Effect of humidity on the wooden beams of the composite section

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Abstract. In wooden structures, the change in humidity affects the deformability and load-carrying capacity of joints and structures according to the criteria of the first and second groups of limiting states. In wooden beams of the composite cross-section with mechanical connections, with an increase in the moisture content of the timber, the structural strength of the structures is affected both by the increment in the deformations of the joints and by the change in the physical and mechanical characteristics of the wood, primarily, the modulus of elasticity of the material.

1 Introduction

It is very important to check the deformability of cross-section wooden elements [1-3]. The theory of compound rods by A.R. Rzhanitsyn is used in this article. Let's consider a beam with a span of 6 m with a cross-section bxh = 150x450 mm from three bars 150x150 mm, the joint operation of which is provided by inclined metal rods of d=20 mm set at an angle of 45 degrees to the longitudinal axis of the beam and to the direction of the fibers in the bars of the composite section. Load on the beam is transmitted in the form of two concentrated forces, applied symmetrically in a ¼ span of the structure. Inclined rods are installed on three pieces from each side of the beam in areas where shear forces act in the joints between the bars of the composite section. The pitch of the rods is 450 mm.

2 Materials and Methods

According to Rzhaynitsyn's theory of compound rods that is mirrored in later works [4-6], the beam design consisting of three separate rods connected by compliant shear links forms a statically indeterminate system, one of the main characteristics of which are deformations of compliant bonds obtained from experiments with constructions of full-scale sizes.
Two differential equations (1) for seams of a three-layer beam

$$\frac{T_1^{''}}{\xi_1} = \Delta_{11} \times T_1 + \Delta_{12} \times T_2 + \Delta_{10} \tag{1}$$

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$$\frac{T_2''}{\xi_2} = \Delta_{21} \times T_1 + \Delta_{22} \times T_2 + \Delta_{20}$$

Coefficients of differential equations

$$\Delta_{11} = \Delta_{22} = \frac{2}{EI} + \frac{h^2}{3EI} = \frac{\left(2 + \frac{h^2}{3i^2}\right)}{EF}$$

$$\Delta_{12} = \Delta_{21} = -\frac{1}{EF} + \frac{h^2}{3EI} = \frac{\left(\frac{h^2}{3i^2} - 1\right)}{EF} \qquad (2)$$

$$\Delta_{10} = \Delta_{20} = -\frac{M^0}{3EI} \times h = -\frac{M^0}{3i^2} \times h/EF$$

where Ti – shearing force in the i-th seam; h – the distance between the centers of gravity of the cross sections of two adjacent rods; M0 – bending moment in a beam that does not have shear bonds; i – radius of inertia of the section of each beam; ξ i – shear rigidity ratio in the i-th seam, ξ i = Tст *m / δc; Tст - shearing force per bond; δc – deformation of mutual displacement of two adjacent bars, obtained in the experiment; m – the number of links per unit length of the seam.

We substitute the coefficients (2) into the system (1):

$$T_1'' \frac{EF}{\xi_1} = \left(\frac{h^2}{3i^2} + 2\right)T_1 + \left(\frac{h^2}{3i^2} - 1\right) \times T_2 - \frac{M^0 h}{3i^2} \qquad (3)$$

$$T_2'' \frac{EF}{\xi_2} = \left(\frac{h^2}{3i^2} - 1\right)T_1 + \left(\frac{h^2}{3i^2} + 2\right) \times T_2 - \frac{M^0 h}{3i^2}$$

We introduce the notation $\beta = \frac{EF}{\xi}, \alpha = \frac{h^2}{3i^2}, \ \mu = -\frac{M^0 h}{3i^2}$.

Then the system (3) takes the form:

$$\beta \times T_1'' = (\alpha + 2)T_1 + (\alpha - 1) \times T_2 + \mu$$

$$\beta \times T_2'' = (\alpha - 1)T_1 + (\alpha + 2) \times T_2 + \mu$$

In our case, for three identical bars $\alpha = \frac{12 \times bh \times h^2}{3 \times bh^3} = 4$.

Then

$$\beta \times T_1'' = 6T_1 + 3T_2 + \mu \qquad (4)$$

$$\beta \times T_2'' = 3T_1 + 6T_2 + \mu$$

Let us compose the sum and difference of these equations.

The system (4) takes the form

$$\beta \times (T_1'' + T_2'') = 9(T_1 + T_2) + 2\mu$$

$$\beta \times (T_1'' - T_2'') = 3(T_1 - T_2)$$

We introduce new constants $T_a = \frac{T_1 + T_2}{2}$, $T_c = \frac{T_1 - T_2}{2}$.

Then we get

$$\beta \times T_a'' = 9T_a + \mu \qquad (5)$$

$$\beta \times T_c'' = 3T_c$$

$$T_a'' = \frac{9}{\beta} T_a + \frac{\mu}{\beta} (5^*)$$

$$T_c'' = \frac{3}{\beta} T_c$$

We denote by $\lambda_1^2 = \frac{9}{\beta}$, $\lambda_2^2 = \frac{3}{\beta}$, $\gamma_1 = \frac{1}{\beta}$. Then

$$T_a'' = \lambda_1^2 \times T_a + \gamma_1 \mu \qquad (6)$$

$$T_c'' = \lambda_2^2 T_c.$$

Border conditions T1(0)=T1(l)=0, T2(0)=T2(l)=0, or Ta(0)=Ta(l)=Tc(0)=Tc(l)=0, where l – half-span.

The solution of system (6) has the form

$$T_a = A \times sh\, \lambda_1 X + B \times ch\, \lambda_1 X + \frac{\gamma_1}{\lambda_1} \int_0^x \mu(\xi)\, sh\, \lambda_1 (X - \xi)\, d\xi$$

$$T_c = C \times sh\, \lambda_2 X + D \times ch\, \lambda_2 X .$$

From the boundary conditions Tc(0)=Tc(l)=0 should be Tc=0, T=Ta = T1 = T2.

Total shearing forces in the joint will be:

$$0 \le X \le a \qquad T_a = \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \left[X - \frac{ch\lambda_1(l-a) \times sh\lambda_1 X}{\lambda_1 \times ch\lambda_1 l} \right], \qquad (7)$$

$$a \le X \le l \qquad T_a = \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \left[a + \frac{sh\lambda_1(x-a)}{\lambda_1^2} - \frac{ch\lambda_1(l-a)}{ch\lambda_1 l} \times \frac{sh\lambda_1 X}{\lambda_1} \right].$$

Running forces in the joints are calculated by formulas $\tau = T'$:

$\tau_1 = \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \left[1 - \frac{ch\lambda_1(l-a) \times ch\lambda_1 X}{ch\lambda_1 l} \right]$ when $0 \le X \le a$,

$\tau_2 = \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \left[ch\lambda_1(x-a) - \frac{ch\lambda_1(l-a)}{ch\lambda_1 l} \times ch\lambda_1 X \right]$

when $a \le X \le l$,

where $\lambda_1^2 = \frac{9}{\beta}$, $\beta = \frac{EF}{\xi}$, $\gamma_1 = \frac{1}{\beta}$, $i^2 = \frac{I}{F} = \frac{h^2}{12}$.

We substitute into equations:

$$\tau_1 = \frac{4P}{9h} \left[1 - \frac{ch\lambda_1(l-a) \times ch\lambda_1 X}{ch\lambda_1 l} \right] \text{ when } 0 \le X \le a, \qquad (8)$$

$$\tau_2 = \frac{4P}{9h} \left[ch\lambda_1(x-a) - \frac{ch\lambda_1(l-a)}{ch\lambda_1 l} \times ch\lambda_1 X \right] \text{ when } a \le X \le l.$$

The displacements (deflections) of the beam are calculated by the formula:

$$\sum_{i=1}^{3} E_i I_i W'' = -M^0 + \sum h_i \times T_i \, ;$$

$$\sum_{i=1}^{3} E_i I_i W = \iint (M^0 - 2hT_1) dX^2 + C_1 X + C_2.$$

For the accepted loading scheme on the section $0 \le X \le a$, where $M^0 = P \times X$

$$\sum_{i=1}^{3} E_i I_i W_1 = -\iint \left[PX \right.$$

$$- 2h \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \times \left(X - \frac{ch\lambda_1(l-a) \times sh\lambda_1 X}{\lambda_1 \times ch\lambda_1 l} \right) \Big] dX^2 + C_1 X$$

$$+ C_2.$$

$$\sum_{i=1}^{3} E_i I_i W_1' = -\frac{PX^2}{2} + \frac{8}{9} P \times \left(\frac{X^2}{2} - \frac{ch\lambda_1(l-a) \times ch\lambda_1 X}{ch\lambda_1 l \times \lambda_1^2} \right) + C_1.$$

$$\sum_{i=1}^{3} E_i I_i W_1 = -\frac{PX^3}{6} + \frac{8}{9} P \times \left(\frac{X^3}{6} - \frac{ch\lambda_1(l-a) \times sh\lambda_1 X}{ch\lambda_1 l \times \lambda_1^3} \right) + C_1 X + C_2. \qquad (9)$$

When $a \le X \le 1$ и $M^0 = P \times a$

$$\sum_{i=1}^{3} E_i I_i W_2 = -\iint \left[Pa \right.$$

$$- 2h \frac{h\,\gamma_1}{3i^2} \times \frac{P}{\lambda_1^2} \times \left(a - \frac{sh\lambda_1(x-a)}{\lambda_1} - \frac{ch\lambda_1(l-a) \times sh\lambda_1 X}{ch\lambda_1 l \times \lambda_1} \right) \Big] dX^2$$

$$+ C_3 X + C_4.$$

$$\sum_{i=1}^{3} E_i I_i W_2' = -PaX$$

$$+ \frac{8}{9} P \times \left(aX + \frac{ch\lambda_1(X-a)}{\lambda_1^2} - \frac{ch\lambda_1(l-a) \times ch\lambda_1 X}{ch\lambda_1 l \times \lambda_1^2} \right) + C_3.$$

$$\sum_{i=1}^{3} E_i I_i W_2 = -\frac{PaX^2}{2} + \frac{8}{9} P \times \left(\frac{aX^2}{2} + \frac{sh\lambda_1(X-a)}{\lambda_1^2} - \frac{ch\lambda_1(l-a) \times sh\lambda_1 X}{ch\lambda_1 l \times \lambda_1^3} \right) +$$

$$C_1 X + C_2. \quad (10)$$

The arbitrary integration constants Ci are determined from the boundary conditions:

$$C_1 = -P \times \left(\frac{a^2}{18} - \frac{8}{9\lambda_1^2} + \frac{al}{9} \right), \quad C_2 = 0, \quad C_3 = \frac{1}{9} Pal \;, \quad C_4 = Pa \times \left(\frac{a^2}{54} - \frac{8}{9\lambda_1^2} \right).$$

3 Results

On the basis of the expressions obtained, let us consider the effect of humidity on the load-bearing capacity of wooden beams according to the criteria of the second group of

limiting states. The results of the calculations are presented in Table 1 and in the graphs in Fig. 1 and Fig. 2.
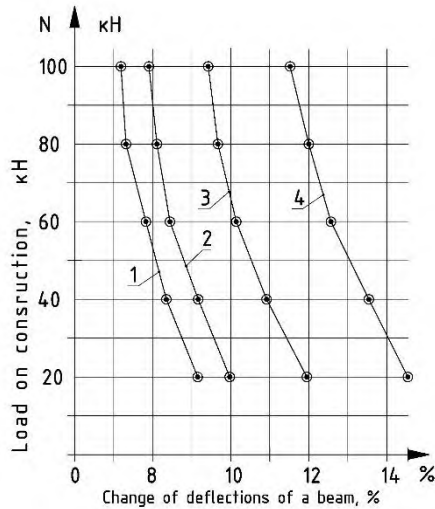
Table 1. Calculated deflections of a wooden beam of a composite section

№	Load on the structure 2*P, кН	The calculated deflections f, mm in the middle of the span, at humidity of wood W%				
		6	12	18	24	30
1	20	5.81	6.34	6.97	7.80	8.93
2	40	12.91	14.02	15.30	16.97	19.25
3	60	20.99	22.65	24.59	27.12	30.55
4	80	29.96	32.19	34.81	38.22	42.82
5	100	39.76	42.62	45.94	50.24	56.04
Elastic modulus			11522	10162	8800	7440

Moisture of wood 6-8% corresponds to the dry mode of premises in which the operation of wooden structures is carried out at relative humidity up to 40%. For the reference point, the deflections of the structure are taken with the standard wood moisture W = 12%. Moisture of wood 18% corresponds to the operation of structures with a relative air humidity of up to 80%, which is typical for wooden structures in unheated ventilated rooms. Humidity 30% corresponds to the hygroscopicity limit (fiber saturation point) of wood.
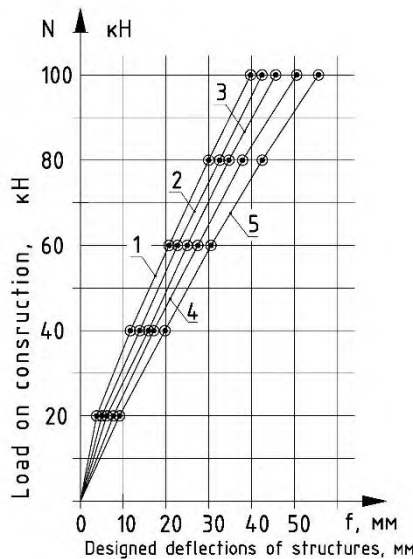
Fig. 1. Estimated deflections of wooden beams of composite cross-section at moisture content of wood: 1 – 6%, 2 – 12%, 3 – 18%, 4 – 24%, 5 – 30%.

Fig. 2. The change in the calculated deflections of wooden beams of the composite section with increasing wood moisture: 1 - 6 to 12%, 2 - 12 to 18%, 3 - 18 to 24%, 4 - from 24 to 30%.

4 Discussion

From the consideration of Table 1 and the graphs in Fig. 1 and 2, we see that as the moisture content of the wood increases, the deflections of the structure at all loading levels increase. With an increase in wood moisture from 6% to 12%, the design deflections within the limits of its design load capacity increase by an average of 8.1%. Further, with an increase in humidity from 12% to 30% in 6% increments, the deflections increase by 8.7%, 10.5%, and 12.8%, respectively. At the same time, with increasing load, the intensity of increment of deflections with increasing moisture content of wood decreases, but by no more than 2.6%. Taking the value of the ultimate deflection fu = 1 / 200L = 30 mm, we can establish that the use of structures from wood of natural moisture, which is one of the advantages of glueless compounds, including connections on inclined metal rods, requires reducing the design load-carrying capacity of structures according to the criteria of the second group of limiting states in 1.33 times. The method used in this article is same to referenced works [7-10].

5 Conclusion

Based on the calculations performed, the following conclusions are drawn:
1. Using the theory of composite rods AR Rzahynitsyn, the effect of wood moisture on the load-carrying capacity of wooden beams of a composite section on compliant bonds in the second group of limit states.
2. The calculated values of deflections are obtained, taking into account the change in the modulus of elasticity of the structural material when the moisture content of the wood changes. Deflections at the level of design load-bearing capacity of the structure from the

condition of normal stresses, which were determined at a standard wood moisture content $W = 12\%$, amounted to $f = 29.96$ mm $= 1 / 200L$ with wood moisture $W = 6\%$ to $f = 42.82$ mm at a wood moisture content $W = 30\%$.

3. The use of structures made of natural moisture wood, which is one of the advantages of glueless compounds, to which the connections on inclined metal rods relate, requires a reduction in the design load-carrying capacity of structures according to the criteria of the second group of limiting states 1.33 times.

4. Construction lift attached to the construction of the composite section with connections on inclined metal rods without the use of glue in the manufacturing stage should be at least 1/200 span.

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