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Geometric modeling of surfaces dependent cross sections in the tasks of spinning and laying

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Abstract. The article is devoted to the development of a geometric model of surfaces of dependent sections to solve the problems of winding by continuous fibers in the direction of the force and its related process of automated winding of composite materials. A uniform method for specifying the surfaces of dependent sections with a curvilinear generator and a method for solid modeling of the shell obtained by winding or calculation methods are described.

1 Introduction

Methods of automated winding and calculations are one of the main methods for obtaining structures from composite materials. In the process of winding, carried out on machines with numerical control, the surface of the mandrel is laid with tension continuous tape composed of unidirectional fibers, threads, strands or bundles impregnated with a binder. After obtaining the required thickness and structure of the shell, polymerization is performed, the final curing of the binder.

2 Problem definition

In the process of winding, various defects may occur: the tape does not fit to the surface or the tape slides from a given curve along which it is laid. You can track these defects in the virtual model of the process for which you want to create a geometric model. The construction of a generalized geometric model of tape laying on a curved surface was described in [1]. In this paper, it is assumed that the simulation of winding ("dry" and "wet") is carried out in a single way – by means of a smooth mapping of a rectangle into a three-dimensional Euclidean space. Also describes methods of analyzing a circuit, the belt laying on the subject of equilibrium filaments Lena and their contact with the surface. All builds use the surface of class C2. It should be noted that in the process of laying the tape on the surface only its first layers are laid on the surface. The remaining layers are placed on the surface formed by the previous layers. Thus, the formed surface is continuously changing. These changes naturally affect both the analysis of the packing patterns of the

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ribbon and the law of motion naturallanguage mechanism of the machine, which was described in the article [2]. To take this fact into account, it is necessary to modify the surface shape in accordance with the variable thickness of the tape [3]. Note that the presented in the paper [4] the method of surface modification is applied to the class of twice continuously differentiable surfaces, the dependent variable of the closed cross sections with curvilinear generatrix, which is in the process of change is incident to the plane parallel to the coordinate plane. We denote this class of surfaces [3] . In this paper [4] we propose a uniform method for describing class surfaces . The surface of technological mandrels often consist of several parts – constructive (surface of the product) and technological (serving for the reversal of the tape). Such surfaces can be specified by different parametric representations. Therefore, to work with such a surface in a virtual model, there is a problem of smooth connection of different parts, coordination of parametrization on the structural and technological parts. With a uniform specification of class surfaces, all these problems are automatically eliminated, since the entire surface of the technological mandrel (all its parts) will be described by one twice continuously differentiable, explicitly given vector function. This greatly simplifies the subsequent computer task of the surfaces of the class in question.

When calculating the parameters of the shell obtained by winding or laying out, there are important geometric problems of constructing intermediate surfaces of deformable solids [6] of a multilayer structure. The article offers a method of modeling a solid body obtained by winding or calculation methods.

3 On one method of approximation of functions

Let [a;b] be a grid $\Delta: a = x_0 < x_1 < ... < x_n = b$. We denote the $\mathbf{S}_{m,\nu}(\Delta)$ linear space of splines of degree m of defect ν with nodes on the grid Δ.. The elements of this space are the functions $s_{m,\nu}(x)$ satisfying the conditions: $s_{m,\nu} \in C^{m-\nu}[a;b]$; on each segment $[x_i; x_n],\ i = 0,1,..., n-1$, the function is $s_{m,\nu}(x)$ a polynomial of degree m. It is known [7] that the dimension of the space $\mathbf{S}_{m,\nu}(\Delta)$ is m+1+ν(n-1). Next, we consider the space $\mathbf{S}_{3,1}(\Delta)$, since the surfaces used must be twice continuously differentiable. In the space $\mathbf{S}_{m,1}(\Delta)$ there is a basis consisting of finite functions $N_{m+1,i}$(x), called B-splines [8]. Extend grid Δ, adding further points $x_{-m} < ... < x_{-1} < a$, $b < x_{n+1} < ... < x_{n+m}$. Then the functions $N_{m,i}(x)$ can be defined by the following relation [4]:

$$N_{m,i}(x) = \frac{x_i - x}{x_i - x_{i-m+1}} N_{m-1,i}(x) + \frac{x - x_{i-m}}{x_{i-1} - x_{i-m}} N_{m-1,i-1}(x); \qquad N_{1,i}(x) = \begin{cases} 1, & x \in [x_{i-1}; x_i); \\ 0, & x \notin [x_{i-1}; x_i). \end{cases}$$

So, any function $s_{3,1}(x)$ can be represented as a linear combination of B–splines $s_{3,1}(x) = \sum_{i=0}^{n+2} \eta_i \cdot N_{4,i+1}(x) \cdot$

Let the values of the function f in the grid nodes be known $f_i = f(x_i),\ i = 0,1,..., n$. Then in the space $\mathbf{S}_{3,1}(\Delta)$ we can find a single function $s_{3,1}(x)$ satisfying the conditions [4]:

$$s_{3,1}(x_i) = f_i,\ i = 0,1,..., n; \quad (s_{3,1})'(x_0) = f'(x_0); \quad (s_{3,1})'(x_n) = f'(x_n). \tag{1}$$

We introduce $\omega(f,\delta)=\max\limits_{\substack{x,x+h\in[a;b],\\ |h|\le\delta}}|f(x+h)-f(x)|,\ \delta\in(0;b-a)$ the notation of the continuity module of the function f and $D^{(\alpha)}f$ the derivative of the order f function α. Then, if $f\in C^k[a;b], k=1,2$ and $\bar{h}=\max\limits_i|x_{i+1}-x_i|$, then the estimates are valid [4]:

$$\left\|D^{(\beta)}s_{3,1}-D^{(\beta)}f\right\|_{C[a;b]}\le M_\beta^{(k)}\bar{h}^{k-\beta}\omega\left(D^{(k)}f,\bar{h}\right),\ 0\le\beta\le k, \tag{2}$$

where $M_0^{(1)}=9/8;\ M_1^{(1)}=4;\ \ M_0^{(2)}=19/96;\ \ M_1^{(2)}=2/3;\ \ M_2^{(2)}=4$.

As shown in [5], if the grid Δ is uniform, $x_i=a+i\cdot h,\ i=0,1,...,n,\ \ h=(b-a)/n$, the function $s_{3,1}(x)$ can be written explicitly:

$$s_{3,1}(x)=\sum_{i=0}^{n+2}\eta_i\cdot N_{4,i+1}(x),$$

$$\eta_j=\begin{pmatrix}\dfrac{\mathrm{Y}_{n+1}(j,1)}{\mathrm{E}(n+1)} & \dfrac{\mathrm{Y}_{n+1}(j,2)}{\mathrm{E}(n+1)} & \cdots & \dfrac{\mathrm{Y}_{n+1}(j,n+1)}{\mathrm{E}(n+1)}\end{pmatrix}\begin{pmatrix}3\cdot f_0+h\cdot f'(x_0)\\ 6\cdot f_1\\ \cdots\\ 6\cdot f_{n-1}\\ 3\cdot f_n-h\cdot f'(x_n)\end{pmatrix},\ j=1,...,n+1,$$

$$\eta_0=\eta_2-2h\cdot f'(x_0),\ \eta_{n+2}=\eta_n+2h\cdot f'(x_n),$$

where

$$\mathrm{Y}_k(i,j)=(-1)^{i+j}\cdot\Lambda(\min(i,j)-1)\cdot\Lambda(k-\max(i,j)),\ \ i,j=1,...,k,$$

$$\mathrm{E}(k)=2\cdot\Lambda(k-1)-\Lambda(k-2),\ \ k\ge3,$$

$$\Lambda(k)=\frac{\left(2+\sqrt{3}\right)^k-\left(2-\sqrt{3}\right)^k}{\sqrt{3}}-\frac{\left(2+\sqrt{3}\right)^{k-1}-\left(2-\sqrt{3}\right)^{k-1}}{2\sqrt{3}},\ k\ge0.$$

It should be noted that the vector $\boldsymbol{\eta}=(\eta_1...\eta_{n+1})^T$ is the solution of the equation

$$A_{n+1}\cdot\boldsymbol{\eta}=\left(3f_0+hf'(x_0),\ 6f_1,...,6f_{n-1},3f_n-hf'(x_n)\right)^T,$$

where the matrix

$$A_{n+1}=\left(a_{ij}\right)_{i,j=1}^{n+1}=\begin{pmatrix}2100...000\\ 1410...000\\ 0141...000\\ ....\ ......\\ 0000...141\\ 0000...012\end{pmatrix}$$

it has a strict diagonal predominance, moreover $\min\limits_{1\le i\le n+1}\left\{|a_{ii}|-\sum\limits_{j\ne i}|a_{ij}|\right\}=1$.

Let the function f be defined on a segment $[a-\varepsilon;\ b+\varepsilon],\ \ \varepsilon>0$ and a uniform grid

$$\Delta:\ x_{-3} < x_{-2} < x_{-1} < x_0 = a < x_1 < ... < x_n = b < x_{n+1} < x_{n+2} < x_{n+3};$$
$$x_i = a + ih,\ i = -3,..., n+3;\ h = (b-a)/n$$

Chosen so, that $x_{-1},\ x_{n+1} \in [a-\varepsilon; b+\varepsilon]$ and $f_i = f(x_i),\ i = -1,..., n+1$. Consider a cubic spline $s_{3,1}(x) = \sum_{i=0}^{n+2} \eta_i \cdot N_{4,i+1}(x)$ that satisfies interpolation $s_{3,1}(x) = f_i,\ \ i = 0,1,..., n$ and boundary conditions

$$(s_{3,1})'(x_0) = (f_1 - f_{-1})/(2h);\ \ (s_{3,1})'(x_n) = (f_{n+1} - f_{n-1})/(2h).$$

Lemma 1. If a function $f \in C^k[a-\varepsilon; b+\varepsilon],\ \ k = 0,1,2$, it is a fair assessment:

$$\left\| D^{(\beta)} f - D^{(\beta)} s_{3,1} \right\|_{C[a;b]} \le Q_\beta^{(k)} h^{k-\beta} \omega(D^{(k)} f, h),\ 0 \le \beta \le k,$$

where

$$Q_0^{(0)} = 6;\ Q_0^{(1)} = 33/8;\ Q_1^{(1)} = 10;\ Q_0^{(2)} = 163/96;\ Q_1^{(2)} = 11/3;\ Q_2^{(2)} = 10.$$

Evidence. If $f \in C[a-\varepsilon; b+\varepsilon]$. Enter the symbol $\boldsymbol{\eta}_{i,j} = (\eta_i ... \eta_j)^T$. Vector $\boldsymbol{\eta}_{1,n+1} = (\eta_1 ... \eta_{n+1})^T$ is the solution of the equation

$$A_{n+1} \cdot \boldsymbol{\eta} = \left(3f_0 + 0{,}5(f_1 - f_{-1}),\ 6f_1,...,6f_{n-1}, 3f_n - 0{,}5(f_{n+1} - f_{n-1})\right)^T ..$$

We introduce the following notations

$$\tilde{f}_0 = f_1, \tilde{f}_1 = f_0, ..., \tilde{f}_{n+1} = f_n,\ \tilde{f}_{n+2} = f_{n-1},\ \tilde{\mathbf{f}}_{i,j} = (\tilde{f}_i, ..., \tilde{f}_j)^T.$$

Then, obviously, there is equality

$$A_{n+1} \cdot (\boldsymbol{\eta}_{1,n+1} - \tilde{\mathbf{f}}_{1,n+1}) = \left(\frac{f_0 - f_1}{2} + \frac{f_0 - f_{-1}}{2}, 2f_1 - f_0 - f_2, ..., 2f_{n-1} - f_{n-2} - f_n, \frac{f_n - f_{n+1}}{2} + \frac{f_n - f_{n-1}}{2} \right)^T$$

Since the matrix of the system has a strict diagonal predominance [6], the estimates are valid $\left\| \boldsymbol{\eta}_{1,n+1} - \tilde{\mathbf{f}}_{1,n+1} \right\| \le 2\omega(f,h)$ (here and in the future for the vector $\mathbf{e} = (e_1, ..., e_n)^T$ its norm $\|\mathbf{e}\| = \max_i |e_i|$).

From equality $\eta_0 = \eta_2 - f_1 + f_{-1},\ \eta_{n+2} = \eta_n + f_{n+1} - f_{n-1}$, follows $\left\| \boldsymbol{\eta}_{0,n+2} - \tilde{\mathbf{f}}_{0,n+2} \right\| \le 4\omega(f,h)$. Since cubic B-splines are non-negative and form a partition $[a;b]$ of one on the segment,

$$|f(x) - s_{3,1}(x)| \le \sum_{i=0}^{n+2} |f(x) - \tilde{f}_i| \cdot N_{4,i+1}(x) + \sum_{i=0}^{n+2} |\eta_i - \tilde{f}_i| \cdot N_{4,i+1}(x) \le \sum_{i=0}^{n+2} |f(x) - \tilde{f}_i| \cdot N_{4,i+1}(x) + 4\omega(f,h)$$

Note that the following is true $x \in [x_j; x_{j+1}]$ for values

$$\sum_{i=0}^{n+2} |f(x) - \tilde{f}_i| \cdot N_{4,i+1}(x) = |f(x) - \tilde{f}_j| \cdot N_{4,j+1}(x) + |f(x) - \tilde{f}_{j+1}| \cdot N_{4,j+2}(x) +$$
$$+ |f(x) - \tilde{f}_{j+2}| \cdot N_{4,j+3}(x) + |f(x) - \tilde{f}_{j+3}| \cdot N_{4,j+4}(x) \le 2\omega(f,h).$$

If $f \in C^1[a-\varepsilon; b+\varepsilon]$. Consider the spline $\sigma_{3,1}(x) = \sum_{i=0}^{n+2} \zeta_i \cdot N_{4,i+1}(x)$ satisfying interpolation conditions $\sigma_{3,1}(x_i) = f_i,\ i = 0,1,\dots,n$ and the boundary conditions $\sigma'_{3,1}(x_0) = f'(x_0),\ \sigma'_{3,1}(x_n) = f'(x_n)$. Then for it to be a fair assessment (2).

If $\boldsymbol{\zeta}_{i,j} = (\zeta_i, \dots, \zeta_j)^T$. Then, obviously, the following equality holds

$$A_{n+1} \cdot (\boldsymbol{\eta}_{1,n+1} - \boldsymbol{\zeta}_{1,n+1}) = \left(h\left(\frac{f_1 - f_{-1}}{2h} - f'(x_0) \right), 0, \dots, 0, h\left(-\frac{f_{n+1} - f_{n-1}}{2h} + f'(x_0) \right) \right)^T;$$

$$\eta_0 - \zeta_0 = \eta_2 - \zeta_2 - 2h\left(\frac{f_1 - f_{-1}}{2h} - f'(x_0) \right); \quad \eta_{n+2} - \zeta_{n+2} = \eta_n - \zeta_n + 2h\left(\frac{f_{n+1} - f_{n-1}}{2h} - f'(x_0) \right)$$

By virtue of Lagrange's theorem

$$h\left| f'(x_0) - \frac{f_1 - f_{-1}}{2h} \right| = h\left| f'(x_0) - \frac{(f_1 - f_0) + (f_0 - f_{-1})}{2h} \right| =$$
$$= \frac{h}{2}\left| (f'(x_0) - f'(x_0 + \theta_1 h)) + (f'(x_0) - f'(x_0 - \theta_2 h)) \right| \le h\omega(f', h),\ \ \theta_i \in (0;1),\ \ i = 1,2.$$

Similarly, we obtain an inequality $h\left| f'(x_n) - \dfrac{f_{n+1} - f_{n-1}}{2h} \right| \le h\omega(f', h)$.

So, $\left\| \boldsymbol{\eta}_{0,n+2} - \boldsymbol{\zeta}_{0,n+2} \right\| \le 3h\omega(f', h)$. Then, using expressions for derivatives of the B-spline [4],

$$\left| s_{3,1}(x) - \sigma_{3,1}(x) \right| \le 3h\omega(f', h),\ \ \left| s'_{3,1}(x) - \sigma'_{3,1}(x) \right| \le 3\sum_{i=1}^{n+2} \left| \frac{\eta_i - \eta_{i-1}}{3h} - \frac{\zeta_i - \zeta_{i-1}}{3h} \right| N_{3,i+1}(x) \le 6\omega(f', h).$$

based on the estimates (2), we obtain

$$\left| f(x) - s_{3,1}(x) \right| \le \left| f(x) - \sigma_{3,1}(x) \right| + \left| \sigma_{3,1}(x) - s_{3,1}(x) \right| \le \frac{33}{8} h\omega(f', h);$$
$$\left| f'(x) - s'_{3,1}(x) \right| \le \left| f'(x) - \sigma'_{3,1}(x) \right| + \left| \sigma'_{3,1}(x) - s'_{3,1}(x) \right| \le 10\omega(f', h).$$

If $f \in C^2[a-\varepsilon; b+\varepsilon]$. Then Taylor's formula

$$f_{\pm 1} = f(x_0 \pm h) = f(x_0) \pm hf'(x_0) + \frac{h^2}{2} f''(\xi_{\pm 1});\ \ \xi_1 \in (x_0; x_1),\ \ \xi_{-1} \in (x_{-1}; x_0)$$

Therefore $h\left| f'(x_0) - \dfrac{f_1 - f_{-1}}{2h} \right| \le \dfrac{h^2}{2} \omega(f'', h)$.

Similarly, the inequality is obtained $h\left| f'(x_n) - \dfrac{f_{n+1} - f_{n-1}}{2h} \right| \le \dfrac{h^2}{2} \omega(f'', h)$.

Therefore $\left\| \boldsymbol{\eta}_{0,n+2} - \boldsymbol{\zeta}_{0,n+2} \right\| \le \dfrac{3}{2} h^2 \omega(f'', h)$. Therefore,

$$\left| s_{3,1}(x) - \sigma_{3,1}(x) \right| \le \frac{3h^2}{2} \omega(f'', h),\ \ \left| s'_{3,1}(x) - \sigma'_{3,1}(x) \right| \le 3h\omega(f'', h);\ \left| s''_{3,1}(x) - \sigma''_{3,1}(x) \right| \le 6\omega(f'', h)$$

From here we obtain the estimates presented in the Lemma. The Lemma is proved.

Lemma 2. If $\left|y_i - z_i\right| \le \delta,\ i = 0,1,\ldots,n$. Then, if $s_{3,1}(x),\ \sigma_{3,1}(x)$ - cubic splines, satisfying the interpolation conditions

$$s_{3,1}(x_i) = y_i,\ \sigma_{3,1}(x_i) = z_i,\ x_i = a + iH,\ H = \frac{b-a}{n},\ \ i = 0,1,\ldots,n$$

and boundary conditions $s'_{3,1}(x_0) = s'_0,\ s'_{3,1}(x_n) = s'_n,\ \sigma'_{3,1}(x_0) = \sigma'_0, \sigma'_{3,1}(x_n) = \sigma'_n$, moreover $\left|s'_0 - \sigma'_0\right| \le \delta',\ \left|s'_n - \sigma'_n\right| \le \delta'$, it is a fair assessment:

$$\left\|s_{3,1} - \sigma_{3,1}\right\|_{C[a;b]} \le 6\delta + 3H\delta';\ \ \left\|s'_{3,1} - \sigma'_{3,1}\right\|_{C[a;b]} \le \frac{12\delta + 6H\delta'}{H};\left\|s''_{3,1} - \sigma''_{3,1}\right\|_{C[a;b]} \le \frac{24\delta + 12H\delta'}{H^2}$$

Evidence. Imagine both the spline in the form

$$\sigma_{3,1}(x) = \sum_{i=0}^{n+2} \beta_i \cdot N_{4,i+1}(x),\ \ s_{3,1}(x) = \sum_{i=0}^{n+2} \alpha_i \cdot N_{4,i+1}(x)\cdot$$

Let him $\boldsymbol{\alpha}_{i,j} = (\alpha_i,\ldots,\alpha_j)^T$, $\boldsymbol{\beta}_{i,j} = (\beta_i,\ldots,\beta_j)^T$, $\mathbf{y} = (y_0,\ldots,y_n)^T$, $\mathbf{z} = (z_0,\ldots,z_n)^T$.

Then

$$A_{n+1} \cdot \boldsymbol{\alpha}_{1,n+1} = \left(3y_0 + Hs'_0,\ 6y_1,\ldots,6y_{n-1}, 3y_n - Hs'_n\right)^T;$$

$$A_{n+1} \cdot \boldsymbol{\beta}_{1,n+1} = \left(3z_0 + H\sigma'_0,\ 6z_1,\ldots,6z_{n-1}, 3z_n - H\sigma'_n\right)^T$$

Therefore,

$$A_{n+1} \cdot \left(\boldsymbol{\alpha}_{1,n+1} - \boldsymbol{\beta}_{1,n+1}\right) = \left(3(y_0 - z_0) + H(s'_0 - \sigma'_0),\ 6(y_1 - z_1),\ldots,6(y_{n-1} - z_{n-1}), 3(y_n - z_n) + H(\sigma'_n - s'_n)\right)^T$$

Note that

$$\alpha_0 = \alpha_2 - 2Hs'_0,\ \ \alpha_{n+2} = \alpha_n + 2Hs'_n,\ \ \beta_0 = \beta_2 - 2H\sigma'_0,\ \ \beta_{n+2} = \beta_n + 2H\sigma'_n$$

Therefore, there is an assessment $\left\|\boldsymbol{\alpha}_{0,n+2} - \boldsymbol{\beta}_{0,n+2}\right\| \le 6\left\|\mathbf{y} - \mathbf{z}\right\| + 3H\delta' \le 6\delta + 3H\delta'$. Hence, given that B-splines are nonnegative and form a partition of one on the segment, we obtain

$$\left|s_{3,1}(x) - \sigma_{3,1}(x)\right| = \left|\sum_{i=0}^{n+2} (\alpha_i - \beta_i) N_{4,i+1}(x)\right| \le 6\delta + 3H\delta'$$

$$\left|s'_{3,1}(x) - \sigma'_{3,1}(x)\right| = \frac{1}{H}\left|\sum_{i=0}^{n+2} \left((\alpha_i - \beta_i) - (\alpha_{i-1} - \beta_{i-1})\right) N_{3,i}(x)\right| \le \frac{12\delta + 6H\delta'}{H};$$

$$\left|s''_{3,1}(x) - \sigma''_{3,1}(x)\right| = \frac{1}{H^2}\left|\sum_{i=0}^{n+2} \left((\alpha_{i+1} - \beta_{i+1}) - 2(\alpha_i - \beta_i) + (\alpha_{i-1} - \beta_{i-1})\right) N_{3,i}(x)\right| \le \frac{24\delta + 12H\delta'}{H^2}$$

The Lemma is proved.

Consider another type of conditions imposed on the spline $s_{3,1}(x)$, which arise in the interpolation of periodic functions ($f_0 = f_n$). These are conditions of the form

$$s_{3,1}(x_i) = f_i,\ i = 0,1,\ldots,n;\quad (s_{3,1})^{(q)}(x_0) = (s_{3,1})^{(q)}(x_n),\ \ q = 0,1,2 \tag{3}$$

As it is known [12], there is a single function $s_{3,1}(x)$ from $S_{3,1}(\Delta)$ satisfying the conditions (3). In addition, if $f \in C^k[a;b]$, $k = 0,1,2$, then the estimates (2) are valid, and for $k = 0$ the constant $M_0^{(0)} = 7/4$.

If the grid Δ is uniform, the spline $s_{3,1}(x)$ can be written explicitly [4]:

$$s_{3,1}(x) = \sum_{i=-1}^{n+1} \eta_i \cdot N_{4,i+2}(x),$$

$$\eta_i = \frac{6}{\Phi(n)} \sum_{j=1}^{n} \Psi_n(i+1, j) \cdot f_{j-1}; \;\; \eta_0 = \eta_n; \; \eta_{-1} = \eta_{n-1}; \;\; \eta_{n+1} = \eta_1 \;,$$

where

$$\Phi(n) = 4 \cdot A(n-1) - 2 \cdot A(n-2) - 2 \cdot (-1)^n, \; n \geq 4;$$

$$A(n) = \frac{\left(2+\sqrt{3}\right)^{n+1} - \left(2-\sqrt{3}\right)^{n+1}}{2\sqrt{3}}, \; n \geq -1;$$

$$\Psi_n(i,j) = \begin{cases} (-1)^{i+1}\left(A(n-i) + (-1)^n A(i-2)\right), \; j = 1; \\ (-1)^{i+j}\left(4 \cdot A(j-2) \cdot A(n-i) - A(j-3) \cdot A(n-i) + \right. \\ \qquad \left. + (-1)^n A(i-j-1) - A(j-2) A(n-i-1)\right), \; 1 < j \leq i; \end{cases} \qquad i = 1,2,\ldots,n.$$

Note 1. Since b-splines [7] form a partition $[a;b]$ of one on the segment, it follows from the condition that Due to the uniqueness of the interpolation spline $f_0 = f_1 = \ldots = f_{n-1}$ it follows that,

$$\eta_{-1} = \eta_0 = \ldots = \eta_{n+2} = f_0 = \ldots = f_{n-1}.$$

Let $f \in C^m([a;b] \times [c-\varepsilon; d+\varepsilon])$, $\varepsilon > 0$, $0 \leq m \leq 2$ it satisfy the condition:

$$D^{(s,0)} f(a+0, v) = D^{(s,0)} f(b-0, v), \;\; 0 \leq s \leq m$$

for any value $v \in [c;d]$ (here $D^{(\alpha,\beta)} f = \partial^{\alpha+\beta} f / \partial^{\alpha} u \partial^{\beta} v$). We choose a uniform mesh [14]

$$\Delta_u : u_{-3} < \ldots < u_{n+3}, \;\; u_i = a + i \cdot h_u, \; i = -3,\ldots,n+3, \;\; h_u = (b-a)/n;$$

$$\Delta_v : v_{-3} < \ldots < v_{k+3}, \;\; v_j = c + j \cdot h_v, \; j = -3,\ldots,k+3, \;\; h_v = (d-c)/k < \varepsilon;$$

and, put $f_{ij} = f(u_i; v_j)$, $i = 0,1\ldots,n$; $j = -1,0,\ldots,k+1$; $\mathbf{F}_j = \left(f_{0j}, f_{1j}, \ldots, f_{n-1,j}\right)^T$.

Define the functions

$$g_j(u) = \sum_{i=-1}^{n+1} \eta_{ij} \cdot N_{4,i+2}(u), \;\; j = -1,0,1,\ldots,k+1, \;\; u \in [a;b]$$

where $\eta_{n,j} = \eta_{0,j}$, $\eta_{-1,j} = \eta_{n-1,j}$, $\eta_{n+1,j} = \eta_{1,j}$, $\eta_{ij} = \dfrac{6}{\Phi(n)} \sum_{s=1}^{n} \Psi_n(i+1, s) \cdot f_{s-1,j}$.

Consider the function

$$g(u,v)=\sum_{j=0}^{k+2}\varphi_j(u)N_{4,j+1}(v),\quad (u,v)\in[a;b]\times[c;d]$$

$$\varphi_j(u)=\left(\frac{\Upsilon_{k+1}(j,1)}{\mathrm{E}(k+1)}\ \frac{\Upsilon_{k+1}(j,2)}{\mathrm{E}(k+1)}\ \dots\ \frac{\Upsilon_{k+1}(j,k+1)}{\mathrm{E}(k+1)}\right)\begin{pmatrix}3\cdot g_0(u)+0.5\cdot(g_1(u)-g_{-1}(u))\\ 6\cdot g_1(u)\\ \dots\\ 6\cdot g_{k-1}(u)\\ 3\cdot g_k(u)-0.5\cdot(g_{k+1}(u)-g_{k-1}(u))\end{pmatrix}$$

Theorem 1. The following assessments take place

$$\left\|D^{(0,\beta)}f-D^{(0,\beta)}g\right\|_{C([a;b]\times[c;d])}\le Q_\beta^{(m)}h_v^{m-\beta}\omega\left(D^{(0,m)}f,h_v\right)+9\cdot 2^\beta\cdot M_0^{(m)}\frac{h_u^m}{h_v^\beta}\omega\left(D^{(m,0)}f,h_u\right),$$

$$\left\|D^{(\beta,0)}f-D^{(\beta,0)}g\right\|_{C([a;b]\times[c;d])}\le Q_0^{(m-\beta)}h_v^{m-\beta}\omega\left(D^{(m-\beta,0)}f,h_v\right)+9M_\beta^{(m)}h_u^{m-\beta}\omega\left(D^{(m,0)}f,h_u\right);$$

$$0\le\beta\le m.$$

For m=2 inequality holds:

$$\left\|D^{(1,1)}f-D^{(1,1)}g\right\|_{C([a;b]\times[c;d])}\le Q_1^{(1)}\omega\left(D^{(1,1)}f,h_v\right)+18M_1^{(2)}\frac{h_u}{h_v}\omega\left(D^{(2,0)}f,h_u\right).$$

Evidence. For each fixed value [15] $u\in[a;b]$, we denote $w_u^{(\alpha)}(v)$ a cubic interpolation spline that satisfies the interpolation conditions a and the boundary conditions

$$\left(w_u^{(\alpha)}\right)'(v_0)=\frac{D^{(\alpha,0)}f(u,v_1)-D^{(\alpha,0)}f(u,v_{-1})}{2h_v},\ \left(w_u^{(\alpha)}\right)'(v_k)=\frac{D^{(\alpha,0)}f(u,v_{k+1})-D^{(\alpha,0)}f(u,v_{k-1})}{2h_v}$$

Have

$$\left|D^{(0,\beta)}f(u,v)-D^{(0,\beta)}g(u,v)\right|\le\left|D^{(0,\beta)}f(u,v)-D^{(\beta)}w_u^{(0)}(v)\right|+\left|D^{(0,\beta)}g(u,v)-D^{(\beta)}w_u^{(0)}(v)\right|$$

By virtue of Lemma 1, the estimate is fair

$$\left|D^{(0,\beta)}f(u,v)-D^{(\beta)}w_u^{(0)}(v)\right|\le Q_\beta^{(m)}h_v^{m-\beta}\omega\left(D^{(0,m)}f,h_v\right),\ 0\le\beta\le m.$$

Note that for a fixed value $u\in[a;b]$, function $g(u,v)$ it is a cubic [16] spline that satisfies the interpolation conditions $g(u,v_j)=g_j(u),\ j=0,\dots,k$ and boundary condition $D^{(0,1)}g(u,v_0)=\dfrac{g_1(u)-g_{-1}(u)}{2h_v}$; $D^{(0,1)}g(u,v_k)=\dfrac{g_{k+1}(u)-g_{k-1}(u)}{2h_v}$.

Since the $g_j(u)$ interpolation spline satisfying the interpolation conditions $g_j(u_i)=f(u_i,v_j),\ i=0,1,\dots,n$ and boundary conditions of the form (3), then, by virtue of the estimates (2), we have

$$\left\|D^{(\beta)}g_j-D^{(\beta,0)}f(\,\cdot\,,v_j)\right\|_{C[a;b]}\le M_\beta^{(m)}h_u^{m-\beta}\omega\left(D^{(m,0)}f,h_u\right),\ 0\le\beta\le m \tag{4}$$

Therefore,

$$\left|g(u,v_j)-w_u^{(0)}(v_j)\right|\le M_0^{(m)}h_u^m\omega\left(D^{(m,0)}f,h_u\right),\ j=0,\dots,k.$$

In addition, we have the inequality

$$\left|\left(w_u^{(0)}\right)'(v_i)-D^{(0,1)}g(u,v_i)\right|\le M_0^{(m)}\frac{h_u^m}{h_v}\omega\left(D^{(m,0)}f,h_u\right),\ i=0,k\cdot$$

From here, on the basis of Lemma 2, we can conclude that

$$\left|D^{(0,\beta)}g(u,v)-D^{(\beta)}w_u^{(0)}(v)\right|\le 9\cdot 2^{\beta}\cdot M_0^{(m)}\frac{h_u^m}{h_v^{\beta}}\omega\left(D^{(m,0)}f,h_u\right),\ 0\le\beta\le m\cdot$$

So, we come to the following assessment

$$\left|D^{(0,\beta)}f(u,v)-D^{(0,\beta)}g(u,v)\right|\le Q_\beta^{(m)}h_v^{m-\beta}\omega\left(D^{(0,m)}f,h_v\right)+9\cdot 2^{\beta}\cdot M_0^{(m)}\frac{h_u^m}{h_v^{\beta}}\omega\left(D^{(m,0)}f,h_u\right)$$

Note that $\sigma_u(v)=D^{(\beta,0)}g(u,v)$ for a fixed value $u\in[a;b]$, it is a cubic spline that [17] satisfies the interpolation $\sigma_u(v_j)=D^{(\beta)}g_j(u),\ j=0,...,k$ and boundary conditions.

$$(\sigma_u)'(v_0)=\frac{D^{(\beta)}g_1(u)-D^{(\beta)}g_{-1}(u)}{2h_v};\ (\sigma_u)'(v_k)=\frac{D^{(\beta)}g_{k+1}(u)-D^{(\beta)}g_{k-1}(u)}{2h_v}.$$

Due to inequality (4) we have

$$\left|\sigma_u(v_j)-w_u^{(\beta)}(v_j)\right|\le M_\beta^{(m)}h_u^{m-\beta}\omega\left(D^{(m,0)}f,h_u\right),\ j=0,...,k\,.$$

Besides,

$$\left|(\sigma_u)'(v_i)-\left(w_u^{(\beta)}\right)'(v_i)\right|\le M_\beta^{(m)}\frac{h_u^{m-\beta}}{h_v}\omega\left(D^{(m,0)}f,h_u\right),\ i=0,k\cdot$$

Based on Lemma 2, we conclude that

$$\left|\sigma_u(v)-w_u^{(\beta)}(v)\right|\le 9M_\beta^{(m)}h_u^{m-\beta}\omega\left(D^{(m,0)}f,h_u\right).$$

By virtue of Lemma 1, there is inequality

$$\left|D^{(\beta,0)}f(u,v)-w_u^{(\beta)}(v)\right|\le Q_0^{(m-\beta)}h_v^{m-\beta}\omega\left(D^{(m-\beta,0)}f,h_v\right).$$

So, get

$$\left\|D^{(\beta,0)}f-D^{(\beta,0)}g\right\|_{C([a;b]\times[c;d])}\le Q_0^{(m-\beta)}h_v^{m-\beta}\omega\left(D^{(m-\beta,0)}f,h_v\right)+9M_\beta^{(m)}h_u^{m-\beta}\omega\left(D^{(m,0)}f,h_u\right)$$

We consider separately the case. By virtue of Lemma 1, inequality holds true

$$\left|D^{(1,1)}f(u,v)-D^{(1)}w_u^{(1)}(v)\right|\le Q_1^{(1)}\omega\left(D^{(1,1)}f,h_v\right)$$

By virtue of the second inequality of Lemma 2, we have

$$\left|D^{(1,1)}g(u,v)-D^{(1)}w_u^{(1)}(v)\right|\le 18M_1^{(2)}\frac{h_u}{h_v}\omega\left(D^{(2,0)}f,h_u\right)\cdot$$

Thus,

$$\left|D^{(1,1)}f(u,v)-D^{(1,1)}g(u,v)\right|\le Q_1^{(1)}\omega\left(D^{(1,1)}f,h_v\right)+18M_1^{(2)}\frac{h_u}{h_v}\omega\left(D^{(2,0)}f,h_u\right)$$

The theorem is proved.

Consequence. Subject to certain relationships between h_u, h_v grid steps, for example, $h_u \le h_v$ you can select a grid sequence and get a sequence of functions $g_p(u,v)$, $p=1,2,...$ such that $\lim\limits_{p\to\infty}\left\|g_p-f\right\|_{C^m([a;b]\times[c;d])}=0$.

4 The application of the results obtained

The theorem proved in section 1 allows to define in a uniform way surfaces of dependent sections with the curvilinear generatrix (class $C^m_{\Pi_1}, 0\le m\le 2$) which at the movement and change remains incident plane of the parallel set coordinate plane Π_1.

Let $\vec r(u,v)$, $(u,v)\in[a;b]\times[c-\varepsilon;d+\varepsilon]$, $\varepsilon>0$ be a parametric representation of such a surface [18] . Each u-line of such a surface is a curvilinear generatrix of the surface, incident plane, parallel to the coordinate plane. Choose $\Delta_u\times\Delta_v$ a uniform grid on the rectangle $[a;b]\times[c;d]$:

$$\Delta_u : u_{-3}<...<u_0=a<...<u_n=b<...<u_{n+3};$$

$$u_i=a+ih_u,\ i=-3,...,n+3;\ h_u=\frac{b-a}{n};$$

$$\Delta_v : v_{-3}<...<v_0=a<...<v_k=b<...<v_{k+3};$$

$$v_i=c+jh_v,\ j=-3,...,k+3;\ h_v=\frac{d-c}{k}\le\min(h_u,\varepsilon).$$

Consider a surface $\Sigma_{n,k}$ with a parametric representation

$$\vec R_{n,k}(u,v)=\sum_{j=0}^{k+2}\vec B_j(u)N_{4,j+1}(v),\ \ (u,v)\in[a;b]\times[c;d], \tag{5}$$

$$\vec B_j(u)=\begin{pmatrix}\frac{\Upsilon_{k+1}(j,1)}{\mathrm{E}(k+1)} & \frac{\Upsilon_{k+1}(j,2)}{\mathrm{E}(k+1)} & ... & \frac{\Upsilon_{k+1}(j,k+1)}{\mathrm{E}(k+1)}\end{pmatrix}\begin{pmatrix}3\cdot\vec G_0(u)+0.5\cdot\left(\vec G_1(u)-\vec G_{-1}(u)\right)\\ 6\cdot\vec G_1(u)\\ ...\\ 6\cdot\vec G_{k-1}(u)\\ 3\cdot\vec G_k(u)-0.5\cdot\left(\vec G_{k+1}(u)-\vec G_{k-1}(u)\right)\end{pmatrix}$$

$$\vec G_j(u)=\sum_{i=-1}^{n+1}\vec M_{i,j}\cdot N_{4,i+2}(u),\ j=-1,0,1,...,k+1,\ u\in[a;b],$$

Where:

$$\vec M_{n,j}=\vec M_{0,j},\ \vec M_{-1,j}=\vec M_{n-1,j},\ \vec M_{n+1,j}=\vec M_{1,j},\ \vec M_{i\,j}=\frac{6}{\Phi(n)}\sum_{s=1}^{n}\Psi_n(i+1,s)\cdot\vec r\left(u_{s-1},v_j\right).$$

Theorem 2. The surface $\Sigma_{n,k}$ belongs to the class $C^2_{\Pi_1}$.

Evidence. As noted, each u-line surface Σ is a curvilinear generatrix of the surface incident plane parallel to the coordinate plane. Therefore, one of the coordinates of the vector $\vec{r}(u,v_j)=x(u,v_j)\vec{i}+y(u,v_j)\vec{j}+z(u,v_j)\vec{k}$ does not depend on the variable. Let for definiteness, this coordinate will be applicate. Then $z(u_{s-1},v_j)=z_j$, $s=1,2,...,n$. By virtue of observation 1, $(\vec{M}_{ij},\vec{k})=z_j$ for everyone i. As a result $(\vec{G}_j(u),\vec{k})=z_j$. Therefore $(\vec{B}_j(u),\vec{k})=const$. Ah, so, $(\vec{R}_{n,k}(u,v),\vec{k})$ does not depend on u. Thus, any u-line surface $\Sigma_{n,k}$ incident on a plane parallel to the *Oxy* coordinate plane.

The theorem is proved.

By theorem 1 for a sequence of vector functions $\vec{R}_{n,k}(u,v)$ defining surfaces of dependent sections of the class $C^2_{\Pi_1}$ will be executed $\lim\limits_{h_u\to 0}\left\|\vec{R}_{n,k}-\vec{r}\right\|_{C^m([a;b]\times[c;d])}=0$. Thus, the geometric part of the determinant [7] of the class surface $C^2_{\Pi_1}$ it consists of a point frame of sections of such a surface [19], and the algorithmic part of the determinant is given by the vector function (5).

The results of section 1 can be applied to the construction of a vector function that determines the winding body for some of its intermediate layers. Moreover, such a vector function can be written out explicitly.

If $\vec{r}(u,v,j)$, $(u,v)\in[a_1;b_1]\times[a_2;b_2]$, $j=0,1,...,s$ the intermediate layers of the body (the surface of the class C^m). Choosing a uniform grid $w_0=a_3<...<w_s=b_3$, $h=(b_3-a_3)/s$, the body itself will set the vector function:

$$\vec{R}(u,v,w)=\sum_{i=0}^{n+2}\vec{B}_i(u,v)\cdot N_{4,i+1}(w),\ \ (u,v,w)\in[a_1;b_1]\times[a_2;b_2]\times[a_3;b_3],$$

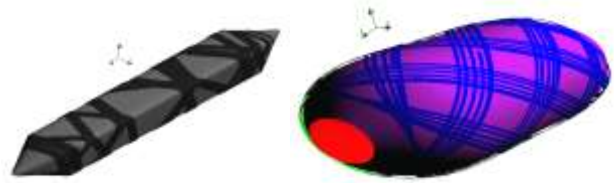
$$\vec{B}_j(u,v)=\begin{pmatrix}\dfrac{\Upsilon_{s+1}(j,1)}{\mathrm{E}(s+1)} & \dfrac{\Upsilon_{s+1}(j,2)}{\mathrm{E}(s+1)} & \cdots & \dfrac{\Upsilon_{s+1}(j,s+1)}{\mathrm{E}(s+1)}\end{pmatrix}\begin{pmatrix}3\cdot\vec{r}(u,v,0)+h_w\cdot\vec{R}'_w(u,v,w_0)\\ 6\cdot\vec{r}(u,v,1)\\ \dots\\ 6\cdot\vec{r}(u,v,s-1)\\ 3\cdot\vec{r}(u,v,s)-h_w\cdot\vec{R}'_w(u,v,w_s)\end{pmatrix},\ j=1,...,s+1,$$

$$\vec{B}_0(u,v)=\vec{B}_2(u,v)-2h_w\cdot\vec{R}'_w(u,v,w_0),\ \vec{B}_{s+2}(u,v)=\vec{B}_s(u,v)+2h_w\cdot\vec{R}'_w(u,v,w_s).$$

Here $\vec{R}'_w(u,v,w_i)$, $i=\overline{0,s}$ are arbitrary twice continuously differentiable vector functions satisfying the condition

$$D^{(\alpha,0,0)}\vec{R}'_w(a_1+0,v,w_i)=D^{(\alpha,0,0)}\vec{R}'_w(b_1-0,v,w_i),\ 0\le\alpha\le m,\ i=\overline{0,s}$$

Figure 1 shows the simulation of the winding surface of a rectangular profile and the surface of rotation. Figure 2 shows the results of solid modeling [20].



a) b)

Fig. 1. Simulation of the winding, where: a) the surface of a rectangular profile, b) the surface of rotation.

a) body b) the layers of the body

c) body d) the layers of the body

Fig. 2. Results of modeling a solid body.

5 Conclusion

The article presents the method of approximation of functions of two arguments, the approximation error is found. The main advantage of the developed method is that the approximating function is written explicitly. The proposed method is applied to the construction of the determinant of the surface of dependent sections with variable generatrix. Also, the method of layer-by-layer modeling of a solid body is presented, the distinctive feature of which is the ability to specify only the points of sections of layers using an explicitly given vector function. The application of the developed methods for geometric modeling of bodies obtained by winding and laying out of composite materials is shown.

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