Thermal calculation of heat exchangers with simplified consideration of axial wall heat conduction

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Abstract. The usual thermal design and rating methods [1, 2] for heat exchangers neglect axial wall heat conduction in the separating walls and external shells of recuperators and in the solid matrix of regenerators. This may lead to undesirable undersizing. In this paper a simplified model is developed for the fast estimation of axial wall conduction effects in counterflow, parallel flow and mixed-mixed cross-flow recuperators. The dispersion model [3] is used to describe the performance deterioration of the exchanger with an effective fluid dispersion Peclet number for the correction of the heat transfer coefficients or mean temperature difference. The method is tested against analytical and numerical calculations for counterflow and parallel flow with good results. It is also shown how the method can be adapted to thermal regenerators and the related thermal calculation methods [1, 2]. An alternative approach is suggested for the consideration of lateral heat conduction resistance in the solid matrix.

1 Introduction

The usual simple heat exchanger design and rating methods [1, 2] neglect the negative effect of axial heat conduction in the separating wall and the outer shells as well as in the solid matrix of regenerators. This may lead to undesirable underdesign. Kroeger [4] was the first to present an analytical closed form solution for balanced and unbalanced counterflow with axial conduction in the separating wall with adiabatic ends. Recently Aminuddin and Zubair [5] published several analytical solutions and calculated results on the performance of counterflow heat exchangers with consideration of axial heat conduction in the separating wall and heat losses from the shell to the vicinity. They assumed various realistic boundary conditions at the wall ends which are useful for the application of the solutions to the cell method or to series connections of exchangers.

In this paper a simple approximation is developed for the fast estimation of the performance deterioration due to axial heat conduction not only in the separating wall but also in the outer shells. Heat losses to the vicinity are neglected. Since axial wall heat conduction has a similar effect as axial fluid dispersion, the dispersion model [3] is used in the approximation method. First counterflow and parallel flow recuperators are considered.

2 System of governing differential equations

The steady state heat transfer process in a counterflow or parallel flow heat exchanger with adiabatic outside surface and axial wall heat conduction can be described with the following system of ordinary differential equations:

$$\frac{\mathrm{d}T_1}{\mathrm{d}x} - N_1\left(T_w - T_1\right) - N_{a1}\left(T_{wa1} - T_1\right) = 0 \qquad (1)$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}x} \pm N_2\left(T_w - T_2\right) \pm N_{a2}\left(T_{wa2} - T_1\right) = 0 \qquad (2)$$

$$\frac{\mathrm{d}^2 T_w}{\mathrm{d}x^2} + N_1 \mathrm{Pe}_{w1}\left(T_1 - T_w\right) + N_2 \mathrm{Pe}_{w2}\left(T_2 - T_w\right) = 0 \qquad (3)$$

$$\frac{\mathrm{d}^2 T_{wa1}}{\mathrm{d}x^2} + N_{a1} \mathrm{Pe}_{wa1}\left(T_1 - T_{wa1}\right) = 0 \qquad (4)$$

$$\frac{\mathrm{d}^2 T_{wa2}}{\mathrm{d}x^2} + N_{a2} \mathrm{Pe}_{wa2}\left(T_2 - T_{wa2}\right) = 0 \qquad (5)$$

The number of transfer units N_i of fluids $i = 1,2$

$$N_i = \frac{\alpha_i A_i}{\dot{W}_i}, \quad N_{ai} = \frac{\alpha_{ai} A_{ai}}{\dot{W}_i} \qquad (6)$$

are formed with the individual heat transfer coefficients alone, where $\alpha_i\ A_i$ describes the heat exchange of fluid i with the separating wall and $\alpha_{ai}\ A_{ai}$ the exchange with the inside surface of the outer wall or shell.

The wall Peclet numbers Pe_w are defined for the separating wall and the outer wall as

$$\mathrm{Pe}_{w,i} = \frac{\dot{W}_i L_i}{\lambda_{w,i} A_{qw,i}}, \quad \mathrm{Pe}_{wa,i} = \frac{\dot{W}_i L_i}{\lambda_{wa,i} A_{qwa,i}}. \qquad (7)$$

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For counterflow and parallel flow $L_1 = L_2 = L$ and $\lambda_{w1} A_{qw1} = \lambda_{w2} A_{qw2} = \lambda_w A_{qw}$. In eq (2) the positive sign is for counterflow and the negative sign for parallel flow.

The boundary conditions and further conditions in brackets are:

Counterflow

$$\begin{aligned} x=0: T_1 = 1 \qquad (T_2 = P_2) \\ x=1: T_2 = 0 \qquad (T_1 = 1 - P_1) \end{aligned} \tag{8}$$

Parallel flow

$$\begin{aligned} x=0: T_1 = 1, T_2 = 0 \\ (x=1: T_1 = 1-P_1, T_2 = P_2) \end{aligned} \tag{9}$$

Counterflow and parallel flow

$$x=0, x=1: \frac{\mathrm{d}T_w}{\mathrm{d}x} = \frac{\mathrm{d}T_{wa1}}{\mathrm{d}x} = \frac{\mathrm{d}T_{wa2}}{\mathrm{d}x} = 0 \tag{10}$$

The system of five ordinary differential equations of first and second order together with the required eight boundary conditions are implemented into the numeric computing environment of the mathematical software Maple for counterflow and parallel flow, respectively. The resulting real-valued two-point (x = 0 and x = 1) boundary value problems are solved by a finite difference method based on the trapezoidal scheme with Richardson extrapolation enhancing the accuracy [6]. The numerical solutions and analytical solutions of Kroeger [4] for special cases are used for the test of the approximation method developed in this paper.

3 Infinite thermal conductivities of the walls

First the most disadvantageous case of infinite thermal conductivities of the walls is considered. The wall Peclet numbers become zero and the system of differential equations, eqs (1) – (4), reduces to eqs (1) and (2). The wall temperatures t_w, t_{wa1} and t_{wa2} take constant values. The following equation has been derived for the calculation of $P_1 = P_{1,0}$ with index 0 for zero wall Peclet numbers.

$$\begin{aligned} \frac{1}{P_{1,0}} = \frac{N_{a1}}{N_1 (N_1 + N_{a1})} + \frac{1}{1 - \exp[-(N_1 + N_{a1})]} + \\ R_1 \left[\frac{N_{a2}}{N_2 (N_2 + N_{a2})} + \frac{1}{1 - \exp[-(N_2 + N_{a2})]} \right] \end{aligned} \tag{11}$$

This equation is valid for counterflow, parallel flow and any flow arrangement. The indices 1 and 2 can be exchanged. In the limiting case $\dot{W}_2 = \infty$, $R_1 = N_2 = N_{a2}$ = 0 the right hand side term in eq (11) turns to $R_1/N_2 = \dot{W}_1 / (\alpha_2 A_2)$. For $N_{a1} = N_{a2} = \infty$, eq (11) applies to the two-sided stirred tank [1, 2].

4 Weak wall heat conduction effects

The limiting case of very low thermal wall conductivities ($\lambda_w \to 0$, $\lambda_{wa1} \to 0$, $\lambda_{wa2} \to 0$) is considered in which fluid and wall temperatures do not change remarkably. The wall temperatures of the outside walls assume the temperature of the adjacent fluid i = 1,2, $T_{wa,i} = T_i$, as the outside surface is adiabatic. So the local axial conductive heat flows in the outer walls $a,i = a$,1 and a,2 are

$$\dot{Q}_{wa,i} = -\lambda_{wa,i} A_{qwa,i} \frac{\mathrm{d}T_{wa,i}}{\mathrm{d}x} = -\frac{\dot{W}_i L}{\mathrm{Pe}_{wa,i}} \frac{\mathrm{d}T_i}{\mathrm{d}x}. \tag{12}$$

With the local temperature of the separating wall

$$T_w = T_1 \frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} + T_2 \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \tag{13}$$

the local conductive heat flow in this wall can be expressed as

$$\begin{aligned} \dot{Q}_w = -\lambda_w A_{qw} \frac{\mathrm{d}T_w}{\mathrm{d}x} = \\ -\frac{\dot{W}_i L}{\mathrm{Pe}_{w1}} \left(\frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} \frac{\mathrm{d}T_1}{\mathrm{d}x} + \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \frac{\mathrm{d}T_2}{\mathrm{d}x} \right). \end{aligned} \tag{14}$$

The substitutive effective axial dispersive heat flows in fluids i = 1,2 are

$$\dot{Q}_{d,i} = -\lambda_{d,i} A_{q,i} \frac{\mathrm{d}T_i}{\mathrm{d}x} = -\frac{\dot{W}_i L}{\mathrm{Pe}_i} \frac{\mathrm{d}T_i}{\mathrm{d}x}. \tag{15}$$

The basic idea of this model is to shift the conductive axial heat flows from the walls into the adjacent fluids. This yields effective Peclet numbers for the dispersion model. Since the separating wall is in contact with both fluids, its heat flow has to be distributed according to the (unknown) fractions φ_1 for fluid 1 and $\varphi_2 = 1 - \varphi_1$ for fluid 2. The substitutive dispersive heat flows are accordingly expressed as

$$\dot{Q}_{d,1} = \varphi_1 \dot{Q}_w + \dot{Q}_{wa,1}, \tag{16}$$

$$\dot{Q}_{d,2} = \varphi_2 \dot{Q}_w + \dot{Q}_{wa,2}. \tag{17}$$

The eqs (16) and (17) represent the basic concept of the model for weak conductive effects. Substituting eqs (12), (14) and (15) into eqs (16) and (17) and rearranging yields

$$\begin{aligned} \frac{1}{\mathrm{Pe}_1} = \\ \frac{\varphi_1}{\mathrm{Pe}_{w1}} \left(\frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} \pm R_1 \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \right) + \frac{1}{\mathrm{Pe}_{wa,1}}, \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{1}{\mathrm{Pe}_2} = \\ \pm \frac{\varphi_2}{\mathrm{Pe}_{w1}} \left(\frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} \pm R_1 \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \right) + \frac{1}{\mathrm{Pe}_{wa,2}}. \end{aligned} \tag{19}$$

The positive sign is for counterflow and the negative sign for parallel flow. The negative sign occurs as for parallel flow $dT_2/dT_1 \leq 0$. The Peclet numbers Pe_1 and Pe_2 could be used if the fractions φ_1 and φ_2 were known. They are determined in the following.

The overall conductive effect can be expressed with the dimensionless dispersive mean temperature difference [3]

$$\Theta_d = \Theta_{\lg} - \left(\frac{P_1}{\mathrm{Pe}_1} + \frac{P_2}{\mathrm{Pe}_2} \right) = \Theta_{\lg} - \frac{P_1 + P_2}{\mathrm{Pe}_{1,2}}. \quad (20)$$

This equation defines also $\mathrm{Pe} = \mathrm{Pe}_{1,2}$ which is equal for both the fluids. The deteriorative conduction effect should yield positive values $\mathrm{Pe} = \mathrm{Pe}_{1,2} > 0$ for all cases of counterflow and parallel flow.

Multiplying eqs (18) and (19) by P_1 and P_2, respectively, summation of both equations and dividing by P_1 yields with $P_2/P_1 = R_1$

$$\frac{1+R_1}{\mathrm{Pe}} = \frac{1}{\mathrm{Pe}_{w1}} (\varphi_1 \pm R_1 \varphi_2) \left(\frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} \pm R_1 \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \right) + \frac{1}{\mathrm{Pe}_{wa,1}} + \frac{R_1}{\mathrm{Pe}_{wa,2}}. \quad (21)$$

This equation reveals that the condition $\mathrm{Pe} > 0$ can only be fulfilled throughout if

$$\varphi_1 = \frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2}, \quad \varphi_2 = \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2}. \quad (22)$$

Substituting eq (22) into eq (21) finally yields

$$\frac{1+R_1}{\mathrm{Pe}} = \frac{1}{\mathrm{Pe}_{w1}} \left(\frac{\alpha_1 A_1}{\alpha_1 A_1 + \alpha_2 A_2} \pm R_1 \frac{\alpha_2 A_2}{\alpha_1 A_1 + \alpha_2 A_2} \right)^2 + \frac{1}{\mathrm{Pe}_{wa,1}} + \frac{R_1}{\mathrm{Pe}_{wa,2}}. \quad (23)$$

The eq (23) remains valid if the indices 1 and 2 are exchanged. The positive sign is for counterflow and the negative sign is for parallel flow. The dispersive Peclet number Pe has to be applied to both fluids.

Substituting eq (22) into eqs (18) and (19) yields formulas for Pe_1 and Pe_2 which have to be applied to fluid 1 and fluid 2, respectively.

With counterflow $\mathrm{Pe}_1 > 0$ and $\mathrm{Pe}_2 > 0$, which means that, owing to wall heat conduction, for fixed inlet and outlet temperatures the mean temperature difference $\theta_m = T_{1m} - T_{2m}$ decreases, the mean temperature T_{1m} decreases and T_{2m} increases.

With parallel flow one of the Peclet numbers may be negative. If e.g. $\mathrm{Pe}_{wa,1} = \mathrm{Pe}_{wa,2} = \infty$ (no outer wall effect) and $N_1 > N_2$, then $\mathrm{Pe}_1 > 0$ and $\mathrm{Pe}_2 < 0$. This means that in this case $\theta_m = T_{1m} - T_{2m}$ decreases, T_{1m} decreases and T_{2m} decreases as well, but less than T_{1m}. This is in qualitative accordance with the real process.

The application of Pe_1 and Pe_2 from eqs (18), (19) and (22) gives the same results as Pe from eq (23). The two Peclet numbers Pe_1 and Pe_2 yield more insight in the process, applying only one Peclet number Pe according to eq (23) is more appropriate for the approximation method developed in the following.

5 Approximation equation for Pe

The relative error of Pe from eq (23) decreases with increasing values of the wall Peclet numbers and the resulting Pe for both fluids. For lower values of Pe eq (23) provides too low values of Pe. In the limiting case $\mathrm{Pe}_w = \mathrm{Pe}_{wa,1} = \mathrm{Pe}_{wa,2} = 0$ eq (23) gives the wrong result $\mathrm{Pe} = 0$. This case has been investigated in section 3 and the correct value of $\mathrm{Pe} = \mathrm{Pe}_0$ can be calculated from eq (20). Substituting in eq (20) Θ_d according to the well known relationship for any flow arrangement with or without dispersion

$$\Theta_d = \frac{P_1}{\mathrm{NTU}_1} = P_1 \left(\frac{1}{N_1} + \frac{R_1}{N_2} \right) \quad (24)$$

yields the formula for the calculation of $\mathrm{Pe} = \mathrm{Pe}_0$:

$$\frac{1+R_1}{\mathrm{Pe}_0} = \frac{\Theta_{\lg}}{P_{1,0}} - \left(\frac{1}{N_1} + \frac{R_1}{N_2} \right). \quad (25)$$

In eq (25) $\mathrm{Pe}_{1,0}$ is determined from eq (11) and $\Theta_{\lg}$ is the dimensionless logarithmic mean temperature difference for counterflow or parallel flow, respectively, calculated with $P_{1,0}$ and R_1.

The eqs (20) and (25) are also exactly valid for the pure cross-flow and are approximations for other one-pass flow arrangements, if $\Theta_{\lg}$ is replaced by the correct mean temperature difference for the flow arrangement under consideration.

With the limiting Peclet number Pe_0 from eq (25) and the Peclet number from eq (23) which is now denoted with Pe_∞ the approximation equation is formed

$$\mathrm{Pe} = \left(\mathrm{Pe}_0^m + \mathrm{Pe}_\infty^m \right)^{\frac{1}{m}} \quad (26)$$

which yields exact values of Pe for $\mathrm{Pe}_\infty = \infty$ and for $\mathrm{Pe}_\infty = 0$.

The comparison of eq (26) with analytical and numerical results from eqs (1) – (10) provides a value of $m = 0.87$. In the range $5 \leq \mathrm{Pe} \leq 23$ for counterflow and $9 \leq \mathrm{Pe} \leq 140$ for parallel flow the mean relative error of Pe is about 5 % and the mean relative error of P_1 or P_2, calculated with Pe, is about 0.5 %.

The temperature change P_1 can be calculated by correcting the overall heat transfer coefficient (kA) according to

$$\frac{1}{(kA)_\lambda} = \frac{1}{(kA)} + \frac{1}{\mathrm{Pe}} \left(\frac{1}{\dot{W}_1} + \frac{1}{\dot{W}_2} \right) \quad (27)$$

and applying the corrected value $(kA)_\lambda$ to the known formulas for counterflow and parallel flow. Other ways are possible as shown in [3].

6 Application of the model to mixed-mixed cross-flow

The eqs (11) and (24) – (26) can directly be applied to mixed-mixed cross-flow. Only the eq (23) has to be replaced by a new equation and in eq (25) Θ_{lg} has to be replaced by the mean temperature difference for mixed-mixed cross-flow.

Now in the separating wall axial heat conduction takes place in the two flow directions and both the flow lengths L_i may be different. So in the wall Peclet numbers usually $L_1 \neq L_2$ and $\lambda_{w1} A_{qw1} \neq \lambda_{w2} A_{qw2}$ (see eq (7)).

Applying again the principle of shifting the conductive heat flows into the fluids leads to the following equation of $\mathrm{Pe} = \mathrm{Pe}_\infty$:

$$\psi_1 = \mathrm{Pe}_{w1}^{1/2}(1+R_1)^{1/2}\left(1+\frac{\alpha_2 A_2}{\alpha_1 A_1}\right) \tag{28}$$

$$\psi_2 = \mathrm{Pe}_{w2}^{1/2}(1+R_2)^{1/2}\left(1+\frac{\alpha_1 A_1}{\alpha_2 A_2}\right) \tag{29}$$

$$\frac{1}{\mathrm{Pe}_\infty} = \left(\frac{1}{\psi_1}\right)^2 + \left(\frac{1}{\psi_2}\right)^2 + \frac{1}{\mathrm{Pe}_{wa1}(1+R_1)} + \frac{1}{\mathrm{Pe}_{wa2}(1+R_2)}. \tag{30}$$

Using eq (28) and (29) the eq (23) for counterflow (+) and parallel flow (-) can be rearranged to a similar form:

$$\frac{1}{\mathrm{Pe}_\infty} = \left(\frac{1}{\psi_1} \pm \frac{1}{\psi_2}\right)^2 + \frac{1}{\mathrm{Pe}_{wa1}(1+R_1)} + \frac{1}{\mathrm{Pe}_{wa2}(1+R_2)}. \tag{31}$$

In the limiting cases of one constant fluid temperature, $\psi_1 = \infty$ or $\psi_2 = \infty$, the eqs (30) and (31) become identical. Since both equations have been derived according to the same model they should reach approximately the same accuracy, also in the general case $\psi_1 < \infty$ and $\psi_2 < \infty$.

Using the Peclet numbers $\mathrm{Pe}_{(30)}$ from eq (30) and $\mathrm{Pe}_{(31,\pm)}$ from eq (31) enables the general presentation:

$$\frac{1}{\mathrm{Pe}_\infty} = \frac{a}{\mathrm{Pe}_{(31,+)}} + \frac{1-a}{\mathrm{Pe}_{(31,-)}} \tag{32}$$

with

$a = 0$: Parallel flow
$a = 1/2$: Mixed-mixed cross-flow
$a = 1$: Counterflow.

Other one-pass symmetrical flow arrangements may be described with individual values of $0 \le a \le 1$.

7 Application of the model to regenerators under steady periodic operation

The method derived for counterflow and parallel flow recuperators can also be adapted and applied to fixed-bed and rotary regenerators. The outer walls are considered as part of the solid matrix and need not be considered separately.

7.1 Infinite axial thermal conductivity of the solid matrix

This limiting case corresponds to the steady state heat transfer process in two mixed-unmixed cross-flow heat exchangers coupled by a circulating transversely mixed flow stream [7, 8]. The temperature change P_1

$$\frac{1}{P_1} = \frac{R_{1s}}{1-\exp\left[-R_{1s}\left(1-e^{-N_1}\right)\right]} + \frac{R_{1s}}{1-\exp\left[-R_{2s}\left(1-e^{-N_2}\right)\right]} \tag{33}$$

with the fluid to solid matrix capacity ratios:

For the fixed-bed regenerator

$$R_{i,s} = \frac{\dot{W}_i \tau_i}{C_{s,i}} \tag{34}$$

and the rotary regenerator

$$R_{i,s} = \frac{\dot{W}_i(\tau_1+\tau_2)}{C_{s,t}} \tag{35}$$

The capacity $C_{s,i}$ in eq (34) is the capacity of one matrix "*i*" and $C_{s,t}$ in eq (35) is the total capacity of the rotating matrix. The time τ_i is the duration of period "*i*" and the sum $(\tau_1 + \tau_2)$ in eq (35) is the duration of one revolution of the matrix.

The numbers of transfer units N_1 and N_2 in eq (33) are formed with the corrected heat transfer coefficients $\alpha_{i,o}$ according to

$$\frac{1}{\alpha_{i,o}} = \frac{1}{\alpha_i} + \frac{1}{\alpha_s} = \frac{1}{\alpha_i} + \phi\frac{\delta}{\lambda_s}\cdot \tag{36}$$

They take the lateral heat conduction resistance inside the solid matrix into account. The estimation of the additional resistance $1/\alpha_s = \phi\,\delta/\lambda_s$ is subject to Hausen's theory of regenerators [9, 1, 2] and will be discussed later in this paper.

Once the temperature change P_1 is determined from eq (33), the Peclet number for $\lambda_s = \infty$ can be calculated. For regenerators the previous eq (25) has to be replaced by

$$\frac{1+R_1}{\mathrm{Pe}}=\frac{\Theta_{\lg}}{P_1^{(33)}}-\left(\frac{1}{N_1}+\frac{R_1}{N_2}\right)\frac{k_0}{k}. \tag{37}$$

For rotary regenerators:

$$R_1=\frac{\dot{W}_1}{\dot{W}_2}=\frac{1}{R_2} \tag{38}$$

For fixed-bed regenerators:

$$R_1=\frac{\dot{W}_1\tau_1}{\dot{W}_2\tau_2}=\frac{1}{R_2} \tag{39}$$

The correction factor k/k_0, which is in reality a correction of the mean temperature difference, takes the deviations of the time averaged longitudinal temperature profiles in the regenerator from those in the equivalent counterflow recuperator into account. This correction has been and still is discussed and calculated by many researchers [9, 10]. It is calculated for a solid matrix with infinite lateral and zero axial thermal conductivity. Results are given in the relevant literature [9, 10].

7.2 Low axial thermal conductivity of the solid matrix

The previous eqs (23) or (31), (28) and (29) can directly be applied using the following definitions of the wall Peclet numbers.

Fixed-bed regenerator:

$$\mathrm{Pe}_{wi}=\mathrm{Pe}_{s,i}=\frac{\dot{W}_i L\tau_i}{\lambda_s A_{qs,i}\left(\tau_1+\tau_2\right)} \tag{40}$$

Rotary regenerator:

$$\mathrm{Pe}_{wi}=\mathrm{Pe}_{s,i}=\frac{\dot{W}_i L}{\lambda_s A_{qs,t}} \tag{41}$$

The cross-section for heat conduction $A_{qs,i}$ in eq (40) belongs to the one solid matrix „i" under consideration. The cross-section $A_{qs,t}$ in eq (41) is the total axial cross-section of the rotating solid matrix.

The products $\alpha_i A_i$ in eqs (23), (28) and (29) have to be substituted according to

Fixed-bed regenerator:

$$\alpha_i A_i=\alpha_{i,0}A_i \tag{42}$$

Rotary regenerator:

$$\alpha_i A_i=\alpha_{i,0}\tau_i \tag{43}$$

7.3 Final correction for axial solid matrix conduction

With Pe_0 from eq (37), considering eqs (33) – (39), and with Pe_∞ from eqs (23) and eqs (40) – (43) the final Peclet number can be calculated using eq (26). Then eq (27) is applied to the correction of the overall heat transfer coefficient (kA).

For the rotary regenerator

$$\frac{1}{kA}=\left(\frac{1}{\alpha_{1,0}A_1}+\frac{1}{\alpha_{2,0}A_2}\right)\frac{k_0}{k} \tag{44}$$

and eq (27) yields the corrected value $(kA)_\lambda$. In eq (44) A_1 and A_2 are the actual heat transfer surfaces in the hot and cold section.
For the fixed-bed regenerator eq (27) has to be adapted to the definitions in Hausen's theory [9, 1, 2] leading to

$$\frac{1}{(kA)_\lambda\left(\tau_1+\tau_2\right)}=\frac{1}{(kA)\left(\tau_1+\tau_2\right)}+\frac{1}{\mathrm{Pe}}\left(\frac{1}{\dot{W}_1\tau_1}+\frac{1}{\dot{W}_2\tau_2}\right) \tag{45}$$

The surface A in eq (45) is the heat transfer surface of one solid matrix.

If inlet and mean outlet temperatures are given, eq (20) can be used for the correction of the mean temperature difference instead of eq (27) [3].

7.4 Consideration of lateral conductive resistance in solid matrix

7.4.1 Equivalent wall thickness

The function ϕ in eq (36) can be calculated according to Hausen [9, 1, 2] for the plane wall of thickness δ, for the cylinder of diameter δ and the sphere of diameter δ. For other geometries Hausen introduced the equivalent wall thickness

$$\delta_{eq}=\frac{\delta}{2}+\frac{V_s}{A} \tag{46}$$

to be used in the equations for the plane wall.

A new formula for the equivalent wall thickness is proposed which gives better agreement with exact calculations for spherical elements than eq (46)

$$\frac{1}{\delta_{eq}}=\frac{2}{3}\frac{1}{\delta}+\frac{1}{6}\frac{A}{V_s}\cdot \tag{47}$$

For the cylinder eqs (46) and (47) yield the same results.

7.4.2 Alternative calculation model

In the theory on thermal regenerators [9, 10] the transient process is calculated under the assumption of zero axial and infinite lateral thermal conductivity or zero wall thickness, $\delta_{eq}/\lambda_s=0$. If $\delta_{eq}/\lambda_s>0$ the additional resistance $1/\alpha_s$ from eq (36) is added.

According to previous investigations on temperature oscillation heat transfer processes [11], not only the heat transfer coefficient has to be corrected (eq (36)) but also the wall thickness or the capacity of the elements. (Index *s* in this paper is index *w* in ref [11].) If $\delta_{eq}/\lambda_s > 0$ the reduced effective capacity $\widetilde{C}_s \le C_s$ should be used for the determination of Hausen's reduced period Π [9, 10]. The reduction $\widetilde{C}_s/C_s = \widetilde{X}/X$ and the related heat transfer resistance $1/\alpha_s$ can be calculated according to the temperature oscillation model [11]. The relevant equations are given in the appendix. The resulting values of α_s are equal to or a few percent lower than those of Hausen's approach. The required capacity reduction yields another minor or negligible correction on the safe side. The limiting case of infinite storage capacity does no longer occur.

8 Conclusions

1. The effect of axial heat conduction in the walls of recuperators and in the solid matrix of regenerators can be described and estimated with the axial dispersion model and an effective dispersive Peclet number.
2. The derived approximation model can be applied to counterflow, parallel flow and mixed-mixed cross-flow recuperators as well as counterflow and parallel flow regenerators.
3. The effect of lateral heat conduction in the solid matrix of regenerators can be described with a temperature oscillation model, developed earlier for the evaluation of temperature oscillation experiments [11]. A new formula for Hausen's equivalent wall thickness [1, 2, 9] can improve and simplify the calculation methods for regenerators.

References

1. VDI Wärmeatlas, Edition 11, VDI-GVC (ed.), Springer 2013
2. VDI Heat Atlas, VDI-GVC (ed.), Springer-Verlag Berlin Heidelberg 2010
3. Roetzel W., Na Ranong Ch., Fieg, G.: New axial dispersion model for heat exchanger design. *Heat and Mass Transfer*, 47 (2011), pp. 1009–1017
4. Kroeger P.G.: Performance deterioration in high effectiveness heat exchangers due to axial heat conduction effects. *Adv. Cryog. Eng.*, 12 (1967), pp. 363–372.
5. Aminuddin M., Zubair S.M.: Analytical solution to counter-flow heat exchanger subject to external heat flux and axial conduction. *Int. Journal of Refrigeration*, 74 (2017), pp. 20–35.
6. Ascher U.M., Mattheij R.M.M., Russell R.D.: *Numerical solution of boundary value problems for ordinary differential equations*. Classics in applied mathematics 13, Society for Industrial and Applied Mathematics, Philadelphia 1995.
7. Martin H.: *Wärmeübertrager*. Georg Thieme Verlag, Stuttgart New York, 1988.
8. Na Ranong Ch., Roetzel W.: Stationäres und instationäres Verhalten von zwei gekoppelten Kreuzstromwärmeübertragern mit quervermischtem Umlaufstrom. In: Wärmeaustausch und erneuerbare Energiequellen, Tagungsband *VIII. Internationales Symposium Szczecin-Leba* 18.-20.9.2000, 269 – 276, Herausg. W. Nowak, TU Szczecin, 2000, ISBN 83-87423-14-9.
9. Hausen H.: *Wärmeübertragung im Gegenstrom, Gleichstrom und Kreuzstrom*. 2. Auflage, Springer-Verlag, Berlin Heidelberg New York 1976. English translation: McGraw-Hill, New York, 1983.
10. Baclic B.S., Dragutinovic G.D.: *Operation of counterflow regenerators*. Computational Mechanics Publications, Southampton, UK and Boston, USA, 1998.
11. Roetzel W., Na Ranong Ch.: Evaluation of temperature oscillation experiment for the determination of heat transfer coefficient and dispersive Peclet number. In Proceedings HTRSE 2016, pp. 229 – 241. Extended version: *Archives of Thermodynamics*, Vol. 39 (2018), No. 1, 91–110, DOI: 10.1515/aoter-2018-0005.

Appendix

1 Calculation of α_s in eq (36) according to the oscillation model [11]

Equivalent wall thickness

$$\delta_{eq} = 2\left(\frac{V_s}{A}\right)_{eq} \tag{A1}$$

Substitution into eq (47)

$$\left(\frac{A}{V_s}\right)_{eq} = \frac{4}{3}\frac{1}{\delta} + \frac{1}{3}\left(\frac{A}{V_s}\right) \tag{A2}$$

Dimensionless equivalent wall thickness

$$X_{eq} = \left(\frac{V_s}{A}\right)_{eq}\sqrt{\frac{\pi\rho c}{(\tau_1+\tau_2)\lambda}}, \quad \left(\Omega = \frac{2\pi}{\tau_1+\tau_2}\right) \tag{A3}$$

Equivalent Nußelt number

$$\mathrm{Nu}_{s,eq} = \frac{\alpha_s}{\lambda}\left(\frac{V_s}{A}\right)_{eq}, \quad \mathrm{Nu}_s = \widetilde{\mathrm{Nu}}_s X/\widetilde{X} \tag{A4}$$

$$\mathrm{Nu}_{s,eq} = \frac{X\left(\sinh^2 2X + \sin^2 2X\right)}{\left(\sinh 2X - \sin 2X\right)\left(\sinh^2 X + \cos^2 X\right)} \tag{A5}$$

Application of eqs (A2), (A3), (A5) and (A4) yields α_w as function of δ and (V_s/A). For the cylinder $\delta_{cyl} = 4(V_s/A)_{cyl}$. For the sphere $\delta_{sph} = 6(V_s/A)_{sph}$.

2 Calculation of the reduced effective capacity $\widetilde{C}_s$ according to ref [11]

Effective capacity ratio

$$\frac{\widetilde{C}_s}{C_s} = \frac{\widetilde{V}_s}{V_s} = \frac{\widetilde{X}}{X} \quad \text{(A6)}$$

For the plane wall

$$\frac{\widetilde{X}}{X} = \frac{\sinh^2 2X + \sin^2 2X}{2X(\sinh 2X + \sin 2X)(\sinh^2 2X + \cos^2 X)} \quad \text{(A7)}$$

Instead of eq (A2) the following eq (A8) is recommended for the application to eq (A7)

$$\left(\frac{A}{V_s}\right)^*_{eq} = \frac{0.87}{0.87 + X}\frac{2}{\delta} + \frac{X}{0.87 + X}\left(\frac{A}{V_s}\right) \quad \text{(A8)}$$

For precise calculations of cylinder and sphere (α_w and $\widetilde{C}_s$) use the equations (A3) and (A4) in the appendix of ref [11].

Nomenclature

A – area, m^2,
a – variable, eq (32),
C – heat capacity, J/K,
k – overall heat transfer coefficient, W/(m^{2}K),
L, l – flow length, m,
m – exponent, eq (26)
N – number of transfer units, formed with heat transfer coefficient, $N = \alpha A / \dot{W}$,
Nu – Nußelt number, $\mathrm{Nu} = \frac{\alpha}{\lambda}\left(\frac{V}{A}\right)$, eq (A4),
NTU – number of transfer units, $kA/\dot{W}$,
P_1 – dimensionless temperature change of flow stream 1, $P_1 = \frac{T_{1,in} - T_{1,out}}{T_{1,in} - T_{2,in}}$,
P_2 – dimensionless temperature change of flow stream 2, $P_2 = \frac{T_{2,out} - T_{2,in}}{T_{1,in} - T_{2,in}}$,
Pe – Peclet number for fluid dispersion, $\mathrm{Pe} = \dot{W}L / (\lambda_d A_q)$,
Pe_w – wall Peclet number, $\mathrm{Pe}_w = \dot{W}L / (\lambda_w A_{qw})$,
$\dot{Q}$ – heat flow stream, W,
R – capacity ratio, $R_1 = \dot{W}_1 / \dot{W}_2 = 1/R_2$, for fixed bed regenerators $R_1 = \frac{\dot{W}_1 \tau_1}{\dot{W}_2 \tau_2} = \frac{1}{R_2}$,
T – temperature, K,
V – volume, m^3,
$\dot{W}$ – heat capacity rate, W/K,
X – dimensionless wall thickness, eq (A3),
x – dimensionless flow length, $x = l/L$

Greek symbols

α – heat transfer coefficient, W/(m^{2}K),
δ – wall thickness or diameter, m,
Θ – dimensionless mean temperature difference, $\Theta = \Delta T_M / (T_{1,in} - T_{2,in})$,
λ – thermal conductivity, W/(m·K),
Π – reduced period of regenerator,
τ – duration or period, s,
ϕ – function, eq (36),
φ – fraction, eqs (16) and (17),
ψ – dummy variable, eqs (28) and (29)

Subscripts and superscripts

a – outside,
cyl – cylinder,
d – dispersive,
eq – equivalent,
i – counter,
lg – logarithmic,
m – mean value,
q – cross-section,
s – solid matrix,
sph – sphere,
t – total,
w – wall,
λ – corrected for heat conduction,
0 – corrected for conductive resistance, or zero wall Peclet number,
∞ – infinite wall Peclet number,
1, 2 – fluid 1, 2,
$\sim$ – effective reduced value