Method for determining the rational parameters of dynamic dampers of low-frequency vibrations

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Abstract. The problems have been considered of natural nonlinear vibrations of an absolutely rigid semiball and a semicylinder on a horizontal plane, assuming that there is no energy dissipation, sliding and tipping on foundation. To adjust the damper to a frequency close to the fundamental tone of vibrations, it is necessary to assess the natural frequency of the damper, which is determined under the assumption on smallness of the vibrations amplitude. This paper represents the comparison of the natural frequency of linearized and nonlinear system. The relative error has been estimated of the natural frequency calculation, which is caused by linearization. It is shown that the ratio of the natural frequency of the linearized system to the natural frequency of the nonlinear system does not depend on the mass and radius. This conclusion made it possible to generalize the results of particular computational solutions and to obtain a formula which takes into account the amplitude influence on the natural vibrations frequency and helps to determine the natural frequency for the initial angles to ninety degrees.

1 Introduction

The dynamic dampers of vibrations, which are characterized by low-frequency natural vibrations (less than 10 Hz, and often less than 1 Hz), are widely used to reduce the loads in different mechanisms and engineering structures [1] when mining operations and underground space development. In this case, the vibration dampers with rolling bodies are used [2–6], which have high reliability. Their overview is represented in the works [2, 6]. The damper parameters must be such that natural frequency is close to the frequency of fundamental tone of vibrations [2]. Therefore, it is important to know the natural frequency of the vibration damper, which, as a rule, is determined under the assumption on smallness of the vibrations amplitude, which makes it possible to linearize the motion equation. At

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high amplitudes, the nonlinear differential motion equation is solved by numerical methods, which, however, allow to find only particular solutions for specific conditions. There is a need to generalize the particular solutions.

The objective of work is to develop a generalization method of numerical results for determining the rational parameters of dynamic dampers of vibrations, which provide the required natural frequency of nonlinear vibrations.

2 Main part

Let us consider one of the simplest dampers [6], made in the form of a semicylinder or a semiball 1, vibrating on a plane 2 (Fig. 1). When solving a number of applied problems related to the vibrational impact on the loose medium [7, 8], the vibrations of a semicylinder and a semiball can serve as a model representation of the solid particles motion.

When a semicylinder swings, we assume that there is no energy dissipation, sliding and tipping on foundation.

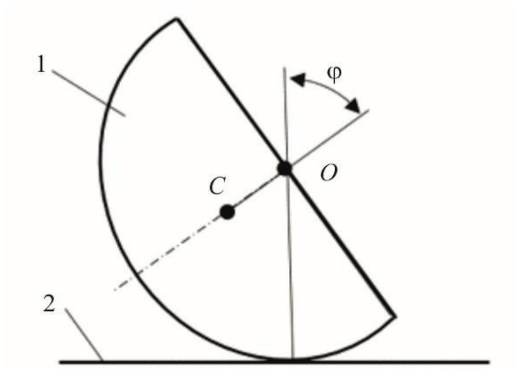

Fig. 1. The computational scheme for damper of vibrations: 1 – a semicylinder or semiball; 2 – plane.

The equation of semicylinder vibrations [9]

$$\left(\frac{3}{2}-\frac{8}{3\pi}\cos\varphi\right)\ddot{\varphi}+\left(\frac{4}{3\pi}\sin\varphi\right)\dot{\varphi}^2+\frac{4g}{3\pi r_1}\sin\varphi=0 \tag{1}$$

where φ – angle; g – gravitational acceleration; r_1 – semicylinder radius (identical symbols used in the description of vibrations of a semicylinder and a semiball, will be recorded with 1 and 2 indices, respectively). A point above the letters means time differentiation. Initial conditions: $\varphi=\varphi_0$ and $\dot{\varphi}=0$. Since the dissipation is not considered, then φ_0 is the amplitude of natural vibrations.

The equation (1) can be represented in a form

$$(9\pi-16\cos\varphi)\ddot{\varphi}+(8\sin\varphi)\dot{\varphi}^2+\frac{8g}{r_1}\sin\varphi=0. \tag{2}$$

The nonlinear equation (2) has no analytical solution. For its linearization, it is assumed that the angle φ is low and take $\sin\varphi\approx\varphi$, $\cos\varphi\approx 1$. As a result, obtain

$$(9\pi - 16)\varphi + \frac{8g}{r_1}\varphi = 0\,. \qquad (3)$$

Dividing (3) by a factor before the second derivative, we get

$$\varphi + \omega_{1,0}^2\varphi = 0\,.$$

where $\omega_{1,0} = 2\cdot\sqrt{\frac{2g}{r_1(9\pi - 16)}}$ – the natural angular frequency.

Then the frequency and period of natural vibrations

$$v_{1,0} = f_{1,0}\sqrt{\frac{g}{r_1}}\,, \qquad (5)$$

$$\mathrm{T}_{1,0} = \frac{1}{f_{1,0}}\sqrt{\frac{r_1}{g}}$$

where $f_{1,0} = \frac{1}{\pi}\sqrt{\frac{2}{(9\pi - 16)}}$

From (5) we determine the radius at which the semicylinder has the natural frequency $v_{1,0}$

$$r_1 = g f_{1,0}^2 v_{1,0}^{-2}\,. \qquad (6)$$

In order to determine the natural frequency of nonlinear vibrations for each specific value r and φ_0, the equation (2) can be solved numerically. However, this makes it difficult to generalize the results. The task may be simplified if to consider that, based on the theorem on the change in the kinetic energy, we have

$$\varphi = 4 f_1(\varphi_0)\sqrt{\frac{g}{r_1}}\,,$$

where $f_1(\varphi_0) = \sqrt{\frac{(\cos\varphi - \cos\varphi_0)}{(9\pi - 16\cos\varphi)}}\,.$

The semicylinder is turned through an angle $d\varphi$ during the time

$$dt = \frac{d\varphi}{\varphi}\,. \qquad (7)$$

By integrating both sides of the equation (7), we determine the vibration period $T_1(\varphi_0)$ and the natural frequency $v_1(\varphi_0)$ with account of the nonlinearity

$$T_1(\varphi_0) = \int_0^{\varphi_0} \frac{1}{f_1(\varphi_0)}\sqrt{\frac{r_1}{g}}\,d\varphi\,,$$

$$v_1(\varphi_0) = \int_0^{\varphi_0} f_1(\varphi_0)\sqrt{\frac{g}{r_1}}d\varphi \ . \tag{8}$$

When integrating, symmetry of a phase portrait with respect to the coordinate axes is taken into account, which made it possible to set the limit values 0 and φ_0; and for obtaining the period, the result should be quadrupled. Figure 2 represents the characteristic form of a phase portrait, which has been obtained by numerical integration of equation (2) using the Runge–Kutta–Fehlberg method of an order 4-5 with $r_1 = 0.1$ m, φ_0 =5°, 10°, 30°, 60° and 90°.

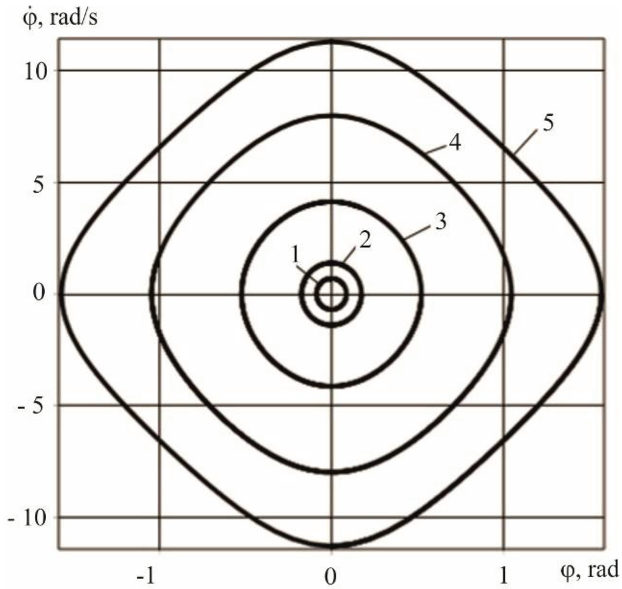

Fig. 2. The characteristic form of a phase portrait of semicylinder vibrations: (r_1=0.1 m, where 1, 2, 3, 4 and 5 – φ_0 =5°, 10°, 30°, 60° and 90°).

Hereinafter, the calculations are performed through mathematical Maple package. Let us find the natural frequencies ratio

$$v_1^*(\varphi_0) = \frac{v_{1,0}}{v_1(\varphi_0)} = \int_0^{\varphi_0} \frac{f_{1,0}}{f_1(\varphi_0)}d\varphi \ . \tag{9}$$

This value will be called the relative natural frequency. Pay attention to the independence $v_1^*(\varphi_0)$ from the mass and radius of the semicylinder - the first provision of the method.

The relative error is caused by linearization of equation (2)

$$\delta_1(\varphi_0) = \frac{v_{1,0} - v_1(\varphi_0)}{v_{1,0}} \cdot 100\,\% = \left(1 - v_1^*(\varphi_0)\right)100\,\% \ .$$

In Figure 3, the round markers represent the results of numerical integration of the expression (9) by Newton-Cotes method, and the triangular markers represent the errors. At amplitudes higher than 60 degrees, the error exceeds 20 %. The lines represent the results of approximation by functions 20°≤ φ_0 ≤ 90°:

$$v_1^*(\varphi_0) = 6 \cdot 10^{-5} \varphi_0^2 + 10^{-4} \varphi_0 + 0.9996\,; \tag{10}$$

$$\delta_1(\varphi_0) = 6.4 \cdot 10^{-3} \varphi_0^2 + 1.38 \cdot 10^{-2} \varphi_0 - 4.4 \cdot 10^{-2}\,. \tag{11}$$

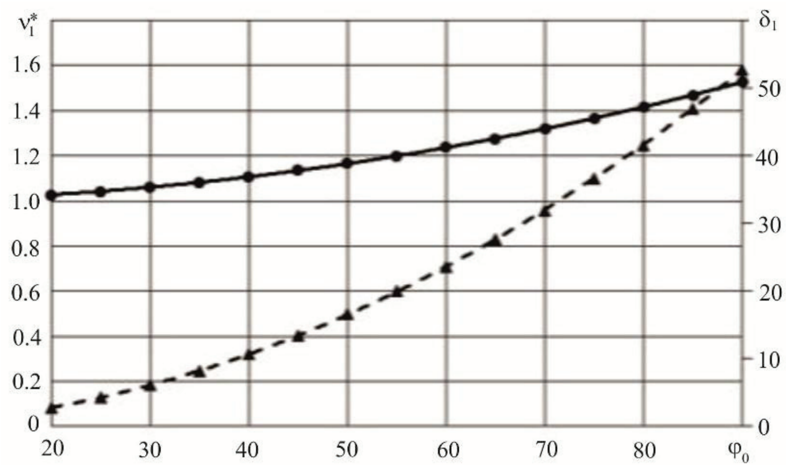

Fig. 3. The dependence of relative natural frequency v_1^* (full line) and error δ_1 (dashed line) on the initial angle φ_0.

The approximation has been performed in the interval 20°≤ φ_0 ≤ 90° by the least square method and ensures high accuracy (determination factor is 0.999). The possibility of approximation of numerical experiments is the second provision of the method.

Formulas (10) and (11) make possible to determine the natural frequency of vibrations of a semicylinder and an error when linearizing the vibration equation. Since, as it shown above, $v_1^*(\varphi_0)$ does not depend on the mass and radius of the semicylinder, taking into account (9) and (10) we obtain the expression for determining the natural frequency of nonlinear vibrations

$$v_1(\varphi_0) = v_{1,0}\left(6 \cdot 10^{-5} \varphi_0^2 + 10^{-4} \varphi_0 + 0.9996\right)^{-1}. \tag{12}$$

Having substituted (5) into (9), and solving relatively $r_1(\varphi_0)$, we obtain the radius at which the semicylinder has the natural frequency $v_1(\varphi_0)$

$$r_1(\varphi_0) = g f_{1,0}^2 \left[v_1(\varphi_0) v_1^*(\varphi_0)\right]^{-2}. \tag{13}$$

In formula (13), in contrast to (6), there is a factor $v_1^*(\varphi_0)$ which takes into account the influence of φ_0 amplitude on the natural frequency v during the nonlinear vibrations.

Let us analyse the vibrations of a damper [6], made in the form of a semiball. When modelling the semiball motion, we use the assumptions and numerical methods described above.

The equation of semiball vibrations [10–12]

$$I_p\ddot{\varphi} + ml^2 \sin\varphi\cos\varphi \cdot \dot{\varphi}^2 + mgl\sin\varphi = 0 . \quad (14)$$

where I_p – the moment of inertia of a semiball relative to the instantaneous centre; m – the semiball mass; l – the distance of the mass centre C from the semiball base O.

The moment of inertia I_p and a distance l are calculated by the formulas:

$$I_p = ml^2\left(\frac{83}{45} + \sin^2\varphi\right),\ l = \frac{3}{8}r_2$$

where r_2 – the semiball base radius.

As a result of the equation (14) linearization, we have

$$I_c\ddot{\varphi} + mgl\varphi = 0 . \quad (15)$$

where $I_c = \dfrac{83}{320}mr_2^2$ – the moment of semiball inertia relative to the horizontal axis, passing through the centre of inertia, and which is perpendicular to the drawing plane.

Having divided (15) by I_c, we get

$$\ddot{\varphi} + \omega_{2,0}^2\varphi = 0 .$$

where $\omega_{2,0} = \sqrt{\dfrac{120g}{83r_2}}$ – the natural angular frequency.

Then the natural frequency of vibrations

$$v_{2,0} = f_{2,0}\sqrt{\frac{g}{r_2}} , \quad (16)$$

where $f_{2,0} = \dfrac{1}{2\pi}\sqrt{\dfrac{120}{83}}$.

From (16) it follows that a semiball has the natural frequency $v_{2,0}$ with a radius

$$r_2 = gf_{2,0}^2 v_{2,0}^{-2} . \quad (17)$$

Having done the same way as when determining the period of nonlinear vibrations of a semicylinder, we have:

$$\phi = 4f_2(\varphi_0)\sqrt{\frac{g}{r_2}} ,$$

$$T_2(\varphi_0) = \int_0^{\varphi_0} \frac{1}{f_2(\varphi_0)}\sqrt{\frac{r_2}{g}}d\varphi ,$$

$$v_2(\varphi_0) = \int_0^{\varphi_0} f_2(\varphi_0)\sqrt{\frac{g}{r_2}}d\varphi , \quad (18)$$

where $f_2(\varphi_0) = \sqrt{\dfrac{(\cos\varphi - \cos\varphi_0)}{3(83/45 + \sin^2\varphi)}}$.

The phase portrait is not represented, since it is qualitatively similar to the phase portrait of semicylinder vibrations.

Determine the relative natural frequency

$$v_2^*(\varphi_0) = \frac{v_{2,0}}{v_2(\varphi_0)} = \int_0^{\varphi_0} \frac{f_{2,0}}{f_2(\varphi_0)} d\varphi . \tag{19}$$

Pay attention to the fact that in this case as well the $v_2^*(\varphi_0)$ does not depend on the mass and radius of the damper (the first provision of the method has been satisfied).

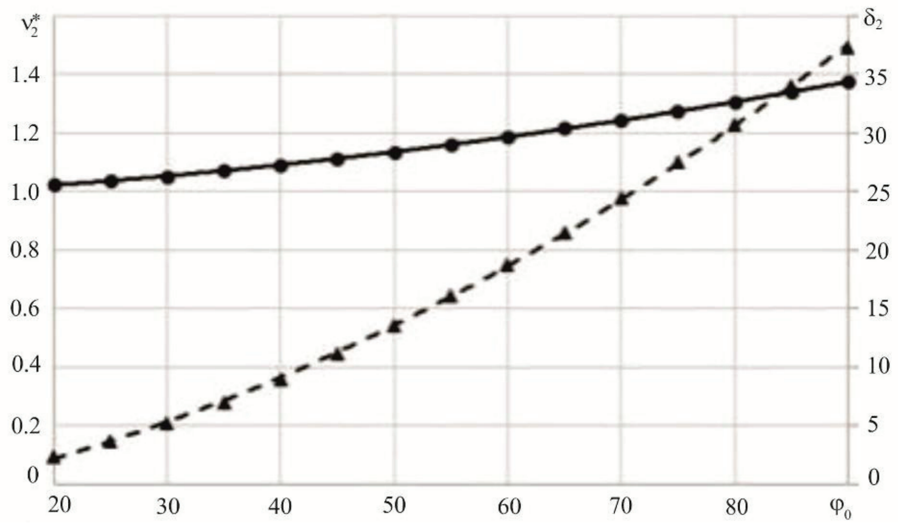

Fig. 4. The dependence of relative natural frequency v_2^* (full line) and error δ_2 (dashed line) on the initial angle φ_0.

The relative error of linearization of the equation (14)

$$\delta_2(\varphi_0) = \frac{v_{2,0} - v_2(\varphi_0)}{v_{2,0}} \cdot 100\% = (1 - v_2^*(\varphi_0))100\% .$$

In Figure 3, the results are represented of the numerical integration of expression (19) and the error $\delta_2(\varphi_0)$, as well as their approximation in the range $20° \leq \varphi_0 \leq 90°$ through functions (determination factor is 0.999):

$$v_2^*(\varphi_0) = 3 \cdot 10^{-5} \varphi_0^2 + 1.6 \cdot 10^{-3} \varphi_0 + 0.9763 ; \tag{20}$$

$$\delta_2(\varphi_0) = 3.1 \cdot 10^{-3} \varphi_0^2 + 0.1621\varphi_0 - 2.3747$$

– the second provision of the method has been satisfied.

With account of (19) and (20), we obtain an expression for determining the natural frequency of nonlinear vibrations

$$v_2(\varphi_0) = v_{2,0}\left(3 \cdot 10^{-5} \varphi_0^2 + 1.6 \cdot 10^{-3} \varphi_0 + 0.9763\right)^{-1}. \tag{21}$$

From (19), with account of (16), we obtain the radius at which the semiball has the natural frequency $v_2(\varphi_0)$

$$r_2(\varphi_0) = g f_{2,0}^2 \left[v_2(\varphi_0) v_2^*(\varphi_0)\right]^{-2}. \tag{22}$$

In formula (22), the factor $v_2^*(\varphi_0)$ accounts for the influence of the amplitude φ_0 on the natural frequency at nonlinear vibrations.

Let us connect the solutions for a semicylinder and a semiball, presenting the formulas (5), (6), (9), (12), (13), (16), (17), (19), (21) and (22) in such a form

$$v_{i,0} = f_{i,0}\sqrt{\frac{g}{r_i}}; \tag{23}$$

$$v_i^*(\varphi_0) = \frac{v_{i,0}}{v_i(\varphi_0)} = \int_0^{\varphi_0} \frac{f_{i,0}}{f_i(\varphi_0)} d\varphi; \tag{24}$$

$$v_1(\varphi_0) = v_{1,0}\left(6 \cdot 10^{-5} \varphi_0^2 + 10^{-4} \varphi_0 + 0.9996\right)^{-1}; \tag{25}$$

$$v_2(\varphi_0) = v_{2,0}\left(3 \cdot 10^{-5} \varphi_0^2 + 1{,}6 \cdot 10^{-3} \varphi_0 + 0.9763\right)^{-1}; \tag{26}$$

$$r_{i,0} = g f_{i,0}^2 v_{i,0}^{-2};\ r_i(\varphi_0) = g f_{i,0}^2 \left[v_i(\varphi_0) v_i^*(\varphi_0)\right]^{-2}, \tag{27}$$

where i – for a semicylinder – 1, and for a semiball – 2;

$$f_{1,0} = \frac{1}{\pi}\sqrt{\frac{2}{(9\pi - 16)}};\ f_{2,0} = \frac{1}{2\pi}\sqrt{\frac{120}{83}}.$$

The proposed method made it possible to obtain formulas (23) - (27) which account for the influence of the radii on the natural frequencies of a semicylinder and a semiball at both low and high vibration amplitudes. In the first case, the vibrations are linear, and in the second – nonlinear, dependent on amplitude.

Conclusions

The method for generalizing the numerical experiments has been proposed in order to determine the influence of radii on the natural frequencies of the nonlinear vibrations of a semicylinder and a semiball, which consists in the following:

1. The vibrations of a nonlinear system are characterized by a relative frequency $v_i^*(\varphi_0)$, depending only on the amplitude. The absence of such body parameters as mass and radius is achieved by representing the natural vibrations frequency as a product of two factors, one of which is $f_{i,0}$, and the second is $\sqrt{g/r_i}$.

2. The results of numerical experiments for determining the relative frequency are approximated by a second degree polynomial of the amplitude.

The rational radii (27) of dynamic dampers of vibrations have been determined.

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