, steady-state output, regular mode, dynamo burst, and as without inversions.

For the second case, when the radial part of the α-effect was specified by the scalar function $\alpha(r) = -sin(\pi(r - r_1))$ the reversions were obtained in the whole considered region, however, only in two modes damped and stationary.

In the model considered, the appearance of magnetic field reversions against the background of a constant or slightly varying amplitude of the velocity field (except for the case of a dynamo burst) is typical, which allows us to consider the change in the velocity field equal to zero in the first equation of the system (3). Then the amplitude $u(t)$ can be expressed from the first equation of the system (3) through the amplitudes of the magnetic modes B_i and the four-mode model is converted to a three-mode model that generates inversions in a magnetic field.

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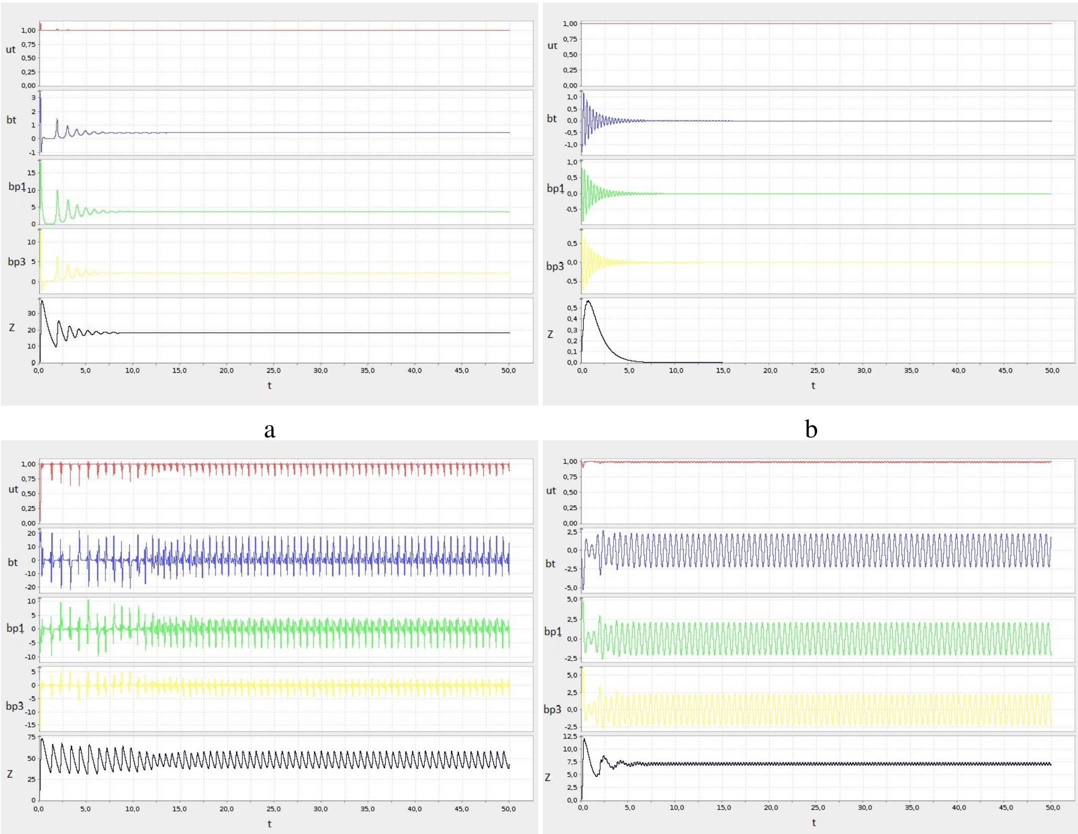

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Figure 1. The dynamo regimes for the case of a radial part α-effect as $\alpha(r) = r$: a) output steady-state regime ($Re_m = 20,\ R_a = 30$), b) the magnetic field with damped oscillations ($Re_m = 250,\ R_a = 30$), c) dynamo burst ($Re_m = 510,\ R_a = 60$), d) the steady regime of the magnetic field ($Re_m = 100,\ R_a = 30$).

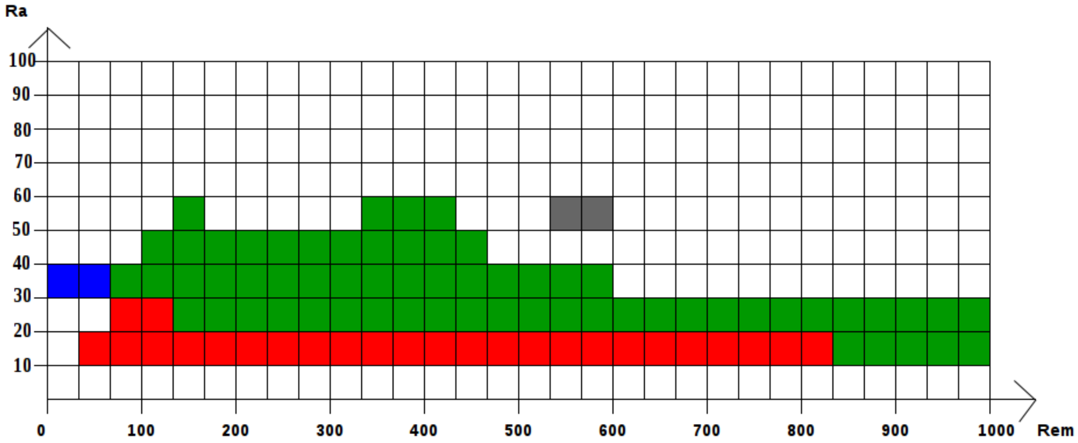

Figure 2. The nature of the generation of the magnetic field as a function of the values of the parameters R_α (turbulent dynamo) and Re_m (large–scale dynamo). Scalar parametrization of the α-effect as a function $\alpha(r\theta) = r\,cos\theta$. The white area is the generation of a magnetic field without inversions, the red one – is the generation of a field with damped oscillations, the blue – is steady–state regime, the grey – is dynamo burst, the green – is the steady regime.