

# Charge accumulation in thunderstorm clouds: fractal dynamic model

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**Abstract.** The paper considers a fractal dynamic charge accumulation model in thunderstorm clouds in view of the fractal dimension. Analytic solution to the model equation has been found. Using numerical calculations we have shown the relationship between the charge accumulation and the medium with the fractal structure. A comparative study of thunderstorm electrification mechanisms have been performed.

## 1 Introduction

Convective clouds inside which thunderstorm processes mainly develop are natural objects with nontrivial fractal structure [1]. One of the most important thunderstorm parameters is the electric field intensity. Once a lightning strike have occurred, the electric field weakens and then recovers. The process of restoring neutralized charges and respectively the electric field after the thunderstorm discharge is the matter of animated debate among scientists.

As is known, a central place in thunderstorm electricity occupies charge generation and separation processes inside convective clouds. About two dozen current mechanisms are known that result in charges generation inside the thunderclouds [2]. They can be divided into two groups.

The first group is associated with elementary processes in the clouds: the electrification within the ionic environment, the electrification caused by frictional contact between two icicles, the electrification accompanying the freezing of water and its solutions, the electrification upon destruction of crystallizing droplets, etc.,

The second group includes the induction electrification mechanism stipulated by the electric field, whose origin is not yet clear. Here we have the contact electrification, the electrification associated with droplets colliding with hailstones, the electrification produced by the melting of hailstones, etc.

None of these mechanisms can currently be considered dominant. Some authors believe that several processes may simultaneously happen in a thundercloud leading to the electrification of hydrometeors. The efficiency of each mechanism depends on the stage of the cloud development. Some mechanisms are valid only in certain parts of the cloud.

Fractal nature of clouds and their specific fractal dynamic characteristics were disregarded in all proposed mechanisms. Meanwhile, the fractal-dynamic properties

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provide reliable description of the charge accumulation process and electric field recovery in the study of thunderstorm electricity. In this regard, this paper proposes a fractal dynamic model of the charge accumulation in thunderstorm clouds along with the fractional integro-differentiation apparatus for this phenomenon study.

## **2 Mathematical modeling for charge accumulation in thunderstorm clouds**

Basing on the gained over many decades knowledge about the thunderstorm clouds properties D. Mason put forward major requirements to more or less rational theory on thunderstorm electricity [2, 3, 4, 5, 6].

1. The average duration of precipitation and electrical activity from a single thunderstorm cell is about 30 minutes.

2. The average electric moment destroyed by a lightning strike is about 110 C-km, and the corresponding charge is 20-30 C. In a typical thunderstorm cell interval between individual lightning flashes is about 20 s. A typical lightning bolt produces the electric current on the order of 1 amps.

3. The magnitude of the charge separated right after the lightning thunderstorm discharge due to falling velocity of various precipitation elements has an order of  $\frac{8000}{v}$  s ( $v$  - particle velocity in air is meters per second) and equals 1000 C.

4. In a powerful cumulonimbus cloud this charge is generated and separated in a volume of about 50 km<sup>3</sup> having a typical radius of 2 km and limited by the levels of isotherms 0 and - 40 °C .

5. The center of the negative charge is close the isotherm -5 °C while the main positive center is a few kilometers above; there can also be an additional positive charge near the base of the cloud centered near the level of isotherms of 0 °C or slightly lower.

6. The processes of charges generation and separation are closely associated with the appearance of precipitation, especially grains. Precipitation should have chances for falling at a speed of several meters per second overcoming the ascending currents.

7. To ensure the first lightning strike 12-20 minutes after the appearance of precipitation particles with a size detecting by a radar, a sufficiently large charge should be generated and separated.

The aforementioned requirements for the theory of thunderstorm electricity are changed and supplemented with new knowledge. According to Chalmers [4] the thunderstorm criteria given above are based on average characteristics while the theory should meet the requirements based on information of the intense thunderstorms since the charge accumulation rate in powerful thunderstorm clouds exceeds the rate in average thunderstorm clouds by approximately two orders of magnitude.

At the same time, not a single thunderstorm electricity theory based on the mechanisms of charges generation and separation known to date, can explain the high rates of charge accumulation in the powerful thunderstorm clouds.

As stated above, once a thunderstorm strike have happened the charges in the cloud begin to be restored as well as the electric field. In the precipitation area after the discharge, the electric field changes by rather complicated laws due to significant space charges. The electric field intensity initially grows fairly quickly, and then slows down, approaching its original value.

Let us consider the scheme of creating an electric moment in a thundercloud proposed by Imyanov I. M. [7]. According to this scheme, provided that the generation of the hydrometeors with the charge  $q$  runs across the entire cloud  $V$  with constant intensity  $I$ , the charge accumulation rate  $Q$  in a column with a cross-section area equal to one and casts into the form of

$$\frac{dQ}{dt} = IqV - \frac{Q\lambda}{\varepsilon_0} - p \frac{Q}{h^2}, \quad (1)$$

where  $\lambda$  - is the conductivity,  $\varepsilon_0$  - is the dielectric constant,  $h$  is the charge accumulation area thickness,  $p$  is the turbulence coefficient.

In equation (1) cloud fractal structure is not taken into account. Since fractal dimensions of the clouds shape does not affect intracloud atmospheric processes, it is appropriate to also consider the fractal structure of the medium when studying the accumulation of charges within thunderclouds. By a fractal medium it is implied a mass dimension distributed in space, (is a measure of how the medium fills the space it occupies). It is less than the dimension of the space being filled. By fractal form, we imply a fractal dimension geometric image.

However, this characteristic feature changes the charge accumulation equation into a fractional-order differential equation for charge accumulation. Therefore, taking into account the fractality of the cloud environment and using the Caputo derivative

$$\partial_{at}^\alpha u(t) = \text{sign}^n(a-t) D_{at}^{\alpha-n} \frac{\partial^n u(t)}{\partial t^n}, \quad n-1 < \alpha \leq n, \quad n \in N, \quad \text{where } D_{at}^\alpha \text{ - is the Riemann-}$$

Liouville fractional integro-differentiation operator, which is defined as follows:

$$D_{at}^\alpha u(t) = \begin{cases} \frac{\text{sign}(t-a)}{\Gamma(-\alpha)} \int_a^t \frac{u(s) ds}{|t-s|^{\alpha+1}}, & \alpha < 0, \\ u(t), & \alpha = 0, \\ \text{sign}^n(t-a) \frac{\partial^n}{\partial t^n} D_{at}^{\alpha-n} u(t), & n-1 < \alpha \leq n, n \in N, \end{cases}$$

where  $\Gamma(z)$  is the Euler gamma function,  $\alpha$  -is the order of integro-differentiation, as well as the notion of the effective rate change for the certain physical quantity  $Q$ , in our case it is the charge accumulation rate defined as [8-10]

$$\left\langle \frac{dQ(t)}{dt} \right\rangle = \frac{1}{\tau} D_{0t}^{\alpha-1} \frac{dQ(t)}{dt} = \frac{1}{\tau} \partial_{0t}^\alpha Q(t), \quad 0 < \alpha < 1, \quad (2)$$

Equation (1) rewrite in the following form:

$$\partial_{0t}^\alpha Q(t) = \left( IqV - \frac{Q\lambda}{\varepsilon_0} - p \frac{Q}{h^2} \right) \tau, \quad 0 < \alpha < 1, \quad (3)$$

where  $\tau$  - the characteristic time for the process.

Equation (3) is the fractal-dynamic equation of charge accumulation in thunderstorm clouds, in the right hand of which, under the bracket there is:

$IqV$  - the linear fractional rate for charge accumulation in a region with thickness  $h$ ;

$\frac{Q\lambda}{\varepsilon_0}$  - the loss due to charging by conduction  $\lambda$  in view of the medium fractality;

$p \frac{Q}{h^2}$  - the turbulent losses.

Rewrite (3) as follows:

$$\partial_{0t}^{\alpha} Q(t) + \beta Q(t) = f(t), \quad 0 < \alpha < 1, \quad (4)$$

where  $\beta = \left( \frac{\lambda}{\varepsilon_0} - \frac{p}{h^2} \right) \tau$ ,  $f(t) = IqV\tau$ .

Consider the Cauchy problem for equation (4) with the initial condition

$$Q(0) = Q_0. \quad (5)$$

The solution to problem (4)-(5) takes the form of [11]:

use the formula

$$\partial_{0t}^{\alpha} Q(t) = D_{0t}^{\alpha} Q(t) - \sum_{k=1}^n \frac{t^{k-\alpha-1}}{\Gamma(1-\alpha)} Q^{(k-1)}(0),$$

get

$$D_{0t}^{\alpha} Q(t) - \sum_{k=1}^n \frac{t^{k-\alpha-1}}{\Gamma(1-\alpha)} Q^{(k-1)}(0) + \beta Q(t) = f(t),$$

hence, we find

$$\begin{aligned} Q(t) &= F * t^{\alpha-1} E_{\alpha,\alpha}(-\beta t^{\alpha}) + \sum_{k=1}^n a_k t^{\alpha-k} E_{\alpha,\alpha-k+1}(-\beta t^{\alpha}) = \\ &= f * t^{\alpha-1} E_{\alpha,\alpha}(-\beta t^{\alpha}) + \sum_{k=1}^n Q^{(k-1)}(0) \frac{t^{k-\alpha-1}}{\Gamma(k-\alpha)} * t^{\alpha-1} E_{\alpha,\alpha}(-\beta t^{\alpha}) = \\ &= IqV\tau t^{\alpha} E_{\alpha,\alpha+1}(-\beta t^{\alpha}) + Q_0 E_{\alpha,1}(-\beta t^{\alpha}), \end{aligned} \quad (6)$$

where  $E_{\alpha,\alpha}(-\beta t^{\alpha}) = \sum_{k=0}^{\infty} \frac{-\beta^k t^{k\alpha}}{\Gamma(\alpha k + \alpha)}$  is a Mittag - Leffler type function. Equation (6)

describes the fractal dynamics for charge accumulation in thunderstorm clouds taking into account the fractal structure of the cloud environment.

### 3 Results of calculations for the model testing

The charge accumulation has been calculated for:  $\varepsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$ ,  $p = 10 \text{ m}^2/\text{s}$ ,  $h = 50 \text{ m}$ ,  $I = 170 \text{ mm/h}$ ,  $\lambda = 10^{-12} \text{ Cm/m}$ ,  $V = 1 \text{ km}^3$  [3]. Figure 1 shows the calculated curves for a relationship between the charge accumulation dynamics in thunderstorm clouds and fractional values, which determine the fractal dimension. It can be seen for small values of  $\alpha$  linear growth rate for the charge occurs while with increase of the fractal dimension such linearity is violated. It can also be seen that the graph's origin does not coincide with the origin of coordinates, this means that the charges do not disappear completely or are not reset once the thunderstorm breakdown have occurred. After the breakdown, an increase in the fractal dimension leads to some relaxation time, after which the restoration of neutralized charges and the electric field strength begins.

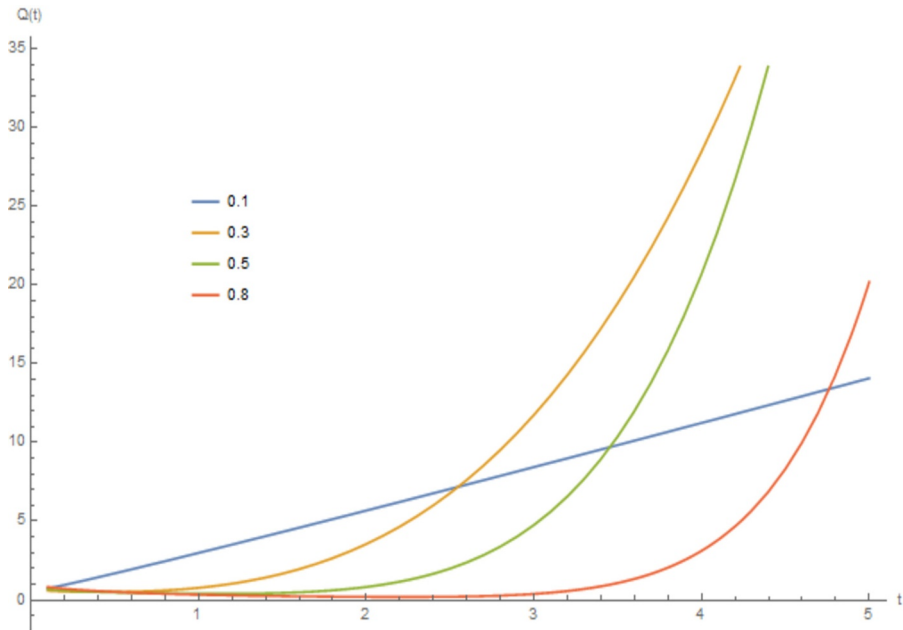


Fig.1. Curves dynamics of charge accumulation in thunderstorm clouds at various  $\alpha$

For a comparative analysis in Fig. 2 there are fractal-dynamic model curves together with the curves of elementary electrification mechanisms in thunderstorm clouds of  $1 \text{ km}^3$  for the limiting case of unipolar charging [3]. It can be seen that for  $\alpha = 1$  the curve from fig.1 similar to curves I, II, III $\delta$ , IV, V, while for other values of  $\alpha$  the curves are similar to the other curves in fig. 2.

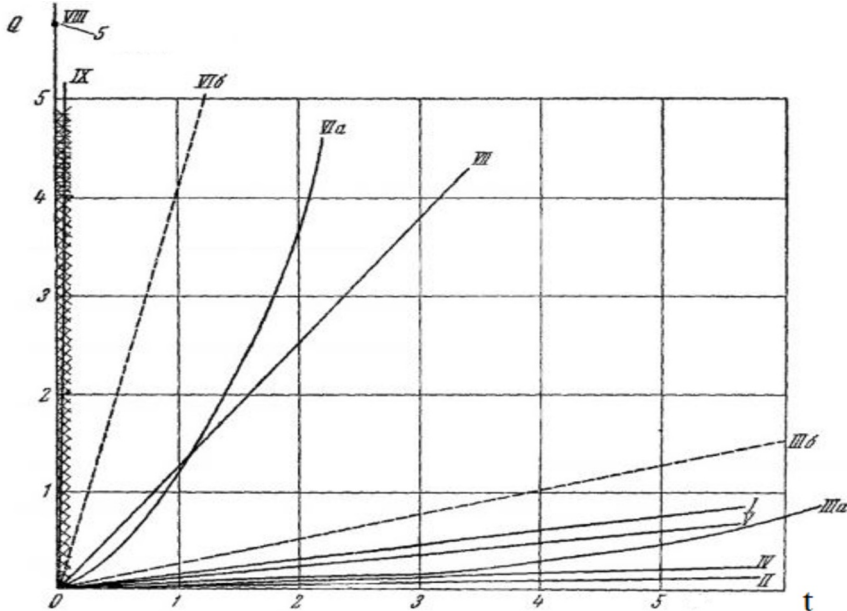


Fig.2. The effect of elementary electrification mechanisms in thunderstorm clouds

Figure 2 shows the curves of various elementary mechanisms of electrification in thunderclouds: I — Electrification by unipolar ion trapping; II — electrification by the Wilson mechanism; III — electrification by Elster and Geytel method; IIIa — initial field strength 1 V/cm; III b — in an electric field of 500 V/cm; IV — electrification by the waterball effect; V — electrification by the ice ball effect; VI — electrification by the droplets destruction in an electric field; VI a — initial field strength 1 V/cm; VI b — in an electric field of 500 V/cm; VII — electrification during inelastic hit of supercooled droplets upon an ice surface; VIII — electrification by the Workman and Reynolds mechanism; IX — electrification via contact potential difference between water and ice.

Charge accumulation values in Fig.1 significantly higher compared to the values in Fig. 2. This implies the importance of the fractal characteristic of the medium. It can be argued that the most favorable range for the rapid recovery of neutralized charges and electric field is when the fractal parameter values are between  $\alpha = 0.3$  and  $\alpha = 0.7$ . Nevertheless, what are the properties of cloud fractal structures, how they depend on the environment, and how the fractal dimension is related to the fractional exponents in the equation for the dynamics of charge accumulation and the restoration of the electric field in thunderstorm clouds — these issues require separate research.

## 4 Conclusion

A detailed study of macro- and micro-processes in thunderstorm clouds should make it possible to resolve the issues of the fractal-dynamic mechanism in thunderstorm clouds. Among the most discussed issues in thunderstorm physics is how the electric field arises and maintained. It is not also clear how lightning discharge is maintained. The question of the nature of ball lightning remains. These issues should be given special attention in view of fractal structure of clouds. The phenomena considered in this work is still at the initial stage of study. In recent years, exploration in this field has advanced forward and the use of the fractional integro-differential calculus in building of the fractal dynamics model for charge accumulation in thunderstorm clouds contributed to this. The obtained formulas can be used to calculate changes in the charge acquired by cloud particles under condensation, sublimation and coagulation processes with specified parameters and the fractal dimension. Numerical experiments to assess the effect of the medium fractality on the charge acquired by cloud particles with various microphysical parameters have shown a general dependence of the charge accumulation on the fractal dimension.

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