

Response of oscillatory system “liquid layer-rod” to driving disturbances

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Abstract. This article deals with main characteristics of the oscillating system "central body – liquid" by means of its well-known representation in the form of a pendulum mathematical model. It makes possible to evaluate spread of specified disturbances at the general physical level and to determine the most dangerous frequencies that lead to increased amplitudes of fluid oscillations. We propose equations for single-frequency pendulums, which influence each other by means of resistance forces and added mass. Several examples with different natural frequencies of the body are considered. The calculation results showed that besides natural frequencies of the components, system has two more natural frequencies. So, system actually responds only to disturbances which frequencies are close to the natural frequency of the liquid layer. In this case, amplitudes of liquid and the body sharply increase. This fact indicates that in real technological processes frequency of dominant disturbance should be as far from the first resonant frequency of the liquid as possible. The further experimental and theoretical studies that take into account the influence of the following modes on the dynamic picture of the process are also of interest.

1 Introduction

A number of chemical-technological operations, including oscillations in tanks with a central body, take place in mineral processing processes [1], as well as in metallurgy, in particular, in the treatment of molten iron in ladles [2]. Here, the strong agitation of the liquid can lead to large dynamic loads on the fasteners of the central body and to splash of liquid. In addition, interest in the oscillations of the fluid it is connected associated with problems of transportation of liquid media in various tanks. Actually it is related to all types of transportation - by rail, by sea or by air cargo. This field of mechanics received a great impulse by development of rocket technology, since fluid fluctuations (fuel and oil) in the tanks cause significant disturbances in rocket dynamics and significantly complicate flight

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control. Beside to practical interest in influence of oscillations on the liquid media, there are also theoretical questions about the occurrence of these oscillations and interactions between oscillating fluid and the vessel or body.

The study of generation and spread of surface waves in fluid layers is one of the main fields in mechanics. There is large number of publications devoted to this problem [3, 4]. Actual part of this scientific direction is the wave motion of fluid in tanks. Here important results were obtained [5, 6], revealing the dynamic characteristics of wave processes in vessels. The basis of these studies is a theory of potential flows of incompressible fluids. At present, dynamic effects of wave motion in layers often prevail over viscous effects, but this theory has not lost its significance. In spite of development of the powerful numerical methods, the application of this theory to applied problems is accompanied by certain difficulties. A fundamentally important simplification in considering the dynamics of bodies with liquid layers, is the representation of oscillating layers in a form of pendulums [7]. This formulation does not fully disclose the process of interaction, but at general physical level shows important details of the motion spread in considered oscillatory system.

In this paper, we use this approach to study the interaction between oscillating fluid layer and central body immersed in it. Let's assume the body and the liquid layer are single-frequency pendulums. As a rule, the first mode practically determines the dynamic characteristics of the system and the loads occurring in the process. In our case, solution will suggest the conditions when the most severe situations arise. However, a multimode (theoretically infinite) oscillation pattern is typical to liquid surface. In this case, a certain frequency range may have significant effect. So, for example, the splashes themselves, of course, are the result of low-frequency oscillations, however, in addition to such relatively rare rises, local high swelling and even cumulative jets generated by much higher frequencies occur [8]. The frequency analysis of the layer surface and the bottom zone in [8, 9] shows a wide range of frequencies that is characteristic of this process. This suggests that considered model unable to cover many interesting aspects of the process, but it can be useful in practice. Thus, let us consider the fluid layer and the body as a single oscillatory system with their own dynamic characteristics. It is exposed to some disturbances that set it in motion. It also contains formulas for evaluation the main departure frequency. It is rather difficult to estimate the force effect of the gas component, therefore, in this formulation, the degree of the oscillatory system susceptibility to a certain disturbance frequency at a constant amplitude will be traced. The concept of system susceptibility is well known in the theory of turbulent boundary layers [10].

2 Mathematical formulation of the problem

Standard equation for pendulum oscillation includes equality of inertial and elastic forces. In our case, it is also necessary to include the forces of interaction between the two bodies: the resistance force and force associated with the added mass [11]. Thus, the equations for fluid and body motion (the tank motion is not considered) can be represented as

$$M_G \frac{dU_G}{dt} = -K_G X_G - M_p \left(\frac{dU_G}{dt} - \frac{dU_F}{dt} \right) - k\rho_G S_F (U_G - U_F) + a \sin(2\pi f_p t), \quad (1)$$

$$M_F \frac{dU_F}{dt} = -K_F X_F + M_p \left(\frac{dU_G}{dt} - \frac{dU_F}{dt} \right) + k\rho_G S_F (U_G - U_F), \quad (2)$$

where t – time; X_G, X_F – movement of fluid and body mass; U_G, U_F – velocities of liquid layer and body; M_G, M_F, M_p – mass of liquid, body and added mass; K_G, K_F – elastic ratios;

ρ_G – liquid density; S_F – the cross-sectional area of the body; k – resistance ratio; f_p – frequency of disturbing force change; a – amplitude of disturbing force. Discard all terms in the right side of equation except the first one, and will have the classic equation of pendulum oscillations, the oscillation frequency is determined by

$$2\pi f_G = (K_G / M_G)^{1/2}, \quad 2\pi f_F = (K_F / M_F)^{1/2}. \quad (3)$$

For a cylinder submerged in a stationary liquid (large vessel compared to the size of the cylinder)

$$2\pi f_F = [K_F / (M_F + M_p)]^{1/2}, \quad (4)$$

so, the natural frequency of the submerged cylinder is reduced. The Archimedes force may be included in this system, but in this case we assume that the movement of the fluid and body is strictly horizontal and the slope of body is not taken into account. If consider the body slope, the equations become somewhat more complicated (vertical components are added). In this case, the elastic force (the first term in the right side) can be supplemented by the $(K_F - K_A)X_F$, where K_A may be related to movement in nonlinear manner, but should be less than K_F .

3 Natural frequencies of the system

As follows from the theory of liquid oscillations in axisymmetric vessels [6], the natural frequencies of a fluid layer are determined by the expression

$$2\pi f_k = \left\{ g \frac{\lambda_k}{R} \frac{[\exp(\lambda_k) - \exp(-\lambda_k)]}{[\exp(\lambda_k) + \exp(-\lambda_k)]} \right\}^{1/2} \quad (5)$$

where H – layer height; R – vessel radius, λ_k – eigenvalues ($\lambda_1 = 1.841$; $\lambda_2 = 5.331$; $\lambda_3 = 8.536$; $\lambda_4 = 11.707$). Let us assume (for a vessel with fluid layer radius and height of 4 m, the first natural frequency is 0.47 Hz) that the natural frequency of the cylinder is an unknown value (it depends on the mounting system), so we will vary it. If the system solution is given in the form

$$X_G = A_G \cdot \exp(2\pi f_s t), \quad X_F = A_F \cdot \exp(2\pi f_s t) \quad (6)$$

where A_G, A_F – amplitudes of layer and body; f_s – natural frequencies of the whole system, so we can determine that

$$f_s^2 = -\frac{1}{2}(1 + m_G + m_F)^{-1} \left\{ \frac{[(1 + m_F)f_G^2 + (1 + m_G)f_F^2] \pm \sqrt{[(1 + m_F)f_G^2 + (1 + m_G)f_F^2]^2 - 4f_G^2 f_F^2 (1 + m_G + m_F)}}{2} \right\} \quad (7)$$

where $m_G = M_p / M_G$, $m_F = M_p / M_F$, so oscillatory system has two more characteristic values - these are the resonant frequencies of the system.

4 Results

Let's show a number of examples of oscillatory movements of the cylindrical body and liquid at different resonance frequencies of the body f_F , driving frequency f_p at a constant frequency f_G of 0.47 Hz, which corresponds to the natural frequency of the fluid layer of 4 meters height. In the figures (Fig. 1A - Fig. 10A), the amplitudes of oscillations are

shown in dimensionless arbitrary units. Absolute values are not important in this problem statement. Here, the comparison of the curves is important, as well as how much the amplitudes of the oscillations of the system change depending on the specified disturbance frequency. Figures 1B – 10B show the spectral densities of the corresponding curves depicted in Figures 1A – 10A, which are obtained using the standard Fast Fourier transform (FFT) algorithm.

4.1 The first example

Resonant frequency f_f of the body is 0.25 Hz. In this case, from formula (7) it follows that $f_{S1} = 0.168$ Hz, a $f_{S2} = 0.469$ Hz. For $f_p = 0.168$ Hz, we obtain the following amplitude-frequency characteristics (Fig. 1 and 2).

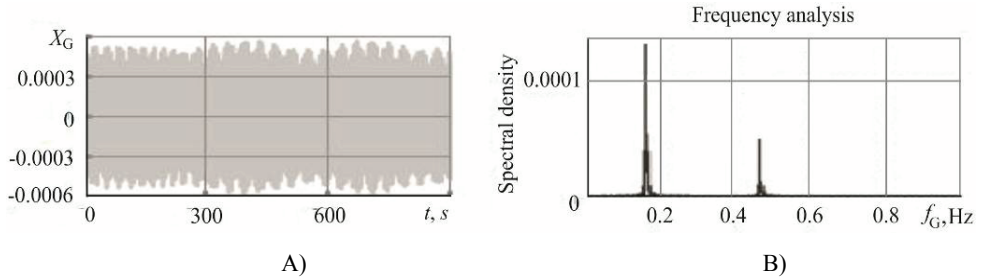


Fig. 1. Oscillations of liquid layer at $f_p = 0.168$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained as a result of Fast Fourier transform (FFT).

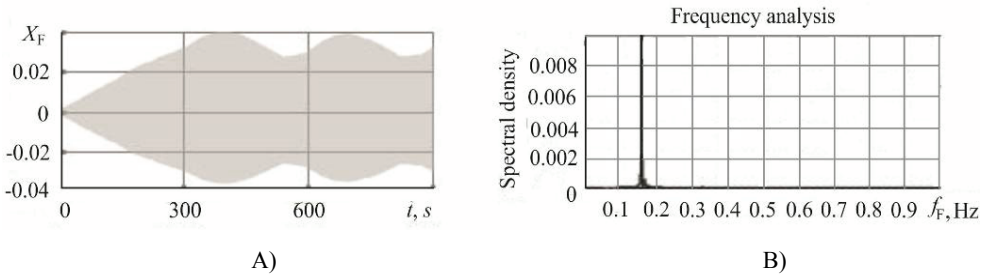


Fig. 2. Oscillations of central body at $f_p = 0.168$ Hz: A – time curve of X_F ; B – amplitude-frequency characteristics of X_F , obtained in FFT.

In this example, the natural frequency of the body is lower than the natural frequency of the liquid. The upper resonant frequency here is close to the resonant frequency of the fluid layer and the frequency of the disturbing force coincides with the lower resonant frequency of entire oscillatory system ($f_p = f_{S1}$). From Figure 1A it follows that the amplitude of layer oscillation is small (in conventional units), i.e. liquid practically does not respond to disturbance, while the body amplitude is higher than the layer amplitude, so the body is involved in the movement, but is strongly inhibited by the fluid. Figure 1B and Figure 2B shows that FFT algorithm responds to disturbing frequency of 0.168 Hz. If the disturbing frequency is somewhat lower or higher than f_{S1} , calculations show that amplitudes of layer and body oscillations are small, i.e. neither liquid nor body is involved in the movement. With increasing frequency, amplitude of the layer slowly rises, and at $f_p = f_G$ we have a completely different picture, presented in Figure 3 and 4.

In this case, as it follows from Figure 3A and Figure 4A, amplitudes of layer and body oscillations are two orders of magnitude higher than in the previous case (Fig. 1A, Fig. 2A). That is, the coincidence of the disturbing frequency with the resonant frequency of the fluid layer results in intensive movement of the layer and the body. In Figure 3B and 4B are

significant peaks corresponding to 0.47 Hz. A further increase of f_p frequency leads to rapid decrease of the movement intensity.

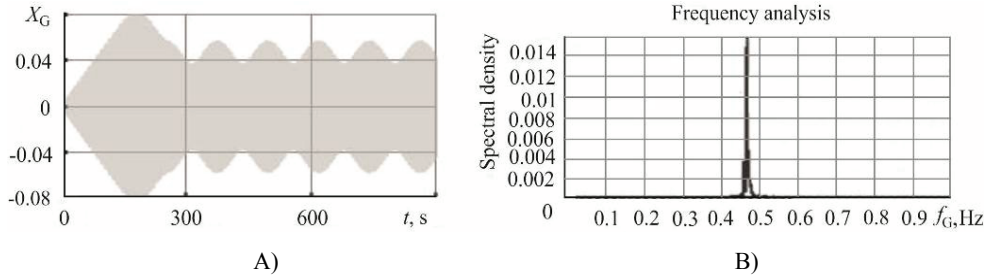


Fig. 3. Oscillations of liquid layer at $f_p = 0.47$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained in FFT.

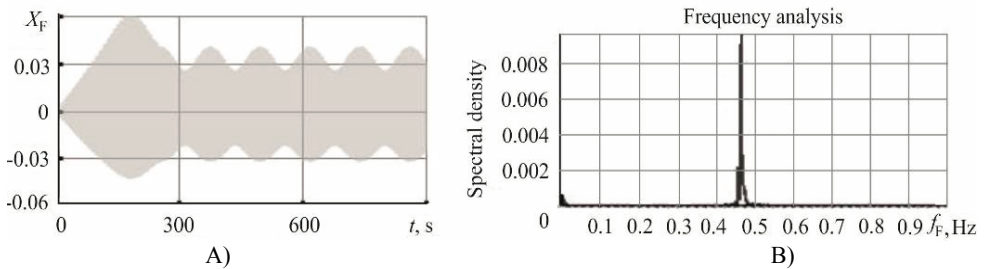


Fig. 4. Oscillations of central body at $f_p = 0.47$ Hz: A – time curve of X_F ; B – amplitude-frequency characteristics of X_F , obtained in FFT.

4.2 The second example

Resonant frequency f_F of the central body is 1.2 Hz (higher than f_G). In this case, from formula (7) it follows that $f_{S1} = 0.468 \Gamma \Pi$, a $f_{S2} = 0.81$ Hz. For $f_p = 0.81$ Hz we obtain the following amplitude-frequency characteristics (Fig. 5 and 6).

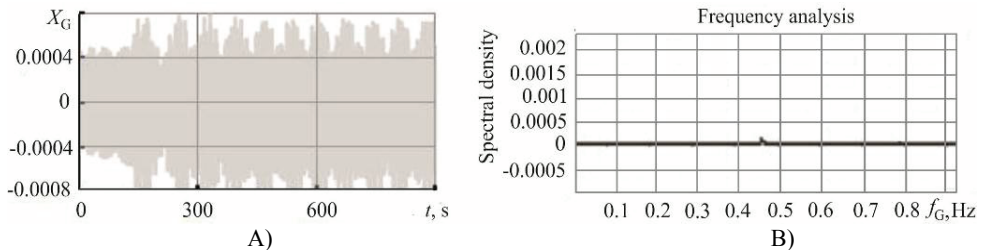


Fig. 5. Oscillations of liquid layer at $f_p = 0.81$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained in FFT.

Unlike the previous example, the lower resonant frequency of the system is close to resonant frequency of the layer, and the upper frequency and f_{S2} are one and a half times higher than f_G . Figure 5 and 6 present a case when the master frequency coincides with the upper resonant frequency of the system. As it is in the first example, the body responds to a disturbance (disturbing frequency presented in Figure 6B), amplitude of its oscillation is slightly increased, fluid layer hardly reacts to this disturbance (there is only a small burst corresponding to f_G in Figure 5B). As the frequency decreases and approaches the resonant frequency of liquid layer, amplitude of layer oscillation slowly grows and as it is in the previous example, in neighborhood of this value oscillations intensity sharply increases (Fig. 7 and 8).

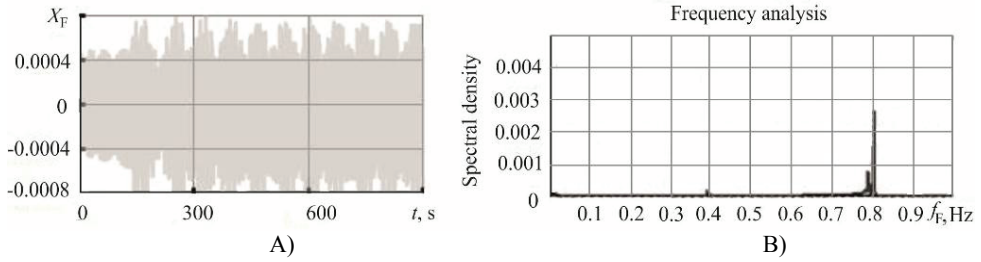


Fig. 6. Oscillations of central body at $f_p = 0.81$ Hz: A – time curve of X_F ; B – amplitude-frequency characteristics of X_F , obtained in FFT.

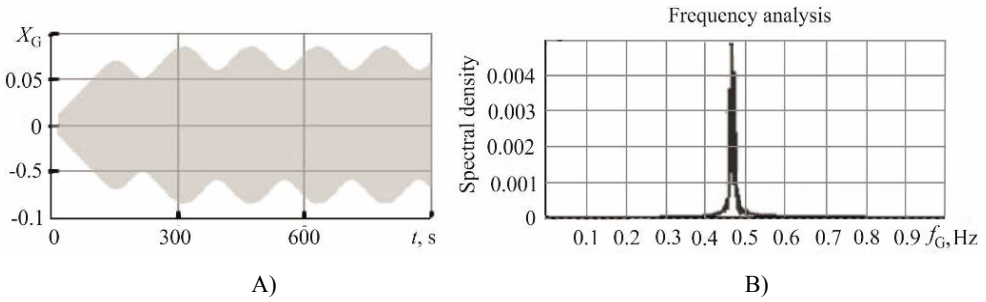


Fig. 7. Oscillations of liquid layer at $f_p = 0.47$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained in FFT.

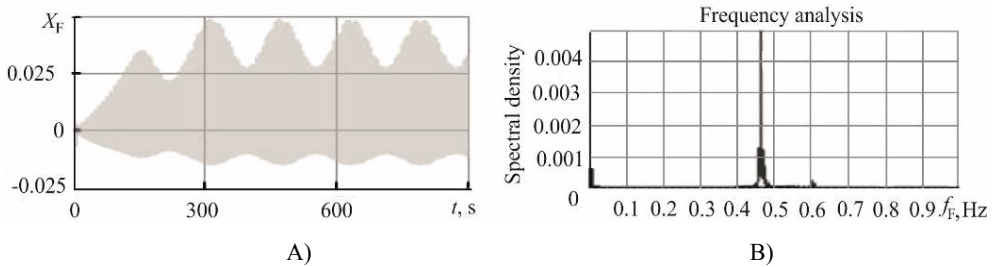


Fig. 8. Oscillations of central body at $f_p = 0.47$ Hz: A – time curve of X_F ; B – amplitude-frequency characteristics of X_F , obtained in FFT.

Figure 7B and 8B show large bursts corresponding to the resonant frequency f_G .

These examples demonstrate that the resonant frequency of liquid layer oscillations in this system actually plays a decisive role because of the fact that mass of the fluid is much greater than the mass of the body. As a rule, in this case one of the natural frequencies of the system is close to the natural frequency of the layer. Therefore, if disturbing frequencies f_p and there f_G are close, there is a strong agitation at the liquid surface. If disturbing frequency coincides with the other natural frequency of the system, the body responds to it, but its movement experiences great resistance from the fluid. The most dangerous situations occur when the natural frequency of the body is close to f_G .

4.3 The third example

Resonant frequency f_F of the central body is 0.45 Hz. In this case from formula (7) it follows that $f_{S1} = 0.302$ Hz, $f_{S2} = 0.47$ Hz. Let's show behavior of the liquid layer at $f_p = 0.46$ Hz (Fig. 9).

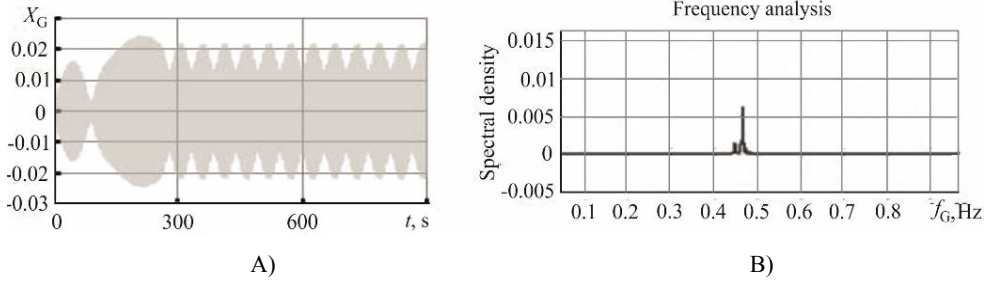


Fig. 9. Oscillations of liquid layer at $f_p = 0.46$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained in FFT.

Calculations show that in this case the body oscillates with the liquid and X_F almost coincides with X_G .

4.4 The forth example

The resonance frequency of central body f_F is 0.49 Hz, that is somewhat higher than f_G . In this case, from formula (7) it follows that $f_{S1} = 0.329$ Hz, $f_{S2} = 0.47$ Hz. This case for $f_p = 0.48$ Hz is shown in Figure 10 for a layer only, since the body oscillations are almost the same.

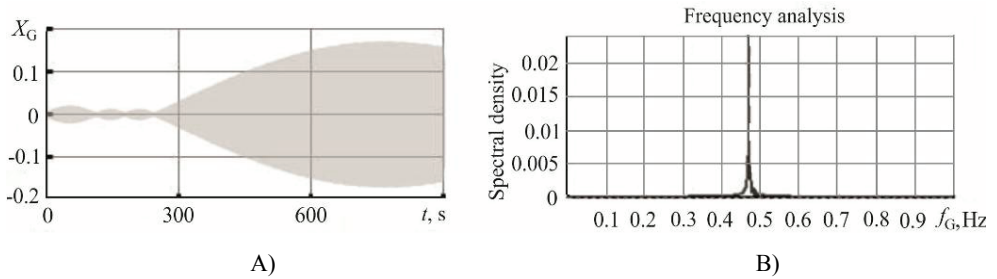


Fig. 10. Oscillations of liquid layer at $f_p = 0.48$ Hz: A – time curve of X_G ; B – amplitude-frequency characteristics of X_G , obtained in FFT.

From the last two examples it follows that if the interval between frequencies f_G and f_F is narrow, the entry of the master frequency into this interval leads to a strong increase of amplitudes of the layer and body oscillations.

Experimental studies of the oscillations of a freely suspended steel rod immersed in a liquid in oscillating vessel were also conducted. The length of the pendulum was chosen in such a way that the frequency of its natural oscillations in the fluid at rest was deliberately less or within the specified frequency range of platform oscillations. Centrally suspended steel cylinders with a diameter of 21 mm and a length of 60 mm (measured natural frequency of oscillations in a fluid at rest is 2 Hz) and 119 mm (measured natural frequency of oscillations in a fluid at rest is 1.577 Hz) were used as a pendulum, the length of the suspension was little.

Figure 11 shows dependence of the frequency of pendulum immersed in a liquid up to the point of suspension on the frequency of platform-liquid system in the range 1.8-2.5 Hz.

It is obvious that when the fluid oscillation frequency approaches the eigenfrequency of a 60-mm pendulum, the frequency of its oscillations in a certain area remains almost constant, while the oscillation frequency of 119-mm pendulum is quite close to vessel frequency.

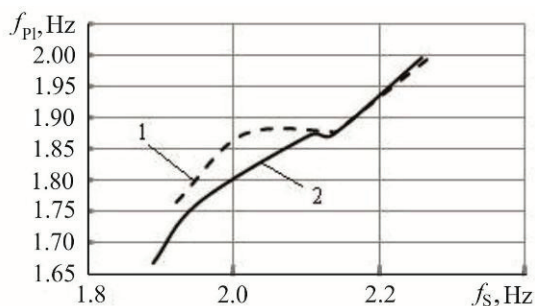


Fig. 11. Frequency dependence of pendulum immersed in a liquid on the frequency of platform-liquid system: 1 – 60 mm pendulum; 2 – 119 mm pendulum.

Conclusions

On the basis of a mathematical model for pendulum oscillations, a system of liquid layer and a central cylindrical body is considered. It was found that in addition to the natural frequencies of these components, the system has two more eigenvalues.

Since the liquid mass is much greater than the body mass, the frequency close to the resonant frequency of the layer practically determines the system behavior when it is disturbed, i.e. the system actually responds only to disturbances which frequencies are close to the natural frequency of the fluid layer. This important result suggests that in practice it is necessary to know the resonant frequency of the liquid layer and to try to damp amplitude with this exciting frequency. This can be done both constructively and technologically. However, it is necessary to point out that the spectrum of oscillation frequencies in the bottom zone is wide and, naturally, contains dangerous low frequencies, that is a challenge for practitioners. It is also desirable that the natural frequency of the central body be as far as possible from the natural frequency of the liquid. Besides, due to dispersion of fluid oscillations, further experimental and theoretical studies, considering the influence of the following modes on the dynamic pattern of the process, are of great interest.

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