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Mathematical modeling of belt and rigid conveyer's drum contact with non-affect Coulomb friction

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Abstract. Conveyor's belt with the drum contact task is paid a great researcher attention. It had solved in stresses with the mix boundary condition and Coulomb friction low action applies. Obtained solutions, as usual, do not satisfy to the displacement on drum surface boundary condition. The attempt to solve the system of Lamb equations in cylindrical coordinates in condition h/R<<1 values where h the belt thin and R drums radius is done. The Prandtl's technique and substitution methods are used to analytical solution obtain. As a result, the displacement expressions for rest and slide drum arcs are obtained. In search of rest arc length, radial and peripheral displacement equalities are used. Applied rest arc efforts are equal in values and have an opposite direction. The equality means the fact that traction efforts do not transfer under rest drum arc. The obtained results are coincided with the ones obtained by Zhukovskiy N.E. for flexible thread model. The fact to simultaneously satisfy to boundary conditions on inner and outer belt sides have to do the conclusion for boundary belt seam existing. Expressions for rest drum arc length without applied friction law are obtained. The displacement and corresponding stresses graphics are demonstrated.

1 Introduction

One of the effective ways to track drive effect increasing on drums surface is achieved with drums cover [1]. It is paid great attention by researchers [1 – 6] in the way of belt and cover drum interaction. Therefore O.V. Andreev [2], using the photo-elastic methods had confirmed slide and slide arc (SA) existing and track transmit on rest arc (RA). Track transmit he explain as belt cross sections deplanations caused with tensile forces. There considerably results in studies belt and cover drum interaction had achieved with scientific school under leading V. I. Mossakovskiy [3; 4]. The cover drum was modeling as Winkler base [3] and one was change more complete Vlasov-Zimmerman base [4]. But, as had found, that for both bases cover drum rigid approaching to infinite, as for coverless drum, occurs belt and drum slide. Besides, therefrom the solving is carrying into stress and strain jumping, that under the choose models is hard to explain.

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Sciences of Ukraine researches, non-experimental had confirmed RA track efforts transmit [5], but had made preposition for boundary layer being in contact zone [6].

The cover drums to belt efforts transmit task had solved under the boundary layer being [6]. The task had solved for stresses under mixed boundary conditions and using Coulomb friction law. Expand L. Prandtl asymptotic technique [7] with the boundary layer allowed to simplify the task and achieved the analytic solving. The solving analysis allowed not to confirm the truck efforts transmit, but explained its mechanism. However, the attempt to explain the boundary conditions in belt and drum contact fulfillment in deflections was failed. Contact task solving for rigid drum and belt interaction in deflections under non-Coulomb friction law using allow explaining mechanism of high molecular (viscous) materials wear under boundary layer absence.

2 Setting the task

The elastic and flexible belt (Fig. 1) surrounds the rigid drum with envelope angle α. The radius of the drum R is considered much larger than the belt thickness h, i.e. $h/R<<1$. Deformations along the cylindrical drum guides are considered small, and they can be neglected. The envelope angle α is divided into two parts - an adjoining zone RA $0<\varphi<\varphi^*$ and the SA $\varphi^*<\varphi<\alpha$, where φ is a current arc variable and φ^* is the φ value that divide the RA and SA parts. The drum rotates in a clockwise direction with a constant angular velocity. Angle counting is from the runoff point $\varphi=0$.

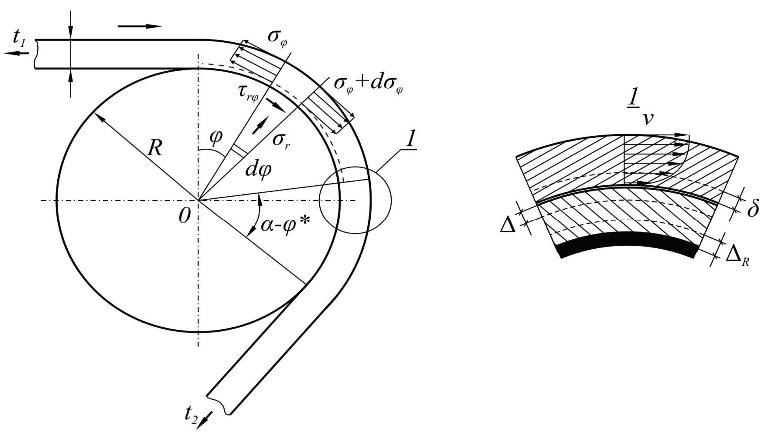


Fig. 1. Scheme of the task.

The coordinate system for the reference of the radial displacement u and the circumferential displacement v has a beginning at the point $\varphi = 0$ and is connected with the drum body.

The problem is to simplify and obtain the solution of equilibrium equations in displacements recorded in cylindrical coordinates (Lamb's equation) for a flat case [8]:

$$(\lambda+\mu)\frac{\partial\varepsilon}{\partial r}-\mu\left(\frac{u}{r^2}+\frac{2}{r^2}\frac{\partial v}{\partial\varphi}\right)+\mu\Delta u=0;\quad (\lambda+\mu)\frac{\partial\varepsilon}{r\partial\varphi}-\mu\left(\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial\varphi}\right)+\mu\Delta v=0, \quad (1)$$

where $\varepsilon=\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\varphi}$; $\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$, λ, μ – elasticity constants [8], r –current radius variable.

3 Problem solving

After system equations (1) components evaluating, with taking into account its smallness h/R<<1, the system of equations (1) will have the form:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial r^2}+(\lambda+\mu)\frac{1}{r}\frac{\partial^2 v}{\partial r\partial\varphi}=0,\quad \mu\left(\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial\varphi}\right)=0 \tag{2}$$

The boundary conditions on the RA may be written as:
under $r=R\quad u=v=0$, under $r=R+h\quad \sigma_r=\tau_{r\varphi}=0$,

$$\text{under } \varphi=0\quad \int_R^{R+h}\sigma_\varphi dr=t_1\text{; under } \varphi=\varphi^*\quad \int_R^{R+h}\sigma_\varphi dr=t_2^*. \tag{3}$$

The boundary conditions on the SA will be written as:
Under $r=R\quad u=0$, under $r=R+h\quad \sigma_r=\tau_{r\varphi}=0$,

$$\text{under } \varphi=\varphi^*\quad \int_R^{R+h}\sigma_\varphi dr=t_1^*\text{; under } \varphi=\alpha\quad \int_R^{R+h}\sigma_\varphi dr=t_2. \tag{4}$$

After evaluating the expressions with taking into account its smallness h/R<<1, we obtain:

$$\sigma_r=\lambda\left(\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial\varphi}\right)+2\mu\frac{\partial u}{\partial r};\quad \sigma_\varphi=\lambda\left(\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial\varphi}\right)+2\mu\frac{1}{r}\frac{\partial v}{\partial\varphi};\quad \tau_{r\varphi}=\mu\frac{\partial v}{\partial r}.$$

4 Problem solving on a RA

Using the methods of substitution to the system of equations (2) with boundary conditions (3) we obtain the solution in the form:

$$u=-D_1(\varphi)\ln(r)+D_2(\varphi)r+D_3;\quad v=-\frac{\lambda+2\mu}{\lambda+\mu}D_1^*(\varphi)\ln(r)+C_2(\varphi) \tag{5}$$

It should be noted, that the subsequent letter D might be marked by the integration of the system. By satisfying the boundary condition $u=0$ for $r=R$ we have:

$$u=D_1(\varphi)\ln(R/r)+D_2(\varphi)(r-R), \tag{6}$$

Satisfying the boundary condition $v=0$ for $r=R$ we have:

$$v=\frac{\lambda+2\mu}{\lambda+\mu}D_1^*(\varphi)\ln(R/r). \tag{7}$$

We now satisfy the second boundary condition (3) $\sigma_r=0$ for $r=R+h$,

$$\begin{aligned}u&=(R+h)D_2(\varphi)\ln(R/r)+D_2(\varphi)(r-R),\\ v&=\frac{\lambda+2\mu}{\lambda+\mu}(R+h)D_2^*(\varphi)\ln(R/r),\end{aligned} \tag{8}$$

where $D_2^*(\varphi)=\int D_2(\varphi)d\varphi$.
We now satisfy the second boundary condition (3) $\tau_{r\varphi}=0$ for $r=R+h$.

Since $\tau_{r\varphi} = \mu \dfrac{\partial v}{\partial r}$ we have

$$\begin{aligned}&\tau_{r\varphi} = \mu \frac{\partial v}{\partial r} \Rightarrow -\mu \frac{(\lambda+2\mu)}{(\lambda+\mu)} \frac{D_2^*}{r} = 0 \Rightarrow D_2^*(\varphi) = 0 \Rightarrow \\ &\Rightarrow D_2^*(\varphi) = \int D_2(\varphi)\mathrm{d}\varphi = 0 \Rightarrow D_1(\varphi) = 0\end{aligned} \tag{9}$$

Since $D_2(\varphi) = 0$, then the solution of system (2) becomes trivial $u = v = 0$.

Consequently, we cannot simultaneously satisfy two boundary conditions v=0 for r=R and $\tau_{r\varphi}$=0 for r=R+h. Let us refuse the boundary condition $\tau_{r\varphi}$=0 for r=R+h and in the future we will analyze the causes of this phenomenon.

By satisfying the boundary conditions for φ=0 and φ=φ^*:

$$\int_R^{R+h} \sigma_\varphi dr = t_1; \quad \int_R^{R+h} \sigma_\varphi dr = t_2^* \tag{10}$$

we will receive:

$$\begin{aligned}&\lambda D_2(0)h = t_1; \lambda D_2(\varphi^*)h = t_2^*; \\ &D_2(0) = \frac{t_1}{\lambda h}; D_2(\varphi^*) = \frac{t_2^*}{\lambda h}.\end{aligned}$$

In the first approximation we assume the function $D_2(\varphi)$ to be linear:

$$D_2(\varphi) = D_2(0) + \frac{D_2(\varphi^*) - D_2(0)}{\varphi^*}\varphi = \frac{t_1}{\lambda h} + \frac{(t_2^* - t_1)\varphi}{\lambda h \varphi^*} \tag{11}$$

Then the solution (8) has the form:

$$\begin{aligned}&u = \left(\frac{t_1}{\lambda h} + \frac{(t_2^* - t_1)\varphi}{\lambda h \varphi^*}\right)((R+h)\ln(R/r) + r - R), \\ &v = \frac{\lambda+2\mu}{\lambda+\mu}(R+h)\left(\frac{t_1\varphi}{\lambda h} + \frac{(t_2^* - t_1)\varphi^2}{\lambda h \varphi^*}\right)\ln(R/r) + D_3.\end{aligned} \tag{12}$$

To determine a stable D_3 we satisfy the condition v=0 for φ=*0*, r=R

$$v = \frac{\lambda+2\mu}{\lambda+\mu}(R+h)\left(\frac{t_1\varphi}{\lambda h} + \frac{(t_2^* - t_1)\varphi^2}{2\lambda h \varphi^*}\right)\ln(R/R) + D_3 = 0 \Rightarrow D_3 = 0.$$

The solution (12) has the form:

$$\begin{aligned}&u = \left(\frac{t_1}{\lambda h} + \frac{(t_2^* - t_1)\varphi}{\lambda h \varphi^*}\right)((R+h)\ln(R/r) + r - R), \\ &v = \frac{\lambda+2\mu}{\lambda+\mu}(R+h)\left(\frac{t_1\varphi}{\lambda h} + \frac{(t_2^* - t_1)\varphi^2}{2\lambda h \varphi^*}\right)\ln(R/r).\end{aligned} \tag{13}$$

The test showed that the solution (13) satisfies both the equation (2) and the boundary conditions (3), except for the indicated.

Expressions for stresses taking into account the smallness h/R<<1 and the resulting solution (13) will have the form:

$$\sigma_r = \frac{\lambda + 2\mu}{\lambda + \mu}\left(\frac{t_1}{\lambda h} + \frac{\left(t_2^* - t_1\right)\varphi}{\lambda h \varphi^*}\right)\left(\left(\lambda + \mu\right)\left(r - \left(R + h\right)\right) - \lambda\left(R + h\right)\ln\left(R/r\right)\right),$$

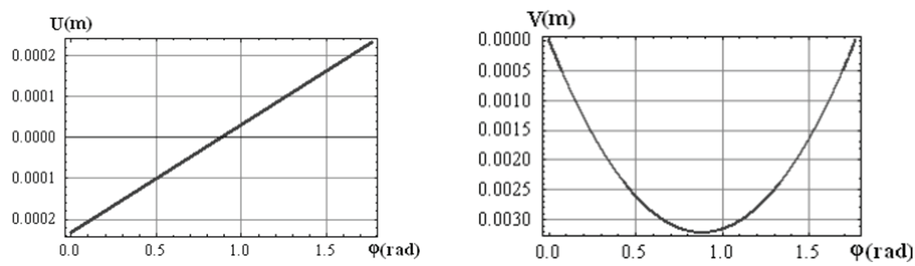
$$\sigma_\varphi = \left(\frac{t_1}{\lambda h} + \frac{\left(t_2^* - t_1\right)\varphi}{\lambda h \varphi^*}\right)\left(\frac{\left(\lambda + 2\mu\right)^2}{\lambda + \mu}\frac{R + h}{r}\ln\left(R/r\right) + \lambda\left(1 - \frac{R + h}{r}\right)\right), \quad (14)$$

$$\tau_{r\varphi} = -\mu\frac{\lambda + 2\mu}{\lambda + \mu}\frac{R + h}{r}\left(\frac{t_1\varphi}{\lambda h} + \frac{\left(t_2^* - t_1\right)\varphi^2}{2\lambda h \varphi^*}\right).$$

It should be noted that at the belt surface $r=R+h$ in the RA under the conditions of the restrictions on the deformation in the circumferential direction v on the surface of the drum $r=R+h$, the shear stresses τ do not equal zero (14).

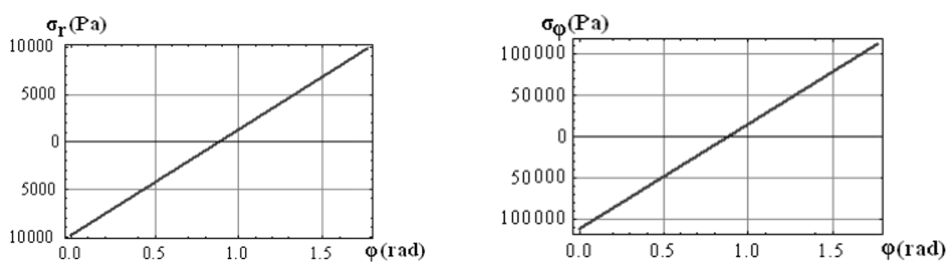
In the absence of such restrictions, the shear stress τ is zero on the tape surface $r=R+h$. Graphic dependences of displacements along the RA are presented in Figure 2 and stresses in Figure 3 and Figure 4.

Graphic dependencies are demonstrated under the output data: $E_m=1.2\cdot10^6$ Pa; $\nu=0.4$; $R=0.25$ m; $h=0.0134$ m; $\alpha=\pi$; $\varphi_0=\alpha$; $t_2=5000$ N; $t_1=4\cdot t_2$; $r=(R+h/2)$ m.



a) b)

Fig. 2. Distribution of displacements: a) radial, b) circular.



a) b)

Fig. 3. Distribution of stresses: a) radial, b) circular.

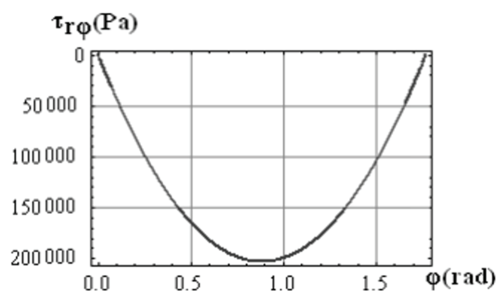
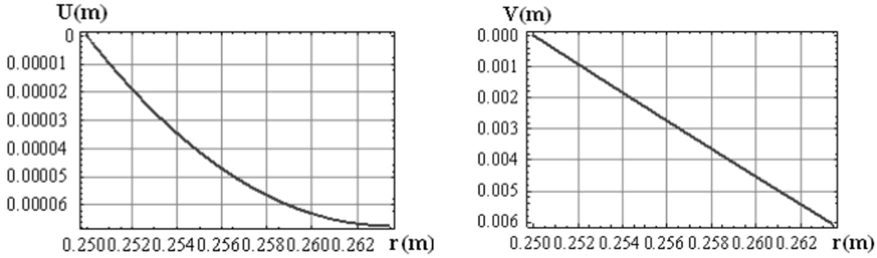


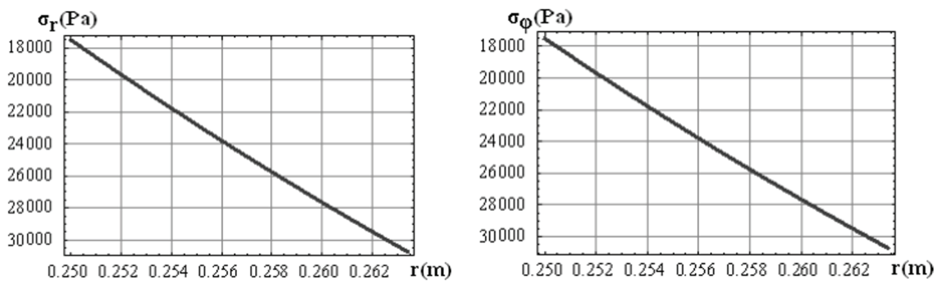
Fig. 4. Distribution of tangential stresses.

The distribution of displacement along the radius on the RA is presented in Figure 5 and stresses in Figure 6 and Figure 7.



a) b)

Fig. 5. Distribution of displacements along the radius ($\varphi=\varphi^*/2$): a) radial, b) circular.



a) b)

Fig. 6. Distribution of stresses: a) radial, b) circular.

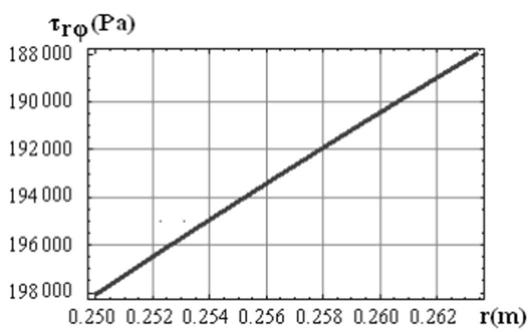


Fig. 7. Distribution of shear stresses.

5 Problem solving on the slide arc

The solution of the system of equations (2) with boundary conditions (4) is presented below:

$$
\begin{aligned}
u_{sl} &= \left(\frac{t_1^*}{\lambda h} + \frac{\left(t_2 - t_1^*\right)\varphi}{\lambda h\left(\varphi_0 - \varphi^*\right)} \right)(r - R), \\
v_{sl} &= -\frac{(\lambda + 2\mu)}{\lambda}(R + h)\left(\frac{t_1^*\varphi}{\lambda h} + \frac{\left(t_2 - t_1^*\right)\varphi^2}{2\lambda h\left(\varphi_0 - \varphi^*\right)} \right) + D_4.
\end{aligned}
\tag{15}
$$

To find a stable D_4 we will satisfy the condition $v=v_0$ for $\varphi=\varphi^*$ and $r=R$, where

$$v_0 = \frac{\lambda + 2\mu}{\lambda + \mu}(R+h)\left(\frac{t_1\varphi^*}{\lambda h} + \frac{(t_2^* - t_1)\varphi^*}{2\lambda h}\right)\ln(R/R) = 0,$$

then $$D_4 = \frac{(\lambda + 2\mu)}{\lambda}(R+h)\left(\frac{\varphi^*}{\lambda h} + \frac{(t_2 - t_1^*)(\varphi^*)^2}{2\lambda h(\varphi_0 - \varphi^*)}\right).$$

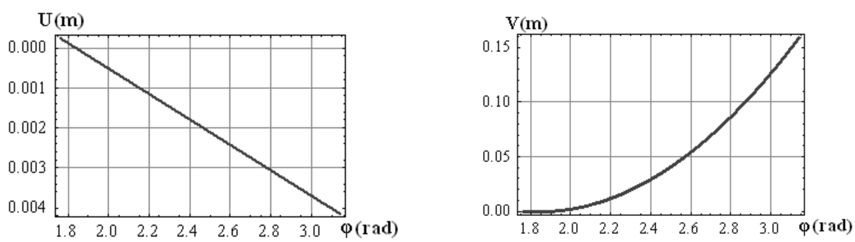
Thus (15) can be written as:

$$u_{sl} = \left(\frac{t_1^*}{\lambda h} + \frac{(t_2 - t_1^*)\varphi}{\lambda h(\varphi_0 - \varphi^*)}\right)(r - R),$$
$$v_{sl} = \frac{(\lambda + 2\mu)}{\lambda}(R+h)\left(\frac{t_1^*(\varphi - \varphi^*)}{\lambda h} + \frac{(t_2 - t_1^*)(\varphi^2 - (\varphi^*)^2)}{2\lambda h(\varphi_0 - \varphi^*)}\right). \tag{16}$$

Expressions for stresses we have in the form:

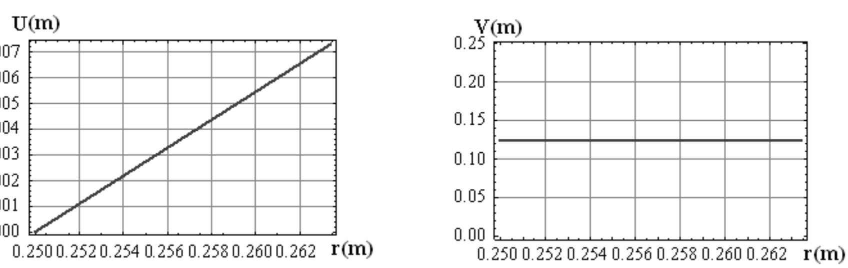
$$\sigma_\varphi = \frac{\lambda^2 r - (\lambda + 2\mu)^2(R+h)}{\lambda r}\left(\frac{t_1^*}{\lambda h} + \frac{(t_2 - t_1^*)\varphi}{\lambda h(\varphi_0 - \varphi^*)}\right),$$
$$\sigma_\varphi = (\lambda + 2\mu)\left(1 - \frac{(R+h)}{r}\right)\left(\frac{t_1^*}{\lambda h} + \frac{(t_2 - t_1^*)\varphi}{\lambda h(\varphi_0 - \varphi^*)}\right), \tag{17}$$
$$\tau_{r\varphi} = 0.$$

The distribution of displacements along the SA and along the radius is shown in Figure 8 and Figure 9 respectively:



a) b)

Fig. 8. Distribution of displacements: a) radial, b) circular.



a) b)

Fig. 9. Displacement distributions along the radius ($\varphi=(\varphi_0+\varphi^*)/2$): a) radial, b) circular.

Graphs of the displacement distributions on envelope angle α is presented in Figure 10.

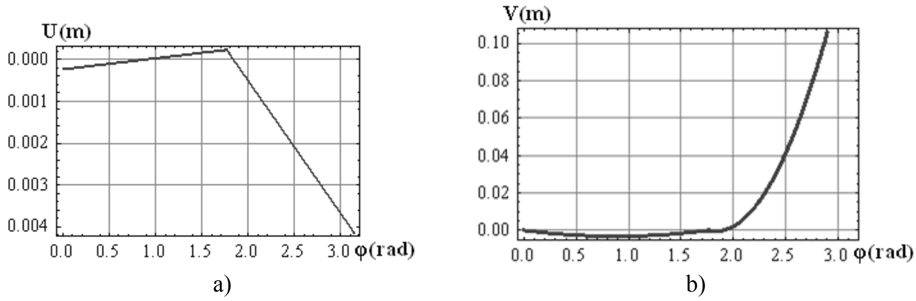


Fig. 10. Displacement distributions: a) radial, b) circular.

It should be noted, that the vast majority of researchers formulate the conditions of work on the RA and SA contacts with taking into account the Coulomb friction law.

The condition of the RA length finding as a rule [3-5], chooses the equality of shear stresses on the rest and slide arcs $\tau_{r\varphi}^{rest}=\tau_{r\varphi}^{slide}$.

Since the problem is solved in displacements, then the condition for finding RA and SA lengths should be in displacements. We find the boundary of the RA based on the conditions of radial and circumferential displacements at $\varphi=\varphi^*$ equality:

$$u_{rest}=u_{slide}\ ;\ v_{rest}=v_{slide} \text{ for } \varphi=\varphi^*$$

or

$$\begin{cases} \left(\dfrac{t_1}{\lambda h}+\dfrac{(t_2^*-t_1)\varphi}{\lambda h\varphi^*}\right)\left((R+h)\ln(R/r)+r-R\right)=\left(\dfrac{t_1^*}{\lambda h}+\dfrac{(t_2-t_1^*)\varphi}{\lambda h(\varphi_0-\varphi^*)}\right)(r-R), \\ \dfrac{\lambda+2\mu}{\lambda+\mu}(R+h)\left(\dfrac{t_1\varphi}{\lambda h}+\dfrac{(t_2^*-t_1)\varphi^2}{2\lambda h\varphi^*}\right)\ln(R/r)= \\ \qquad =\dfrac{(\lambda+2\mu)}{\lambda}(R+h)\left(\dfrac{t_1^*(\varphi-\varphi^*)}{\lambda h}+\dfrac{(t_2-t_1^*)\left(\varphi^2-(\varphi^*)^2\right)}{2\lambda h(\varphi_0-\varphi^*)}\right), \end{cases}$$

where u_{rest}, v_{rest}, u_{slide}, v_{slide} – displacements u and v on the RA and SA respectively. Since the conditions of two unknowns must be two: φ^* and t_2^*. We receive:

$$\begin{aligned} &t_2^*=-t_1, \\ &\varphi^*=\frac{(t_1/t_2)\varphi_0\left(2(r-R)+(R+h)\ln(R/r)\right)}{(r-R)\left(3(t_1/t_2)-1\right)+(R+h)\ln(R/r)}. \end{aligned}$$

The condition for RA existence in the form $t_2^*=-t_1$ corresponds to the condition of the RA existence, which was obtained by M.E. Zhukovskiy for a belt model in the form of a flexible thread. Equality of track efforts in RA of the $t_2^*=-t_1$ means that the track effort is not transferred to the RA. However, as proved theoretically [5], and shown experimentally [6], in RA, efforts are still transmitted. This fact can only be explained by the efforts transmit in the boundary layer.

Graphic dependences of the RA length on the ratio of forces t_1/t_2 for $\varphi_0=\pi$ by:

The author, Kyria R.V. [5], and N.E. Zhukovsky are given in Figure 11.

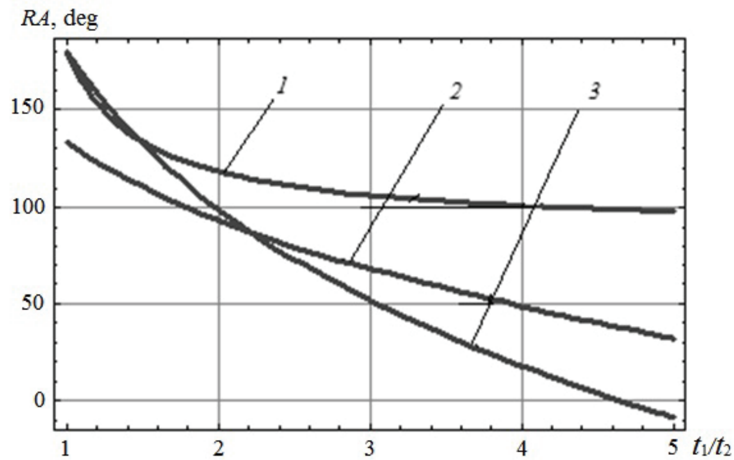


Fig. 11. The length of the RA from the ratio of the traction forces t_1/t_2: 1 – Author; 2 – Kyria; 3 – Zhukovsky.

Obtained graphic dependencies have a similar behavior, that confirm the correct way to solving the problem. Investigations in this direction are conducting.

Conclusions

The problem of the displacement and stresses distributions on the RA (2), (3) and SA (2), (4) is solved.

Satisfaction with all boundary conditions (3) leads to a trivial solution. That is, there is some contradiction between the boundary conditions on the contact of the belt and the drum and on the free surface of the belt. Thus, the exclusion of circumferential displacements on the contact of the belt and the drum leads to shear stresses on the surface of the belt. Nevertheless, we know from the practice that they are not there. It remains to conclude that there is a layer in the belt that takes on the action of shear stresses. In these circumstances, the contradictions in the boundary conditions on the inner and outer surfaces of the belt are eliminated, and then a nontrivial solution of the problem (2), (3) will exist. Thus, we can assume the existence of a boundary layer in the body of the bent, which implements the action of shear stresses. The layer of the belt, with the exception of the boundary one, is in simple stretch condition - the distribution of radial and circumferential stress along the RA is linear (Figure 3a, b). However, the circumferential displacements (Figure 2 a, b), and even shear stresses (Figure 4), take their maximum values in the middle of the RA.

The discontinuous behavior of radial and circumferential stresses can be explained by the displacements behavior when passing from the RA side to the SA. Expression for stresses (14), (17) contains partial derivatives of these displacements, and this fact leads to jumps in stresses. The presence of discontinuous in the behavior of circumferential and radial stresses, when moving from the RA to the SA, can be explained by the existence of a transition zone by φ, in which the jump is reduced.

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