Optimization of hydraulic modes of single-source tree-like heat networks

Alexander V. Lutsenko[1,*], *Nikolay N.* Novitsky[1]

[1] Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences, 130, Lermontov st., Irkutsk, 664033, Russia

Abstract. The article deals with the problem of optimizing the hydraulic modes of radial heat supply systems with a single heat source and pumping stations. The features of the problem under consideration are the presence of several target functions and a fixed flow distribution. The topology of the networks under consideration allows using the equivalent branching methods of branches connected in parallel and in series to reduce the design schemes to single equivalent branches. To solve this problem, the original method proposed by the authors earlier is adapted. The proposed method was tested on an aggregated scheme of a real heat supply system. Computational experiments have shown the performance and computational efficiency compared with the methods proposed earlier.

1 Introduction

Heat supply systems (HSS) have significant reserves of energy saving [1 - 3], which can be realized by optimizing their operation modes. The construction of new residential areas, the closure of industrial facilities leads to significant changes in the structures of HSS loads and an increase in the risk of emergencies. The emergence of new types of units raises questions of their effective use. In practice, the task of planning HSS operation modes is solved by multivariate calculations [4], which does not guarantee the optimality of the obtained modes. Automating the solution of these problems is hampered by several factors: the high dimensionality of the flow distribution models [5, 6], their nonlinearity, the presence of several objective functions, etc. For these reasons, there are no methods and software packages suitable for practical use. This determines the relevance of the development of methods and software systems for calculating the optimal HSS modes.

The literature discusses the problem of optimizing the HSS modes, but most of the work has drawbacks. A large number of papers consider small-scale HSS, for example, [7, 8]. Some works (for example, [9, 10] and others) consider approximation of the relationship between the value of the objective function and the mode parameters chosen as the basis, which makes it difficult to account for discrete mode parameters. In the remaining works, the HSS aggregated schemes are used [11, 12], which does not guarantee obtaining optimal solutions with the required accuracy. The practice of using ready-made software packages for solving systems of equations and inequalities ([13, 14]) is widespread, which leads to

[*] Corresponding author: luc_alex@mail.ru

the impossibility of adapting methods to problems and too high computational costs. Even greater computational costs are required by the use of genetic algorithms ([15]). Private objective functions are often considered. For example, in [16] , the total fuel consumption at heat sources (HS) is minimized, and in [9 , 10] – power consumption at pumping stations (PS) is minimized. The operational management is mainly considered, for example, [16 – 18], and there are practically no works devoted to the planning tasks of the established HSS operation modes in preparation for the heating season.

The article provides the formalization of the optimization problem for the steady-state hydraulic modes (HM) of a HSS of a radial structure, having one HS and PS for the typical case of parallel operation of pumps of the same type. To solve this problem, the previously proposed loop reducing dynamic programming method (LRDP) [19] is adapted. This method has several advantages: an increase in computational costs linear in the dimension of the problem; guaranteed finding optimal HM; providing the possibility of optimization for several objective functions at the same time, etc.

2 Problem statement

The task of optimizing HM HSS is to find a HM that meets the requirements of admissibility and optimality according to a given system of objectives. Energy saving requirements is laid down in the economic objective function. Technological objective functions are associated with the desire to reduce the labor intensity of adjustment activities, to reduce leakages and the risks of accidents.

Pipeline sections, PS and consumers are the main elements of the HSS. Denote the sets of branches that model the elements of the first, second and third types as I_{PL}, I_{PS}, I_C. Then $I_{PL} \cap I_{PS} = I_{PL} \cap I_C = I_C \cap I_{PS} = \varnothing$ and $I_{PL} \cup I_{PS} \cup I_C = I$ – set of all branches, $|I| = n$. Let J be the set of all nodes, $|J| = m$. As the hydraulic characteristics of the i-th branch can be taken [20]:

$$h_i(x_i, z_i, \gamma_i, \kappa_i) = z_i s_i x_i |x_i| / \kappa_i^2 - \gamma_i^2 H_i . \quad (1)$$

Here: i – branch number; h_i, x_i – pressure drop and coolant flow to the branches; s_i, z_i – hydraulic resistance and its relative increase (for example, increased by throttling); κ_i, γ_i – the number of pumps operating at the PS and the relative rotation frequency of their impellers. $H_i > 0$ if $i \in I_{PS}$ and $H_i = 0$ if $i \in I_{PL} \cup I_C$. In (1), to simulate a control that is absent or forbidden to change, one can equate the corresponding parameter to a constant. Also (1) can be considered as a generalization of the hydraulic characteristics of the pipeline section.

Part of the parameters of HM, depending on the external environment, we call the boundary conditions. As such, for HSS, pressure in the feed nodes and flows in the remaining nodes are usually indicated. The considered HSS have features: 1) tree in the single-line representation and multi-loop in the two-line one; 2) supply and return pipelines are symmetrical to each other with the exception of PS; 3) supply and return pipelines are connected through HS and consumers with fixed flows. From these features and the form of setting the boundary conditions follows a fixed flow distribution. In this case, the model of controlled flow distribution used in [20], will take the form:

$$\begin{pmatrix} A^T \mathbf{P} - \mathbf{y} \\ \mathbf{y} - \mathbf{h}(\mathbf{x}, \mathbf{z}, \boldsymbol{\gamma}, \boldsymbol{\kappa}) \end{pmatrix} = \mathbf{0} , \quad (2)$$

where: A is the incidence matrix of the HSS calculated scheme; $\mathbf{P}$ is the nodal pressure vector; $\mathbf{y}$ is the vector of pressure drops on the branches; $\mathbf{h}(\mathbf{x},\mathbf{z},\boldsymbol{\gamma},\boldsymbol{\kappa})$ - vector function with components $h_i(x_i,z_i,\gamma_i,\kappa_i)$.

The requirements of acceptability and realizability of the mode are usually written as [20] $\underline{P}_j \le P_j \le \overline{P}_j$, $\underline{y}_i \le y_i \le \overline{y}_i$, $\underline{z}_i \le z_i \le \overline{z}_i$, $\underline{\gamma}_i \le \gamma_i \le \overline{\gamma}_i$, $\kappa_i \in \mathrm{K}_i$, $\kappa_i\gamma_i\underline{\chi}_i \le x_i \le \kappa_i\gamma_i\overline{\chi}_i$, $i \in I$, $j \in J$. $\mathrm{K}_i = \{0,1,2,...,K_i\}$, K_i – number of pumps on PS; $\underline{\chi}_i, \overline{\chi}_i$ – acceptable limits of x_i when $\gamma_i = 1$, $\kappa_i = 1$; $\underline{P}_j$, $\overline{P}_j$ – acceptable limits of P_j; $\underline{z}_i, \overline{z}_i$, $\underline{y}_i, \overline{y}_i$ и $\underline{\gamma}_i, \overline{\gamma}_i$ – acceptable limits of z_i, y_i и γ_i. We introduce the vector of Boolean variables $\boldsymbol{\delta}$, $|\boldsymbol{\delta}| = n$, and inequality $\underline{z}_i \le z_i \le \underline{z}_i + (\overline{z}_i - \underline{z}_i)\delta_i$. Then the restrictions will take the form

$$\begin{gathered}\underline{P}_j \le P_j \le \overline{P}_j,\ \underline{y}_i \le y_i \le \overline{y}_i,\ \underline{z}_i \le z_i \le \underline{z}_i + (\overline{z}_i - \underline{z}_i)\delta_i,\\ \underline{\gamma}_i \le \gamma_i \le \overline{\gamma}_i,\ \kappa_i \in \mathrm{K}_i,\ x_i/\overline{\chi}_i \le \kappa_i\gamma_i \le x_i/\underline{\chi}_i,\ i \in I,\ j \in J.\end{gathered} \tag{3}$$

The variable component of the cost of maintaining the mode consists of the cost of electricity for the PS and the cost of fuel. In the case of HSS with one HS, fuel costs can not be changed due to the redistribution of load between HS. The electric power consumed on a separate PS is equal to $N_i(x_i,\gamma_i,\kappa_i) = \beta_{0,i}\kappa_i\gamma_i^3 + \beta_{1,i}\gamma_i^2 x_i + \beta_{2,i}\gamma_i x_i^2/\kappa_i$ [20]. Here β_0, β_1, β_2 - the coefficients of a square triple approximating the dependence of the power consumed by one pump on the flow rate at $\gamma = 1$. The economic objective function is: $F_C(\mathbf{x},\boldsymbol{\gamma},\boldsymbol{\kappa}) = \sum c_i^{EP} N_i(x_i,\gamma_i,\kappa_i)$, c_i^{EP} - electricity tariff on the i-th PS.

As a target function, the number of throttling points will be taken $F_z(\boldsymbol{\delta}) = \sum \delta_i$ [21], and $F_P(\mathbf{P}) = \sum_{j\in J} P_j/m$ - as an indicator of the level of pressure [21].

It is proposed to take into account several objective functions on the basis of their lexicographical ordering: 1) economic (F_C); 2) discrete technological (F_z); 3) continuous technological (F_P).

Known parameters are: the topology of the design scheme; border conditions; coefficients of hydraulic and power characteristics; permissible limits of change of continuous parameters; the set of possible values of integer parameters; the cost of electricity on each PS. It is necessary to determine the places of application and the values of throttling, the values of rotor speeds on the PS and the number of pumps in operation, pressure and pressure drops under constrains: (2), (3), $F_C = F_C^*$, $F_z = F_z^*$ и $F_P = F_P^*$. Here: $F_C^* = \min F_C$ under constrains (2), (3); $F_z^* = \min F_z$ under constrains (2), (3) and $F_C = F_C^*$; $F_P^* = \min F_P$ under constrains (2), (3), $F_C = F_C^*$ and $F_z = F_z^*$. We write this problem as:

$$\min_{lex}(F_C, F_z, F_P) \text{ under constrains (2), (3).} \tag{4}$$

3 Method

To use LRDP, the following conditions are necessary and sufficient: 1) additivity of the objective functions; 2) the ability to reduce the design scheme of the network to one branch by the equivalent of serial and parallel branches; 3) fixed flow distribution. The topology of the considered HSS differs from the topology of the distribution heat networks for which

LRDP was developed in that the hanging nodes are connected by a branch corresponding to HS. Obviously, the other conditions are also satisfied. Thus, in order to apply the LRDP to solve problem (4), it is necessary and sufficient to ensure correct consideration of the work of the PS.

The intervals of allowable change in nodal pressure are divided into subintervals of ε (pockets). Renumber the pockets so that their numbers represent discrete pressure readings with a certain multiplier. For definiteness, assume that all branches are downstream. The initial node of the *i*-th branch is denoted as f_i, l_i – the final one. Each branch (i) is associated with a set of segments of possible piezometric graphs $L_i = \bigcup\left\{g_i^k\right\}$, k is the index of segment. Each segment g_i^k begins at some pocket of the initial node of the branch (initial pocket, ϕ_i^k) and ends at some pocket of the end node (final pocket, φ_i^k). Each segment g_i^k is associated with the increment of the objective function $\Delta F_i^k = \left(\Delta F_{Ci}^{\ k}, \Delta F_{zi}^{\ k}, \Delta F_{Pi}^{\ k},\right)$. Represent F_P in the form $F_P(\mathbf{P}) = 1/m\sum_j \left|I_j\right| P_j / \left|I_j\right| = 1/m\sum_i \left(P_{f_i}/I_{f_i} + P_{l_i}/I_{l_i}\right)$, I_j – a set of branches, incident to node j. Then $\Delta F_{Ci}^{\ k} = N_i(x_i, \gamma_i^k, \kappa_i^k)$, $\Delta F_{zi}^{\ k} = \delta_i^k$, $\Delta F_{Pi}^{\ k} = \left(\phi_i^k / \left|I_{f_i}\right| + \varphi_i^k / \left|I_{l_i}\right|\right)\varepsilon / m$, where $\gamma_i^k, \kappa_i^k, \delta_i^k$ – model parameters providing g_i^k.

When creating a set of piezometric graphs, it is necessary to take into account all possible control actions. If the branch models the pipeline section of the network, for each segment of the piezometric graph satisfying (2), the value z_i is calculated, ensuring the implementation of this segment. If such a z_i violates (2), this segment is discarded.

For PS, it is proposed to first generate a set of possible pressure drops (y_i^r, r is the pressure drop index). Then, for each drop, the values of κ_i^r, γ_i^r and z_i^r (satisfying condition (2)) are found, providing this drop and the minimum of the objective function F_C. In case pressure drop cannot be maintained, it is discarded. Then, on the basis of the found set of pressure drops, a set of piezometric graphs is constructed that satisfy condition (2). The optimal combination of κ_i^r, γ_i^r and z_i^r for a given pressure drop is as follows. For each possible $\kappa_i^r \in \mathrm{K}_i$, $\underline{\gamma}_i^r\left(\kappa_i^r\right) = \max\left\{\underline{\gamma}_i, x_i \big/ \left(\overline{\chi}_i \kappa_i^r\right), \sqrt{\left(s_i x_i \left|x_i\right| / \kappa_i^r - y_i^r\right) \big/ H_i}\right\}$ and $\overline{\gamma}_i^r\left(\kappa_i^r\right) = \min\left\{\overline{\gamma}_i, x_i \big/ \left(\underline{\chi}_i \kappa_i^r\right), \sqrt{\left(s_i \overline{z}_i x_i \left|x_i\right| / \kappa_i^r - y_i^r\right) \big/ H_i}\right\}$ are computed. If $\underline{\gamma}_i^r\left(\kappa_i^r\right) > \overline{\gamma}_i^r\left(\kappa_i^r\right)$, the given κ_i^r is discarded; otherwise, the value of $\gamma_i^r\left(\kappa_i^r\right)$ is sought on the interval $\left[\underline{\gamma}_i^r\left(\kappa_i^r\right), \overline{\gamma}_i^r\left(\kappa_i^r\right)\right]$ according to the $\min_\gamma N_i\left(x_i, \gamma_i^r, \kappa_i^r\right)$. Then, a search for $z_i^r\left(\kappa_i^r\right)$ is performed for κ_i^r and γ_i^r found. After that, the best is selected from all such (κ_i^r, γ_i^r, z_i^r).

The forward stroke of LRDP consists in the «convolution» of the HSS design scheme with discarding of unacceptable and non-optimal variants of piezometric graphs due to the methods of equivalence of serial and parallel branches to one branch, on which the optimal piezometric graph will remain.

To equivalent parallel branches, the following is done. If there are a pair of piezometric graph segments (g_{i1}^{k1}, g_{i2}^{k2}) on two parallel-connected branches ($i1$, $i2$), for which $\phi_{i1}^{k1} = \phi_{i2}^{k2}$ and $\varphi_{i1}^{k1} = \varphi_{i2}^{k2}$, this pair becomes the segment of the piezometric graph on the equivalent branch, which begins and ends in corresponding pockets. The increments of the objective function are summed up. All segments that are not included in one pair are discarded.

When the serial connection of two branches ($i1$, $i2$) is equivalent (let $f_{i2} = l_{i1}$), all pairs of piezometric graph segments are on these branches (g_{i1}^{k1}, g_{i2}^{k2}), for which $\varphi_{i1}^{k1} = \phi_{i2}^{k2}$. If two pairs of segments are found that connect the same pockets, the pair that has the worst value of the increment of the objective function is discarded. Each of the pairs of segments found on the original HSS fragment turns into a piezometric plot of the equivalent branch with the corresponding initial and final pockets. The increments of the objective function are summed up.

To restore the optimal piezometric graph, it is necessary, for each equivalent branch, to memorize which fragment it is equivalent to, and for each equivalent segment of the piezometric graph, to memorize which segments it is equivalent to.

4 Method testing

The efficiency of the presented method was tested using the example of solving the problem (4) for the aggregated HSS of the city of Baikalsk (Fig. 1) in two cases: 1) changing γ is prohibited on all PS; 2) allowed. On the PS-1, the pumps are installed on the supply pipe, on the rest of the PS - on the return. Since all PSs receive electricity at the same cost, the total power consumption is minimized. Considered two options loads. The first corresponds to the winter mode of operation HSS. In the second mode (lightweight), all loads were reduced by half. PS of this network have a different number of pumps. Table 1 shows their number per PS, type, power and hydraulic factors.

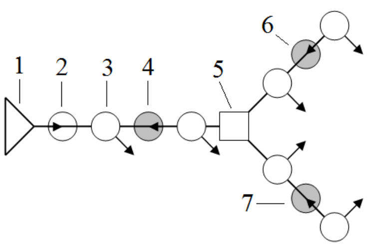

Fig. 1. Scheme of aggregated HSS. 1 - HS; 2 - PS-1; 3 - generalized consumer; 4 - PS-2; 5 - splitter unit; 6 - PS-3; 7 - PS-4.

Table 1. Pumps on PS

PS	Number of pumps (K_i)	Type of pumps	H	s	β_0	β_1	β_2
1	3	SE-800-100-11	120	0.0000375	119.4	0.238	-0.000091
2	5	1X 200-150-500	80	0.00004	40	0.16	0
3	3	TsN-400-105	120	0.000125	60	0.24	0
4	2	D 320-50	60	0.0001	30	0.09	0

The following approach was used as a reference method. First, the values of all Boolean variables were set equal to one and fixed, then the HSS HM with the economic objective function was optimized using the method presented in [20]. After that, to fix the values of F_C, the values of κ_i and γ_i, $i \in I$ were fixed and optimization was performed for the technological objective functions using the method described in [21].

The results of the calculation of LRDP coincided with the results of the calculation by the reference method. Controls on the passive branches are not required. Table 2 shows the number of pumps in operation. In modes marked with an asterisk, changing γ is allowed. The LRDP speed exceeds the reference speed by more than 2 orders of magnitude.

Table 2. Optimization results. Number of pumps included in the PS

Mode	PS-1	PS-2	PS-3	PS-4	Objective function, kW
Winter	3	4	2	0	1445
lightweight	1	2	0	1	426
Winter*	3	4	2	0	1396
lightweight*	1	2	0	1	409
Total pumps	3	5	3	2	

5 Conclusions

1. The article summarizes the original method of multi-objective discrete-continuous optimization of hydraulic modes of distribution thermal networks in the case of radial thermal networks with a single heat source and pumping stations.
2. The new method is implemented in the form of a research program and tested on an aggregated scheme of a real HSS. Calculational experiments confirmed its performance and high computational efficiency.

The study was carried out within the framework of the SB RAS Basic Research Program (AAAA-A17-117030310437-4, project III.17.4.3) and with the financial support of the RFBR and the Government of the Irkutsk Region (project No. 17-48-380021).

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