Simulation modelling of the rolling stock test-bench

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Abstract. Wheelset axles are essential parts of railway and mine site rolling stock. For fatigue testing of axles, various test-benches are designed to implement the cyclic loads. The effectiveness of test-bench vibration analysis grows with the use of numerical approach and simulation models created with the aid of visual programming tools. The purpose of the work is to develop and assess the proposed simulation models of test-bench dynamics created with the aid of visual programming tools. Based on mathematical models, the test-bench simulation models of the lever system vibration have been developed. Simulation models are created with the aid of Simulink visual programming tools running under Matlab system. For modelling the components of Simulink, the SimMechanics and DSP System Toolbox Libraries are used. The comparative analysis of proposed models has been made. For the first time, with the aid of Simulink visual programming tools, the set of test-bench vibration simulation models has been obtained in steady-state and transient motion modes for linear task formulation. The proposed S-models allow automation and visualization of the motion dynamics study for test-bench components in order to determine their rational elastic-weight, kinematic and dynamic behavior. Simulation of vibrations was carried out using design parameters of the test-bench metal framework.

1 Introduction

Axles represent principal parts that determine reliability and safety of railway rolling stock for mining, metallurgical and transportation industries. The railway rolling stock axles are to meet strict requirements to mechanical-and-physical properties, such as strength, stiffness and fatigue strength [1]. The checking-out of these parameters is performed on special-purpose test-benches [2 – 4]. Rates of cyclic loads, their application pattern and methodology of axle testing are regulated by industry state documents [5, 6]. Description of various railway axle fatigue testing methods, as well as the effect of steel composition on fatigue strength, is presented in the article of foreign engineers [7]. In the course of testing axles are loaded with alternating, symmetric or intermittent forces. Designing such test-benches assume mathematical and simulation modelling of their operation in steady-state and transient modes. When modelling the kinematic behaviour of the test-bench, static and

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dynamic loads, acting on its components and kinematic pairs are determined. Results of modelling serve as input data for development of both the best-bench and its parts and units. Simulation modelling significantly intensifies the selection process for test-bench rational layout and design parameters under reliability requirements, and under strength, stiffness and vibration rigidity criteria.

2 Objective and task description

The purpose of the work is to develop simulation models for axle test-bench component motion in steady-state and transient modes of operation. The model-based study of test-bench operation allows designing components and assembly thereof under strength, stiffness and vibration rigidity criteria. In accordance with the objective the following tasks are set:

1) do a review of existing structural designs of wheelset axle fatigue test-benches;

2) develop and configure simulation models of the test-bench design;

3) develop a frequency equation for the vibration system, required to determine its cutoff frequencies;

4) perform calculations of test-bench amplitude frequency response in steady-state and transient modes of operation;

5) carry out the quality assessment for mathematic and simulation models of axle test-bench component vibration.

The methodology of operational task execution is based on fundamental research of mechanical system dynamics with several degrees of freedom. The application of Simulink [8, 9] being an extension of the Matlab [10, 11] interactive system is taken as a tool for simulation modelling. For the purpose of mathematic model calculation the Mathcad engineering calculation system has been selection.

The Simulink-aided visual modelling have several advantages as compared with conventional methods for mathematic model creation. First, when developing a model the equations for equilibrium, dynamics or other expressions are not composed; second, it is not required to use algorithmic programming languages for the purpose of software code writing; third, the process of creating a model is significantly accelerated.

3 Structural designs of axle fatigue test-benches

For the purpose of fatigue testing of axles two basic patterns of their fastening are used: cantilever and on two supports. The cyclic loading of axles can be created with symmetric alternating harmonic forces, direct intermittent forces and using the method of step-wise force value change [12]. Cantilever restraint of the axle is made on a rigid or elastic cushion [12, 13] with application of inertial harmonic disturbance (Fig. 1). Loading of the axle takes place due to imbalance located on its cantilever end. The imbalance weight through the cardan shaft is set in rotation by electric motor. Testing the axle on such a test-bench develops a strong vibration impact on the test-bench foundation and on the premises where it is in operation. Vibration impact reduction on the foundation is reached by means of installation of elastic elements under the test-bench frame.

In order to reduce the vibration caused by the test process the test-bench structural design was suggested for simultaneous testing of several axles [13] installed symmetrically on the platform (Fig. 2). The vectors of forces created by imbalances on axles are contradirectional and are compensated. The group drive ensures a preset synchronization of unbalanced weights with required phase shift. Simultaneous loading of several axles allows reducing the time of testing.
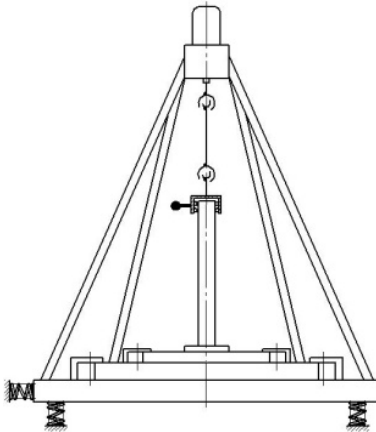
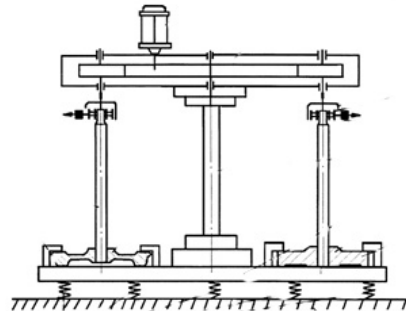**Fig. 1.** The axle test-bench with load application on cantilevers.

Fig. 2. The axle test-bench with load application on cantilevers.

With another test pattern [2] the axle of a part thereof is loaded as a free-ended beam (Fig. 3). There are no any particular advantages of such loading patterns as compared to each other. However, the installation of the axle on two supports is somewhat more suitable as compared to vertical cantilever-like installation with a wheel.

The cyclic intermittent loading is applied between two supports using a hydraulic, inertial and lever-type exciter. In order to determine the rate of such load the factor is introduced for its adjusting to equivalent symmetrical alternating loading.
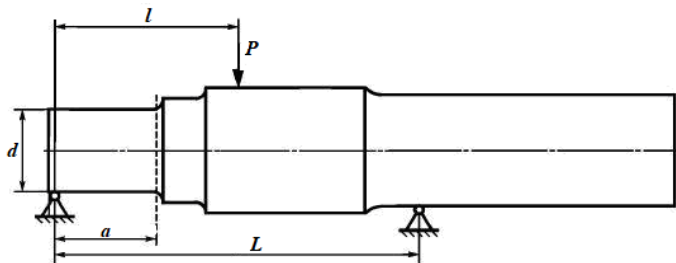

Fig. 3. Cyclic load application pattern under axle fatigue testing.

Hydraulic exciters of dynamic loads are quire complex and there are costly. Inertial exciters are less expensive regarding the hardware configuration, but the same implemented using the complex designs. The simplest and inexpensive are the lever-type mechanisms of axle loading.

The lever-type exciter contains several rods (beams) forming a transfer mechanism axle loading (Fig. 4). In accordance with acting workloads the test-bench mechanism shall create cyclic load on the axle with the amplitude of not less than 1300 kN and the frequency within 5 – 10 Hz and asymmetry parameter of 0.1 [14].
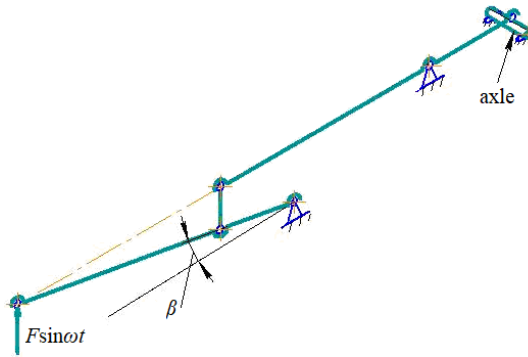
Fig. 4. Test-bench pattern with the lever-type exciter of axle loading.

   In its design the test-bench is a spatial mechanism, however, if to place a support in the test-bench lever contact point the test-bench can be considered as a planar mechanism. The test-bench can provide a force action on this support corresponding to design force of 1300 kN.

4 Mathematic models of the test-bench

The planar structure of the test-bench (Fig. 5) is the system of double-bar pendulum type with evenly distributed weight along bars and elastic linkage between them due to their flexibility. By the type of acceptable relative mobility the beam *1* interacting with the loading drive represents a pendulum, and the beam *3* loading the wheelset axle being tested plays a role of a yoke. The beams are connected to each other with a short link – pusher *2*.
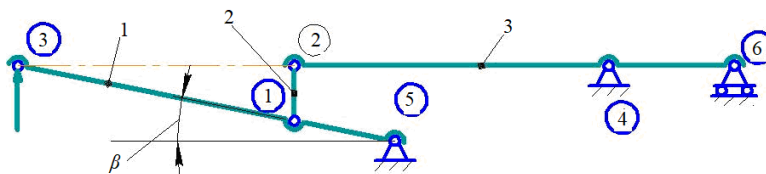

Fig. 5. Test-bench pattern.

   In order to determine the support reaction and forces in test-bench kinematic pairs the test-bench simulation model is designed (Fig. 6). For the purpose of model development the Simulink-package SimMechanics library visual programming tools are used running as part of Matlab environment [15, 16]. This library is intended for modelling mechanisms with rigid links, elastic and kinematic linkage between components.

   The blocks *body*, *body1* and *body2* simulate geometric and inertial parameters of test-bench power components. Upon determination of static loads the inertial parameters in properties of these blocks can be set as elementary. Linkage of component imitators is made using sol called "primitive attributes" that imitate kinematic pairs allowing one or another type of relative motion [17].

   The initial loading is preset in the block called *body actuator*. The settings block allows presetting the force with three coordinates and the torque in relation to the same coordinate axes. If several values of loading are needed to be applied then using the block *constant* the *body actuator* is conveyed a vector or an array of load values.
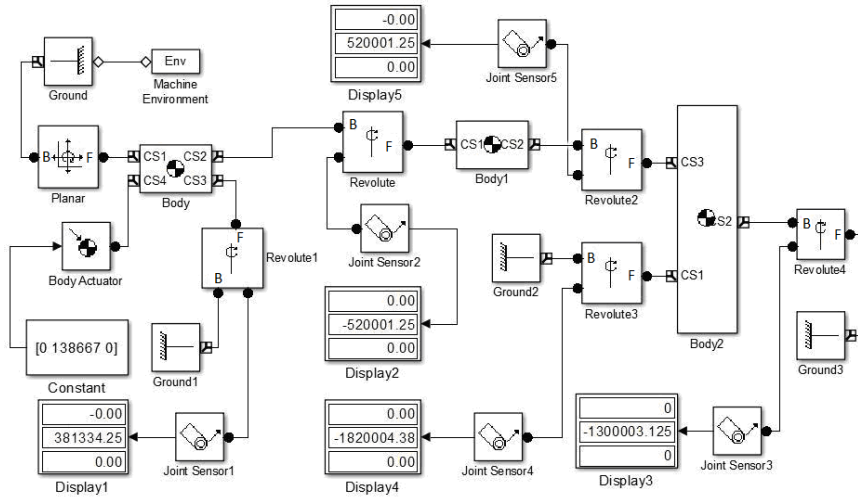
Fig. 6. Simulation model for calculation of test-bench forces.

The response registration in supports and kinematic pairs is made usingthe block *joint sensor* or *body sensor* accordingly. These block can be set to measure kinematic and power parameters of the test-bench structural design being modeled. The blocks *display* from the Sinks Simulink library are used to present the results of modelling.

The results of simulation modelling of force interaction between components and test-bench supports (Table 1) allowed to determine the value of mechanism transfer-function coefficient of as a ratio of the load acting on the axle being tested (node 6) and the load created by the test-bench drive (node 3). With design geometric parameters of the test-bench this coefficient equals to 9.375.

Table 1. Loads on test-bench kinematic pairs.

Node number	1	2	3	4	5	6
Display No.	2	5	–	4	1	3
Force, kN	-520	520	138.7	-1820	381.3	1300

The values of loads acting on test-bench levers are quite high. Therefore, for manufacturing of levers the high-quality structural carbon steel is preferred, such as steel 40. Upon fatigue strength criteria the known methods of the strength of materials determined inertia moments and resistance moments of enclosed rectangular sections of test-bench levers.

For the first approximation the construction of the mathematic model of the test-bench vibrations was made as for a simple vibration system with weights of m_1 and m_2 (Fig. 7). Linear movements of these weights take place in generic coordinates q_1 and q_2.

The Lagrange differential equations of motion are composed with the following assumptions:
– small vibrations of the test-bench lever system are being considered;
– the system is conservative;
– the weights of test-bench levers are lumped;
– the weight of the pusher 2 (see Fig. 7) is disregarded due to its nullity;
– the system elastic elements feature linear characteristics;
– stiffness of elastic elements are equal to rigidness of corresponding test-bench levers.
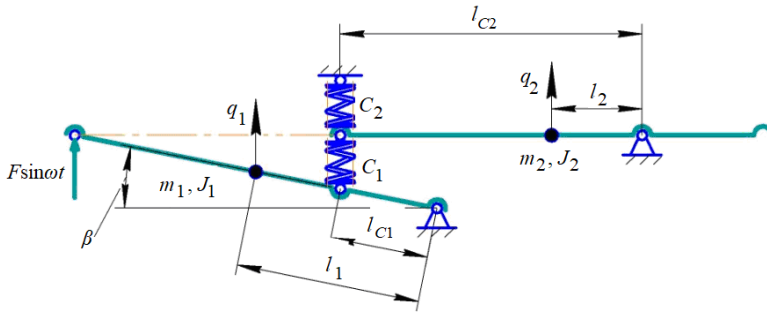
Fig. 7. Test-bench design pattern.

In accordance with accepted assumptions for steady-state operation mode the following system of equations for motions of test-bench levers was obtained:

$$\begin{cases} a_{11}\ddot{q}_1 + c_{11}q_1 + c_{12}q_2 = F'\sin \omega t; \\ a_{22}\ddot{q}_2 + c_{21}q_1 + c_{22}q_2 = 0, \end{cases} \quad (1)$$

where $a_{11} = m_1 + \dfrac{J_1}{(l_1 \cos \beta)^2}$, $a_{22} = m_2 + \dfrac{J_2}{l_2^2}$ are inertial coefficients of the system;

$c_{11} = c_1\left(\dfrac{l_{C1}\cos\beta}{l_1}\right)^2$, $c_{12} = -c_1\dfrac{l_{C1}l_{C2}\cos\beta}{l_1 l_2}$, $c_{21} = c_{12}$; $c_{22} = (c_1 + c_2)\left(\dfrac{l_{C2}}{l_2}\right)^2$ are elastic

coefficient of the system ; m_1, m_2 are weights of drive and axle levers; J_1, J_2 are inertia moments of drive and axle levers; c_1, c_2 are stiffness of drive and axle levers; l_1, l_2 are distance from lever rotation axes to their centers of mass; l_{C1}, l_{C2} are distance from lever rotation axles to elastic elements; β is drive lever tilt angle; $F' = 2F$ is loading force amplitude reduced to lever center of masses; ω is frequency of system loading ; t is time.

Frequency equation of the system (1):

$$a_{11}a_{22}p^4 - (a_{11}c_{22} + a_{22}c_{11})p^2 + c_{11}c_{22} - c_{12}^2 = 0. \quad (2)$$

The biquadratic equation positive roots (2) represent the test-bench eigen frequencies:

$$p_1 = 428\frac{1}{s} ; p_2 = 2636\frac{1}{s}.$$

The eigen frequencies are obtained using the Mathcad [18, 19] software package for test-bench design parameters: $m_1 = 151$ kg; $m_2 = 467$ kg; $J_1 = 29.5$ kg·m^2; $J_2 = 119.2$ kg·m^2; $l_1 = 0.765$ m; $l_2 = 0.375$ m; $l_{C1} = 0.408$ m; $l_{C2} = 1.25$ m; $c_1 = 5.525 \cdot 10^8$ N·m^{-1}; $c_2 = 2.033 \cdot 10^8$ N·m^{-1}; $\beta = 11.3°$.

For more accurate model of test-bench component vibrations the actual deviations and angular displacements of lever centers of masses from equilibrium position are to be taken into consideration (Fig. 8).

For refined model the general appearance of equations for motion remain unchanged, but its inertial and elastic coefficients will be different:

$$\begin{cases} a'_{11}\ddot{q}_1 + c'_{11}q_1 + c'_{12}q_2 = F'\sin \omega t; \\ a'_{22}\ddot{q}_1 + c'_{21}q_1 + c'_{22}q_2 = 0, \end{cases} \quad (3)$$

where $a'_{11} = m_1 + J_1 k_{\theta m1}^2$, $a'_{22} = m_2 + J_2 k_{\theta m2}^2$ are inertial coefficients of the system;

$c'_{11} = c_1 \left(\dfrac{k_{F1}}{\cos \beta} \right)^2$, $c'_{12} = -c_1 \dfrac{k_{F1} k_{F2}}{\cos \beta}$, $c'_{21} = c'_{12}$, $c'_{22} = (c_1 + c_2) k_{F2}^2$ are elastic coefficients of

the system; $k_{\theta m1} = \dfrac{\theta_1}{q_1} \ \text{m}^{-1}$, $k_{\theta m2} = \dfrac{\theta_2}{q_2} \ \text{m}^{-1}$ are coefficients of centers of masses

displacements impacting on their angular displacements; $k_{F1} = \dfrac{\Delta_{C1}}{q_1}$, $k_{F1} = \dfrac{\Delta_{C2}}{q_2}$ are

coefficients of centers of masses displacements impacting on the strain of elastic elements of the system.
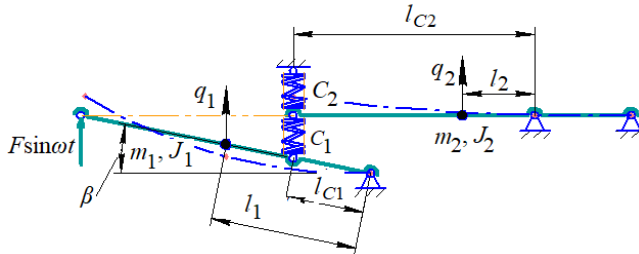

Fig. 8. Test-bench design pattern.

Frequency equation of the system (3):

$$a'_{11} a'_{22} p^4 - (a'_{11} c'_{22} + a'_{22} c'_{11}) p^2 + c'_{11} c'_{22} - c''^2_{12} = 0. \tag{4}$$

With the initial data of the equation (2) the roots of the equation are (4):

$$p'_1 = 777 \frac{1}{s} \ ; p_2 = 3657 \frac{1}{s}.$$

The first eigen frequency of the system (3) is greater as for the system (1) by 81.5%, and the second one is greater by 37.7 %. cThe analysis of eigen frequency calculation data gives grounds to significantly increase of the frequency of loading of the axle being tested and conduct accelerated testing.

More accurate values of eigen frequencies can be calculated if the weight of each test-bench lever is taken for a component with several lumped masses evenly distributed lengthwise. However, under such idealization of the vibration system the number of degrees of freedom and equations of motion will increase severalfold.

In order to ensure long-term operation of the test bench it is necessary to select a rational range of frequencies for the force created by the drive. The test-bench amplitude frequency response allows forecasting the amplitudes and their dynamic loads in the structure. For the system (3) the formulas for amplitudes A_1, A_2 are known [20]:

$$A_1 = \frac{F'(c'_{22} - a'_{22}\omega^2)}{a'_{11} a'_{22} \omega^4 - (a'_{11} c'_{22} + a'_{22} c'_{11})\omega^2 + c'_{11} c'_{22} - c''^2_{12}};$$

$$A_2 = \frac{F' c'_{12}}{a'_{11} a'_{22} \omega^4 - (a'_{11} c'_{22} + a'_{22} c'_{11})\omega^2 + c'_{11} c'_{22} - c''^2_{12}}. \tag{5}$$

On the amplitude frequency response (Fig. 9) the small values of vibrational amplitudes can be observed between two resonant frequencies of the test-bench. Vibrations in tenths of a millimeter can be observed within the range from 2700 to 3500 s^{-1}.
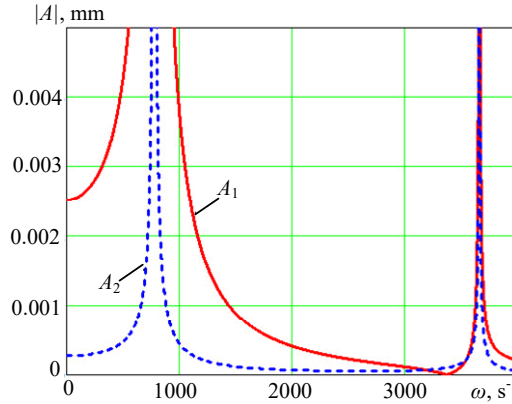
Fig. 9. Amplitude frequency response of the test-bench.

To analyze the test-bench operation in transient modes the system was introduced (3) with a structural damping of vibrations proportional to the rate:

$$
\begin{cases}
a'_{11}\ddot{q}_1 + b_{11}\dot{q}_1 + b_{12}\dot{q}_2 + c'_{11}q_1 + c'_{12}q_2 = F'\sin\left(\varphi_0 + \omega_0 t + \dfrac{\varepsilon t^2}{2}\right); \\
a'_{22}\ddot{q}_2 + b_{21}\dot{q}_1 + b_{22}\dot{q}_2 + c'_{21}q_1 + c'_{22}q_2 = 0,
\end{cases}
\tag{6}
$$

where $b_{11} = \alpha_1\left(\dfrac{k_{F1}}{\cos\beta}\right)^2$, $b_{12} = -\alpha_1\dfrac{k_{F1}k_{F2}}{\cos\beta}$, $b_{21} = b_{12}$, $b_{22} = (\alpha_1 + \alpha_2)k_{F2}^2$ are dissipative

coefficients of the system; α_1, α_2 are system viscous resistance coefficients; φ_0 is initial phase of loading ($\varphi_0 = 0$ is accepted); ω_0 is initial value of the system loading frequency($\omega_0 = 0$ is accepted); ε is angular acceleration.

Values of system viscous resistance coefficients are accepted for the condition of vibration amplitude attenuation by 90% per 8 periods of eigen partial frequencies of the test-bench.

The sequence of simulation model development with the aid of Simulink visual tools is described in [21]. Let's transform the system to (6) modelling-friendly appearance:

$$
\begin{cases}
\ddot{q}_1 = \dfrac{F'\sin\left(\varphi_0 + \omega_0 t + \dfrac{\varepsilon t^2}{2}\right) - b_{11}\dot{q}_1 - b_{12}\dot{q}_2 - c'_{11}q_1 - c'_{12}q_2}{a'_{11}}; \\
\ddot{q}_2 = -\dfrac{b_{21}\dot{q}_1 + b_{22}\dot{q}_2 + c'_{21}q_1 + c'_{22}q_2}{a'_{22}}.
\end{cases}
\tag{7}
$$

The system solving (7) is made using basic Simulink blocks implementing mathematic operations, integration of functions and solution visualization (Fig. 10). The flow lines of block have signal designations corresponding to system coefficients (2).

The vibration amplitudes of centers of masses of levers upon acceleration (Fig. 11) scarcely differ from values corresponding to its amplitude frequency response (see Fig. 9), because the operating frequency of axle loading (62.8 s^{-1}) is well below the test-bench cutoff frequencies.
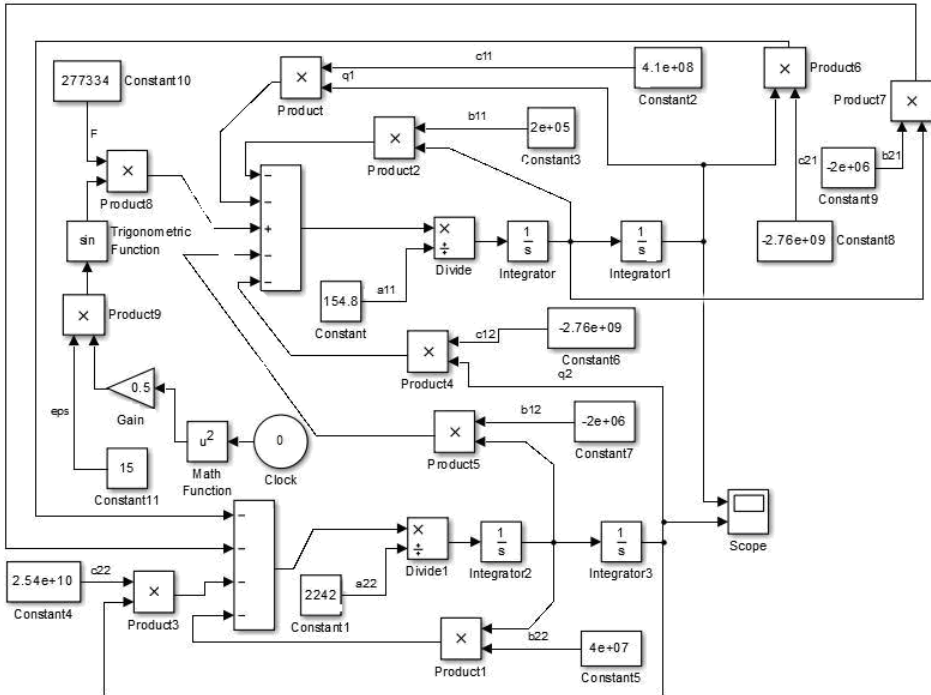
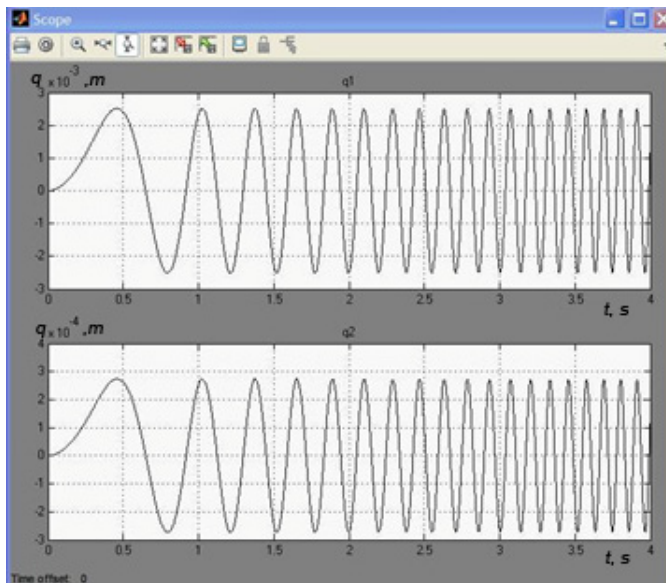**Fig. 10.** Simulation model for test-bench acceleration.

Fig. 11. Vibration amplitudes of test-bench levers q_1 and the axle q_2 in the operating range of frequencies.

With the increase of axle loading frequency up to 4000 s^{-1} the vibration amplitudes of lever centers of masses will reduce by more than tenfold (Fig. 12). The increase of the operating frequency of test bench loading will allow conducting accelerated testing of axles.
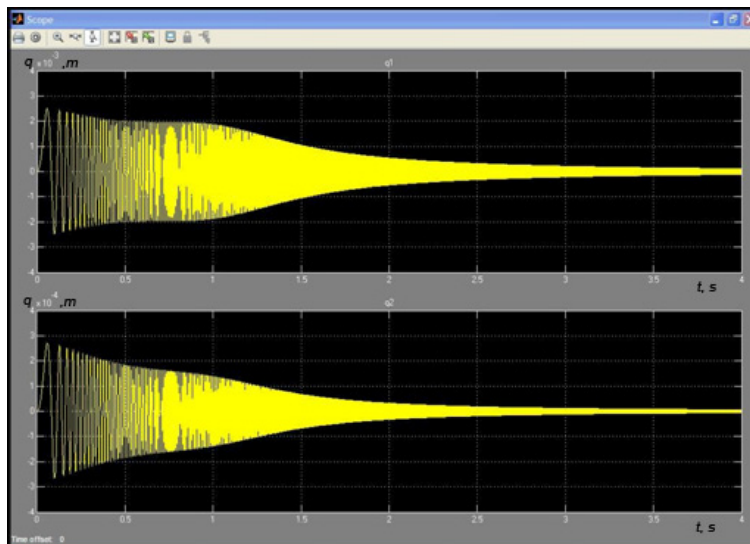
Fig. 12. The vibration amplitude of loader levers q_1 and the axle q_2 in extended range of test-bench frequencies.

5 Conclusions

A review and analysis of existing structural designs of axle test-benches, as well as methodologies of cyclic load application to wheelset axles of terrain and subterrain railway rolling stock was performed.

The linear simulation model or designing the forces on components and kinematic pairs of test-bench lever-type design was developed. The transfer-function coefficient of the mechanism force equals to 9.375.

The linear mathematical model describing vibrations of absolutely rigid test-bench levers in steady-state operation modes as for a conservative system with two weights.

A refined mathematical model describing vibrations of test-bench levers was developed with consideration of their elasticity. An amplitude-frequency response under actual elastic-weight and geometric test-bench parameters was plotted. The test-bench has two eigen frequencies: $p_1 = 777$ s^{-1}; $p_2 = 3657$ s^{-1}.

The test-bench transient operation modes in operating and extended frequency ranges of load action were studied using the refined simulation model. After passing the cutoff frequency p_1 the test-bench lever vibration amplitudes attenuate quickly.

The developed simulation models are proposed to accelerate research of the impact of test-bench parameters on vibrations of its levers in the course of testing wheelset axles of subterrain, mainline and industrial transport.

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