E3S Web of Conferences **123**, 01041 (2019) https://doi.org/10.1051/e3sconf/201912301041
Ukrainian School of Mining Engineering - 2019

Ensuring the sustainability of the motorway subgrade in the zones of underground mining operations

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Abstract. The article describes a variational method for calculating the motorway subgrade stability. The application of this method allows to quickly and accurately determine the most dangerous sliding surface with a minimum coefficient of stability. The purpose of the article is to improve the variational method for calculating the motorway subgrade stability. The article proposes to apply this method in the calculation of the transport earthwork structures, located in the area of mining operations, where there is a danger of failed subsidence. Underground mining operations have a significant impact on the vertical bent curves of a motorway, that leads to the modification of the subgrade geometric dimensions in the negative direction. This is the cause of sharp fluctuations in the values of local curvature and the radii of vertical curves. The use of variational method for calculating the motorway subgrade stability will allow to take into account the velocity of the saturated soil mass movement and the change in the geometric parameters caused by the underground mining operations. The advantages of this method include the possibility to consider the efforts from geosynthetic materials, recommended to use for strengthening the basement and slopes of the subgrade, the rheological properties of the soil, and the load from the vehicle moving along the surface. In the process of research, regulatory documents on the calculation of the stability of road structures reinforced with geosynthetic materials were studied. The identified shortcomings of the existing regulatory documents allowed to conclude that it is necessary to consider the velocity of the saturated soil mass movement caused by the underground mining operations when calculating the stability coefficient of soil structures.

1 Introduction

Currently, the problem of construction and operation of transport facilities in the areas with developed mining industry is the most acute one. As a result of underground mining method, large cavities are formed in the earth interior, which are filled with rocks being collapsed under the influence of gravity and the rock mass forming the mine working roof. Thus, a funnel of subsidence is formed on the earth surface – a shift trough, on the area of

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which the earthen structures are exposed to various deformations. Therefore, when using the motorway subgrade in the zones of underground mining operations, the negative factors should be taken into account, arising from the earth's surface shifts. The degree of impact of underground mining operations depends not only on the motorway subgrade position in the trough, but also on its size. The motorway surface along with the earth's surface undergo subsidence and deformation during mining operations, and have almost the same subsidence values. I.V. Shylin has studied the influence of mining operations on the geometric parameters of the longitudinal profile of the road and determined [1], that trough of the earth surface subsidence under the development of a single horizontal seam with a capacity of 1 m, developed at a depth of 600 m, has the following parameters: maximum subsidence $\eta_{\max} = 0.536$ m, the subsidence trough length $L_{tr} = 621.54$ m. However, besides the geometric changes in the longitudinal profile, the time factor should be taken into account, as well as possible changes within a given time both of the geometric road dimensions and the parameters of undermining [2].

In view of the investment flow reduction in the coal industry and the development of previously abandoned protective pillars, the number of facilities located in the areas of undermined territories is increasing. Currently, about a quarter of the deposits reserves is located in the abandoned protective pillars. Prospects for the development of coal mining are linked with an increase in the total earth surface area located in the undermined territories. On the other hand, the development of pillars with the backfilling of the mined-out space leads to a significant increase in the cost of the extracted mineral. Thus, saving the cost of coal and ore can be achieved by reducing the earth surface shifts impact on motorway facilities located on the surface in the undermined area, by means of application of modern technologies for backfilling the mined-out space with geoplastics and geomaterials.

Reinforcement with geosynthetic materials allows to influence the stress-strain state of both the embankment and the undermined basement. New design solutions reduce the impact of the earth's surface shifts, and the motorway subgrade is strengthened both in the slope area and at the basement [3, 4]. Up to date, sufficient experience has been accumulated in the use of such structures [5, 6]. Constructive solutions for using the reinforced soil can increase the basement load-bearing capacity, as well as increase the embankment slopes stability. The use of various geomaterials makes it possible to prevent the consequences of underground mining operations.

The intensity of the stress-strain state of the motorway subgrade reinforced with geosynthetic materials [7, 8] is increased with displacements of soil masses moving at different speeds. As a result of such movements, the loss of not only the internal stability of the soil structure is possible, namely: rupture of the reinforcing material, stretching or slipping, as well as the loss of the external stability of the soil mass. The movement of soil masses according to the reinforced soil structures changed in the plan and profile causes additional efforts that must be taken into account when designing the new structures and reconstructing the facilities in operation.

2 Relevance

Predicting the end values and velocity of the earth's surface movement in the areas of mining operations is of great interest because it allows to evaluate the motorway subgrade stability. The complexity of the problem is due to the multifactority of the surface shifts dependence on both mining [9, 10] and motor road factors. Empirical methods are widely used to determine displacements and deformations of the surface in the areas of underground mining operations. The stability of the soil slopes is assessed by the limit equilibrium method [11, 12].

Theoretical methods of calculation have been developed with the use of continuum mechanics methods for solving problems [13]. Subsequently, some researchers on the basis of a number of assumptions about the relationship between the derivatives of displacements with respect to coordinates and the dependence between displacements and stresses [14], have obtained differential equations for the finite values of point displacements of the rock massif and the earth's surface.

The next step in the development of the methods theory of the earth's surface movement caused by mining operations was the method in which the calculation of surface displacement is based on the application of the equations of probabilistic processes [15]. These equations contain a number of indefinite functional coefficients, the determination of which is quite a difficult process.

Field observations, studies on models and theoretical assumptions indicate the presence in the rock above the mined-out space of at least two areas in which the rock deformation occurs according to different laws. The existing theoretical methods for calculating surface displacements do not take this fact into account [16], since the rock massif is considered as homogeneous and isotropic. The rock deformation at any point of the massif is described by the same equations, which do not take into account the influence of rock creeping on the velocity and duration of the massif deformation.

Later, the attempts were made to develop a theory of calculating the earth's surface movements, caused by underground mining operations based on continuum mechanics. In accordance with the fact that different areas are formed in the rock massif, the plane problem is considered as a mixed elastic one [17]. This method can be used to calculate the earth's surface movements when conducting the stope mine workings at an angle to the horizon, as well as in the tunnelling the metro main line.

The movement of a basement in the areas of underground mining operations have the following values, which are difficult to approximate and impossible to determine the unambiguous law of their distribution. This thesis is conditioned by the multifactority of the displacements dependence on constructive, technological and other reasons. A probabilistic method for assessing the slopes stability has been developed on the basis of field measurements using statistical analysis [18]. To assess the impact of movements in various directions, it is reasonable to use the principle of superposition, to consider the shifts influence in certain directions on the operational capabilities of soil structures, reinforced with geosynthetic materials, and definitely on the stability and strength of separate elements of the system and the entire structure [19]. The reinforced soil structures are calculated on the stability with peculiarities connected with differences in the operation of such structures in real conditions. In accordance with the current regulatory documents [20, 21], when calculating soil structures reinforced with geosynthetic materials, two forms of stability loss are considered: external and internal, and the operational limit state of the structure is also assessed. The criteria for calculating the internal and external stability of the embankment on a reinforced basis is to ensure: internal stability of the embankment soil, lateral spread of the embankment, stability of the basement against squeezing, stability of the embankment against displacement with rotation, or overall embankment stability. The operational limit state of the embankment is assessed according to the following criteria: subsidence of the embankment and deformability of the reinforcing bed.

Internal stability characterizes the work of the soil and geosynthetic beds in the reinforced part of the structure. In this case, it is believed that the plane of possible collapse passes through reinforcing beds. External stability characterizes the work of the reinforced structure as a whole, without taking into account the method of reinforcement, and the plane of possible collapse passes outside or under the reinforced part. Combined stability loss is also possible when the collapse curve passes simultaneously outside the reinforced part and directly through the reinforced part.

Based on the results of calculations according to the criteria of internal and external stability, the required strength rating of geosynthetics and the minimum length of the beds anchoring are determined. Separately, an assessment is made of the effectiveness of the "soil-geosynthetics" interaction, which provides for calculations of the resistance against the extraction of geosynthetics from the soil massif, resistance against slipping, as well as the reinforcing material rupture. The calculation of reinforced geosynthetic materials of soil structures in accordance with current regulatory documents requires performing a large amount of calculations in order to determine, first of all, the most dangerous sliding surface by the method of circular-cylindrical sliding surface. Therewith, the peculiarities of the soil bed work in the areas of underground mining operations, where the soil mass displacement may occur at different speeds, are not considered. In our opinion, it is advisable to consider the possibility of accounting the velocity of soil masses displacement when calculating the stability coefficient of the soil bed, reinforced with geosynthetic materials by the method [20, 21], and also to propose a variational method for determining the most dangerous sliding surface. The use of this method makes it possible, when calculating the most dangerous sliding surface, to take into account additional efforts from: reinforcing materials, soil rheological properties [22 – 24], as well as dynamic loading [25].

3 Results and discussion

According to the current regulatory documents [20, 21], the stability of the embankment is assessed based on the method of circular-cylindrical sliding surfaces. The method provides for breaking the body of the embankment and the basement into blocks. Determination of the position of the most dangerous sliding surface is performed by the traditional method. Reinforcing layer provides additional holding moment to ensure the overall embankment stability. The required effort in the reinforcement for the most dangerous critical sliding surface is determined by the formula:

$$T_{Rc} = \frac{\left(\sum N_{di} - \sum F_{di}\right) R}{a_T}, \tag{1}$$

where $\sum F_{di}$ is the resultant of holding forces of all blocks constituting the collapse compartment; $\sum N_{di}$ is the resultant of landslide forces of all blocks constituting the collapse compartment, taking into account the signs; R is the radius of the most dangerous critical sliding surface; a_T is the moment arm of the reinforcing bed (Fig. 1) for the considered block.

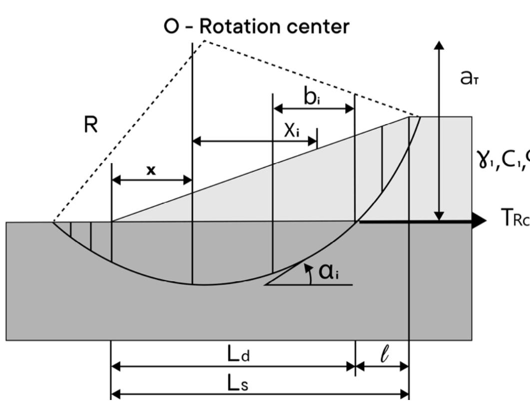

Fig. 1. The scheme for calculating the stability of the embankment by the method of circular-cylindrical sliding surface [20].

The resultant of holding forces of the *i*-th block F_{d_i} is determined by the formula:

$$F_{d_i} = \left(P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}\right)\cos\alpha_i \cdot tg\varphi_d + c_d \cdot b_i \cdot \sec\alpha_i , \quad (2)$$

where P_{d_i} is the weight per long meter of the block under consideration; q_Q is the design intensity of the rolling loading on the embankment surface; b_i is the width of the block under consideration; q_G is an intensity of the external constant load; b_{i_q} is the load band width of the external constant load within the block under consideration; α_i is an angle of inclination of the sliding surface of the block, which is under consideration to the horizontal; $tg\varphi_d$ is the calculated value of an internal friction angle of the soil, taken depending on the position of the segment of the critical sliding surface within the calculated block (the soil of embankment or basement); c_d is the calculated value of the soil adhesion, taken depending on the position of the segment of the critical sliding surface within the calculated block (the soil of embankment or basement).

The resultant of landslide forces is determined by the formula (3):

$$N_{d_i} = \left(P_{d_i} + q_{Q_d} \cdot b_i + \gamma_f q_G \cdot b_{i_q}\right)\sin\alpha_i , \quad (3)$$

where P_{d_i} is the calculated weight of the long meter of the block under consideration; q_{Q_d} is the calculated intensity of the rolling loading on the embankment surface; b_i is the width of the block under consideration; q_G is an intensity of the external constant load; γ_f is partial assurance coefficient to the load in accordance with [20]; b_{i_q} is the load band width of the external constant load within the block under consideration.

According to the method of circular-cylindrical sliding surfaces, the normal component of the *i*-th block stress is determined by the formula:

$$N_i = G_i \cdot \cos\alpha. \quad (4)$$

According to [20]: $G = P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}$, where $P_{d_i} = mg$.

The soil masses displacement, which occurs as a result of underground mining operations, is proposed by the authors to consider using:

$$N_i = G_i \cdot \cos\alpha - m \cdot a \cdot \sin\alpha, \quad (5)$$

where m is the block weight; $a = v^2/R$ is an acceleration.

Thus, we obtain

$$N_i = \left(P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}\right)\cos\alpha - \frac{mv^2}{R}\sin\alpha. \quad (6)$$

The tangential component of the stress of the *i*-th block is determined by the formula:

$$T_i = G_i \cdot \sin\alpha. \quad (7)$$

According to [20] $T_i = \left(P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}\right)\sin\alpha.$

By [20], the actual coefficient of stability of the unreinforced slope K_U is determined by the formula (8):

$$K_U = \frac{M_R}{M_D} , \quad (8)$$

where M_R is the moment of holding forces relative to the center of rotation.

Rotational moment of landslide forces relative to the center of the rotation curve M_D is determined by the formula [20]:

$$M_D = R \cdot \Sigma(P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}) \sin\alpha_i . \quad (9)$$

where R is the radius of the most dangerous critical sliding surface; P_{d_i} is the weight per long meter of the block under consideration; q_Q is an intensity of rolling loading on the embankment surface; b_i is the width of the block under consideration; q_G is an intensity of external constant load; b_{i_q} is the load band width of the external constant load within the block under consideration.

The moment of holding forces relative to the center of rotation, taking into account the velocity of soil masses displacement is determined by the formula [20] proposed by the authors:

$$M_R = R \cdot \Sigma(((P_{d_i} + q_Q \cdot b_i + q_G \cdot b_{i_q}) \cos\alpha_i - \frac{mv^2}{R} \sin\alpha_i) \cdot tg\varphi + c \cdot b_i \cdot \sec\alpha_i), \quad (10)$$

where R is the radius of the most dangerous critical sliding surface; P_{d_i} is the weight per long meter of the block under consideration; q_Q is an intensity of rolling loading on the embankment surface; b_i - is the width of the block under consideration; q_G is an intensity of external constant load; b_{i_q} is the load band width of the external constant load within the block under consideration, m; α_i is an angle of inclination of the sliding surface of the block, which is under consideration to the horizontal; $tg\varphi$ is the value of an internal friction angle of the soil, taken depending on the position of the segment of the critical sliding surface within the calculated block (the soil of embankment or basement); c - is the soil adhesion, taken depending on the position of the segment of the critical sliding surface within the calculated block (the soil of embankment or basement).

The total reinforcing force T_s, required to ensure an assigned internal stability of the slope, is determined by the formula (11) according to [20]:

$$T_s = \left([K_R] - K_U\right) \cdot \frac{M_D}{R} \quad (11)$$

where T_s is the required total reinforcing force, which should be created by geosynthetic beds to stabilize the slope; M_D is the rotational moment of landslide forces relative to the center of the rotation curve; R is the radius of the rotation curve and the moment arm from the force T_s relative to the center of rotation according to Fig. 2; $[K_R]$ is the minimum acceptable coefficient of reinforced slope stability, for the I and II categories of roads is 1.5, III – V categories – 1.3.

By the method of circular-cylindrical sliding surfaces, it is necessary to find the most dangerous one with the minimum coefficient of stability. The method is approximate and requires a large number of calculations. Therefore, the authors propose the use of the method of variational calculus [17]. This method allows to find immediately the most dangerous slip curve with a minimum coefficient of stability, take into account the reinforcement and the velocity of soil masses displacement as a result of underground mining operations.

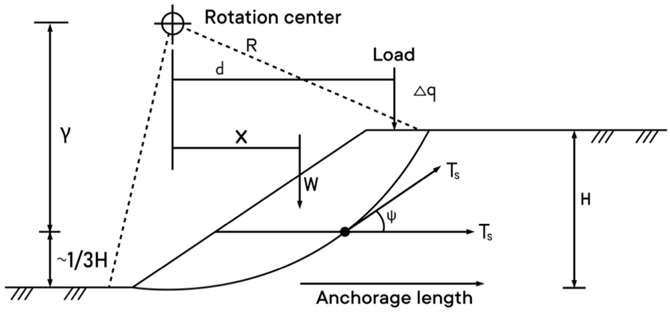

Fig. 2. Scheme for determining the required strength of reinforcing geosynthetics [20].

Consider a single block in the diagram. The area on the sliding surface is denoted as *ds*.

The adhesion force is *dc*:

$$dc = cds = c\sqrt{dx^2 + dy^2}, \tag{12}$$

where *c* is the specific soil adhesion.

The vertical pressure on the oblique plane *ds* is equal to the weight *dG* of the element with width *dx* and height *y*, referred to the middle of *ds* element.

$$\gamma \cdot y \cdot dx = dG. \tag{13}$$

The force vector of the vertical pressure *dG* is decomposed into the tangential dT_G and the normal dN_G components at the point in the middle of the element *ds* of the sliding surface, which is expected. Thus,

$$dT_G = dG \sin\alpha = \gamma \cdot y \cdot dx \cdot \sin\alpha, \tag{14}$$

$$dN_G = dG \cos\alpha = \gamma \cdot y \cdot dx \cdot \cos\alpha. \tag{15}$$

The value of strength from reinforcement *dT*, acting on the *ds* element, can be determined:

$$dT = \gamma \cdot y \cdot l \cdot tg\varphi \cdot dy, \tag{16}$$

where *l* is the length of the reinforcement contact with the soil, which is located in the passive zone.

The value of *dT*, acting on the element *ds*, is decomposed:

into tangential dT_T :

$$dT_T = dT \cdot \cos\alpha, \tag{17}$$

and normal dT_N:

$$dN_T = dT \cdot \sin\alpha. \tag{18}$$

The normal component dT_N of the friction force from the reinforcement is perpendicular to the shift site and presses the side wedge to the fundamental massif. Therefore, when $\varphi \neq 0$, it causes the occurrence of the friction force dF_T over the element of the sliding surface, which interferes with the shift:

$$dF_T = dT_N \cdot tg\varphi = dT \cdot \sin\alpha \cdot tg\alpha = \gamma \cdot y \cdot l \cdot tg\varphi \cdot dy \cdot \sin\theta \cdot tg\varphi = \gamma \cdot y \cdot l \cdot tg^2\varphi \cdot \sin\alpha \cdot dy. \tag{19}$$

The tangential component dT_T of the friction force from reinforcement is directed along the shift site and will hold the wedge in the equilibrium position:

$$dT_T = dT \cdot \cos\alpha = \gamma \cdot y \cdot l \cdot tg\phi \cdot \cos\alpha \cdot dy \,. \tag{20}$$

Since the shifting of the vertical slope is possible towards the free surface, the total component T_G of the force of the vertical pressure of massif along the slip curve is a cause contributing to the shift. Then, also dT_G (in any part of the shift curve, or in any case for the greater its part) is a shifting factor, that can be seen from decomposition of the force dG.

The normal component dN_G of the vertical pressure force is directed to the shift site perpendicularly and presses the compartment to the massif: therefore, when $\varphi \neq 0$, it causes the occurrence of friction force dF_G behind the sliding surface element, which interferes with the shift and is equal to:

$$dF_G = dN_G \cdot tg\varphi = dG \cdot \cos\alpha \cdot tg\varphi = \gamma \cdot y \cdot \cos\alpha \cdot tg\varphi \cdot dx. \tag{21}$$

The sliding body is assumed to be absolutely rigid, therefore vertical forces from the soil masses, shifted as a result of mining operations, can be transferred to the surface of the slip line without reducing.

Decompose the vertical force from the soil mass shifting N_v:

into normal

$$dN_\upsilon = \frac{m\upsilon^2}{R \cdot x} \cos\alpha \cdot dx, \tag{22}$$

and tangential

$$dT_\upsilon = \frac{m\upsilon^2}{R \cdot x} \sin\alpha \cdot dx. \tag{23}$$

Then

$$dF_{N_v} = dN_v tg\varphi = tg \frac{m\upsilon^2}{R \cdot x} \cos\alpha \cdot dx. \tag{24}$$

The stability functional [17] of the slope of the motorway subgrade along the slip curve $y = y(x)$ will be taken as:

$$R = \int_{x_0}^{x_n} \left(F - \Phi\right) dx, \tag{25}$$

where F and Φ are functions, determining appropriate holding and shifting forces by the slip curve.

Obviously, if, as a result of studying the functional R on an extremum [17], it turns out that $R > 0$, then the slope of the motorway subgrade is stable; if $R < 0$, then unstable; when $R = 0$ we have a critical case (state of limit equilibrium), corresponding to the stability coefficient, which is equal to one, if we determine the stability coefficient as the ratio of the holding factor to the shifting one:

$$k = \frac{\int_{x_0}^{x_n} F dx}{\int_{x_0}^{x_n} \Phi dx}. \tag{26}$$

If $c \neq 0$, $\varphi \neq 0$, $l \neq 0$ (the latter condition corresponds to the slope reinforcement), $N_v \neq 0$ the functions F and Φ are determined by the following expressions in accordance with the calculation scheme (see Fig. 1):

$$F=\frac{d(c+F_G+F_T+T_T+F_{N_\upsilon})}{dx}=c\sqrt{1+y'^2}+\frac{\gamma\cdot y\cdot tg\varphi}{\sqrt{1+y'^2}}+\frac{l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi}{\sqrt{1+y'^2}}+$$

$$+\frac{l\cdot\gamma\cdot y\cdot y'^2\cdot tg^2\varphi}{\sqrt{1+y'^2}}+tg\varphi\frac{m\upsilon^2}{R\cdot x\sqrt{1+y'^2}};\tag{27}$$

$$\Phi=\frac{d(T_G+T_\upsilon)}{dx}=\gamma\cdot y\cdot\sin\alpha+\frac{m\upsilon^2}{R\cdot x}\cos\theta=\frac{\gamma\cdot y\cdot y'}{\sqrt{1+y'^2}}+\frac{m\upsilon^2 y'}{R\cdot x\sqrt{1+y'^2}}.\tag{28}$$

Given the above ratios, the stability functional of the vertical reinforced slope by the slip curve $y = y(x)$ takes the form:

$$R=\int_{x_0}^{x_n}\Bigg[\left(\gamma\cdot y\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'^2\cdot tg^2\varphi-\gamma\cdot y\cdot y'\right)/\sqrt{1+y'^2}+$$

$$+c\sqrt{1+y'^2}+tg\varphi\frac{m\upsilon^2}{R\cdot x\sqrt{1+y'^2}}-\frac{m\upsilon^2 y'}{R\cdot x\sqrt{1+y'^2}}\Bigg]dx.\tag{29}$$

The extremals equation for the functional R can be represented as the L. Euler equation:

$$(F-\Phi)_y-\frac{d}{dx}(F-\Phi)_{y'}=0,\tag{30}$$

where the indices y and y' denote differentiation by y i y', respectively.

If the function $(F-\Phi)$ depends explicitly on y and y', then $(F-\Phi)_{xy'}=0$ and the differential equation of extremals can be represented as:

$$(F-\Phi)_y-y'(F-\Phi)_{yy'}-y''(F-\Phi)_{yy'}=0,\tag{31}$$

If to multiply equation (30) by y', then, it is easy to check, its left side is transformed into an exact derivative:

$$\frac{d}{dx}\left[(F-\Phi)-y'(F-\Phi)_{y'}\right].\tag{32}$$

Thus, the L. Euler equation for extremals has the first integral in the form:

$$(F-\Phi)-y'(F-\Phi)_{y'}=c_1.\tag{33}$$

where c_1 is the constant of integration.

Curve $y = y(x)$ bounded on one side by a horizontal line along the x axis (in this case before $x = x_n$), then the transversality condition is written:

$$\left[(F-\Phi)-y'(F-\Phi)_y\right]_{x=x_n}=0.\tag{34}$$

By comparing the expressions (33) and (34), we get that $c_1 = 0$.

Differentiating expressions and making transformations, we obtain the expression (34) in the form:

$$\Bigg[\Bigg(c\sqrt{1+y'^2}+\frac{1}{\sqrt{1+y'^2}}(\gamma\cdot y\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'^2\cdot tg^2\varphi-\gamma\cdot y\cdot y'+$$

$$+tg\varphi\frac{m\upsilon^2}{R\cdot x}-\frac{m\upsilon^2y'}{R\cdot x})-\frac{y'^2\cdot\gamma\cdot y\cdot tg\varphi}{(1+y'^2)\sqrt{1+y'^2}}+\gamma\cdot y\cdot y'\sqrt{1+y'^2}-$$

$$-l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi\cdot\sqrt{1+y'^2}-l\cdot\gamma\cdot y\cdot y'\cdot tg^2\varphi\left(2y'+y'^3\right)\cdot\sqrt{1+y'^2}-$$

$$-\frac{y'^2c}{\sqrt{1+y'^2}}-\frac{m\upsilon^2y'}{R\cdot x}(y''(1+y'^2)-y'^2)\sqrt{1+y'^2}-\frac{m\upsilon^2y'^2tg\varphi}{R\cdot x(1+y'^2)\sqrt{1+y'^2}}\Bigg]_{x=x_n}=0. \quad (35)$$

Taking the common denominator of expression (35) $\sqrt{1+y'^2}\left(1+y'^2\right)$, we have the first integral of the equation of extremals in the general case, when $c\neq 0$, $\varphi\neq 0$, $l\neq 0$, $\upsilon\neq 0$, $m\neq 0$, $R\neq 0$:

$$[c(1+y'^2)^2+\left(1+y'^2\right)\cdot(\gamma\cdot y\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi+l\cdot\gamma\cdot y\cdot y'^2\cdot tg^2\varphi+$$

$$+tg\varphi\frac{m\upsilon^2}{R\cdot x}-\frac{m\upsilon^2y'}{R\cdot x}-\gamma\cdot y\cdot y')-y'^2\cdot\gamma\cdot y\cdot tg\varphi-l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi(1+y'^2)^2-$$

$$-\gamma\cdot y\cdot y'(1+y'^2)^2-y'^2\cdot c\cdot(1+y'^2)-l\cdot\gamma\cdot y\cdot y'\cdot tg^2\varphi(2y'+y'^3)(1+y'^2)^2+$$

$$+\frac{m\upsilon^2y'}{R\cdot x}(1+y'^2)(y''(1+y'^2)-y'^2)+\frac{m\upsilon^2y'^2tg\varphi}{R\cdot x}\Bigg]_{x=x_n}=0. \quad (36)$$

The solution of the differential equation (36) is more convenient to perform in the numerical form. To do this, the values of the constants γ, c, φ, l, υ, m, R should be first substituted in it, which correspond to the particular case of the motorway subgrade slope under consideration, in respect to the case of reinforced slope and then solve the equation by a numerical method, which ensures sufficient accuracy of the results.

For this case, the coefficient of stability can be obtained in the form:

$$k=\frac{\int_{x_0}^{x_n}Fdx}{\int_{x_0}^{x_n}\Phi dx}=\left[\int_{x_0}^{x_n}\left(c\sqrt{1+y'^2}+\frac{\gamma\cdot y\cdot tg\varphi}{\sqrt{1+y'^2}}+\frac{l\cdot\gamma\cdot y\cdot y'\cdot tg\varphi}{\sqrt{1+y'^2}}+\frac{l\cdot\gamma\cdot y\cdot y'^2\cdot tg^2\varphi}{\sqrt{1+y'^2}}+\right.\right.$$

$$\left.\left.+\frac{m\upsilon^2tg\varphi}{R\cdot x\sqrt{1+y'^2}}\right)dx\right]\Bigg/\left[\int_{x_0}^{x_n}\left(\frac{\gamma\cdot y\cdot y'}{\sqrt{1+y'^2}}+\frac{m\upsilon^2y'}{R\cdot x\sqrt{1+y'^2}}\right)dx\right]. \quad (37)$$

The obtained formula (37) for determining the coefficient of stability by the variational method allows to simultaneously take into account the soil strength characteristics, the reinforcement with geosynthetic materials, as well as the velocity of soil masses displacement. The characteristics of the reinforcing material strength, its length and the contact interaction of geosynthetics with the soil is determined by the current regulatory documents [20, 21]. To assess the long-term stability of the slope, composed of clay soils, the values c, φ, obtained from laboratory studies should be substituted into the formula (37), with account of the rheological soils properties. The soil masses velocity is assessed by geodetic monitoring methods. A variational method for determining the stability coefficient of the motorway subgrade slope, reinforced with geosynthetic materials, taking into account the velocity of soil masses displacement as a result of underground mining operations, makes it possible to determine the intervals of use of reinforced structures to strengthen the motorway subgrade.

4 Conclusions

The research results allowed to improve the method of calculating the stability coefficient of the motorway subgrade reinforced with geosynthetic materials, which are located in the area of underground mining operations. The authors propose to take into account the velocity of soil masses displacement when calculating the coefficient of motorway subgrade stability. It is proposed to calculate the internal stability with account of the degression factor for vertical loads, since failure to take into account this circumstance leads to an invalid account of the holding forces impact as compared with the calculations for the overall stability. Monitoring of the earth's surface shifts in the area of underground mining operations allows predicting the local stability of structures and setting their safe interval for calculation by limit states. Determining the direction of earth's surface movement allows a differentiated approach to accounting for the dynamic influence on the internal stability of reinforced soil structures located in the zone of spatial movements of the basement. The study of patterns of the earth's surface shifts allows to provide a reliable interval of the soil structures stability with setting a safe velocity of the soil masses movement.

To reduce the amount of calculations, when determining the most dangerous sliding surface by the method of circular-cylindrical sliding surfaces, it is proposed to apply the variational method. The use of this method, when determining the most dangerous sliding surface, allows to take into account additional efforts from: reinforcing materials, the rheological soil properties, as well as the load from the vehicle moving along the surface.

As a prospect for further research, it is proposed, when calculating the stability coefficient, to take into account the multilayered soil structure in case of composing it with several heterogeneous elements. Each layer is considered as homogeneous, and multi-layering can be taken into account by the conditions of the slip curve passage in the limiting areas, due to the so-called adhesion conditions. The form of local stability loss of such structures is described by the variational calculus as the sum of the functionals of each layer, in accordance with the limit state of the separate structure elements.

The authors express deep gratitude to L.M. Timofieieva and O.A. Ruban for assistance in the chosen field of research.

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