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Solar-Terrestrial Relations and Physics of Earthquake Precursors

Interaction of elastic waves with ice layer in shelf zone

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Abstract. A theoretical model for the propagation of elastic waves of arbitrary wave sizes from 0.5 to 20 units in an ice layer has been developed. The calculation was based on Green's function theory for Helmholtz equation. Special "directed" Green's functions were introduced. They make it possible to anayze wave fields in closes volumes limited by different-angle impedances. The developed calculation algorithms allow one to anayze fields on medium-powered computers for 15 minutes. The suggested methods are capable of estimating elastic wave interactions with different impedances in bays, lakes and other volumes with limited wave sizes.

1 Theory

Earthquakes and tsunamy are accomponied by the distribution of different elastic waves in ocean bottom, in water and in ice. The elastic wave distribution laws for water and for the earth crust were considered in detail in [1], [2], [3].

However, the laws for elastic wave distribution in ice have been insufficiently studied owing to the mathematical awkwardness of the obtained algorithms. In the present work we have developed a correct mathematical model based on the theory of directed Green's functions [4].

It has been proved that application of waves propagating in ice allows one to improve field characteristics such as elastic wave propagation distance, noise immunity of communication and data processing systems.

2 Mathematical model

As an initial model we consider harmonic fields which are described by the following equation:

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$$\Delta\Phi(r)+k^2\Phi(r)=-4\pi q(r_0), \quad (1)$$

where Φ is the oscillating velocity potential; $q(r_0)$ is the density of source distribution in domain r_0; r is the distance from coordinate origin to observation point M; k is the wave number.

Equation (1) is solved by directed Green's functions. Function $G_{Hl}(M, M_0)$ has a sector-shaped pattern in the defined angle interval $\Delta\varphi$

$$G_{Hl}=\begin{cases}1, & \text{when} \quad \varphi_{\min}\le\varphi\le\varphi_{\max}, \\ 0, & \text{in the rest angle interval.}\end{cases} \quad (2)$$

where G_{Hl} is the directed Green's function.

Green's function of a point-source radiator has the form

$$G(r,r_0)=G(M,M_0)=\sum_{l=1}^{L}G_{Hl}(M,M_0)=\frac{e^{ikR}}{R}, \quad (3)$$

where l is an angle sector where Green's function equals a unit; L is the number of sectors overlapping the angle 4π.

Ice surface is approximated by a plane set as long as a plane wave is Helmholtz equation solution in Cartesian coordinate system (Fig. 1).

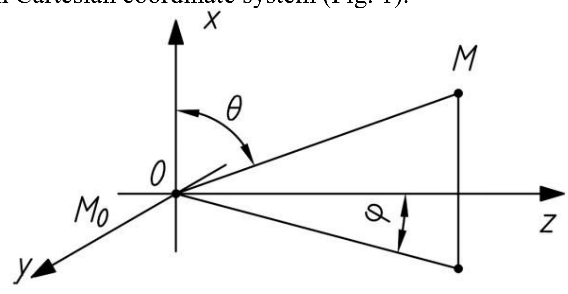

Fig. 1. Generalized angle geometry.

We represent spherical wave (3) in the form of Rayleigh integral (4)

$$\Phi(x,y,z)=\frac{e^{ikR}}{R}=\frac{i}{2\pi}\int\int_{-\infty}^{+\infty}\frac{1}{\sqrt{k^2-U_1^2-U_2^2}}e^{i\left[U_1x+U_2y+z\sqrt{k^2-U_1^2-U_2^2}\right]}dU_1dU_2, \quad (4)$$

The spherical wave in the form of plane waves superposition. Helmholtz equation in the Cartesian system has the form where $\bar{k}=k_x\bar{i}+k_y\bar{j}+k_z\bar{k}$; $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$; $k^2=k_x^2+k_y^2+k_z^2$.

Real variables U_1 and U_2 change within the interval $[-\infty;+\infty]$ where

$$\begin{cases}U_1=\pm k_x=k\cos\theta, \\ U_2=\pm k_y=k\sin\theta\sin\varphi, \\ U_3=\pm k_z=k\sin\theta\cos\varphi.\end{cases}$$

The following notations are introduced:

$$\begin{cases} \cos\theta = V_1, \\ \sin\theta\sin\varphi = V_2, \\ \sin\theta\cos\varphi = V_3. \end{cases}$$

Visible angle zone (circle) is $k\sin\theta\cos\varphi = k_z = \pm\sqrt{k^2 - k_x^2 - k_y^2} = \pm\sqrt{k^2 - U_1^2 - U_2^2}$.

Expression (4) may be used to calculate Green's directed functions. In representation (3) Green's function has the form

$$G(M, M_0) = \frac{e^{ikR}}{R} = \sum_{l=1}^{L} \int\int_{-\infty}^{+\infty} \frac{F(U_1, U_2) e^{i\left(U_1 x + U_2 y + z\sqrt{k - U_1^2 - U_2^2}\right)}}{\sqrt{k - U_1^2 - U_2^2}} dU_1 dU_2, \quad (5)$$

$k = \dfrac{2\pi}{\lambda}$ is the wave number;

$$F_H(U_1, U_2) = \begin{cases} 1, \text{ when } U_{1\min} \le U_1 \le U_{1\max}, U_{2\min} \le U_2 \le U_{2\max}, \\ 0, \text{ on the rest } U \text{ values,} \\ f(U_1, U_2), \text{ in imaginary angle zone.} \end{cases}$$

Function (5) is the basic one to make further numerical experiments.

Hereinafter, the coefficient of wave reflection from interface of a plane layer is denoted as *V*. Transmission (transparency) coefficient is *W*.

Green's functions for position mediums: medium 1, 3 (Fig.2) is assumed in the form

$$G_3(r, r_0) = \frac{e^{ik_3 R}}{R} + V\frac{e^{-ik_3 R}}{R}, \quad (6a)$$

$$G_1(r, r_0) = \frac{e^{ik_1 R_{pr}}}{R_{pr}} W, \quad (6b)$$

where $R = r - r_0$, $R_{pr} = r_{pr} - r_0$ are the observation point coordinates relatively a source.

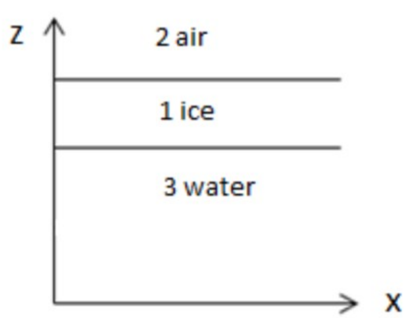

Fig. 2. Layer medium geometry taking into account the ice.

As we introduce the expression in general form which allows us to take into account the reflection from a layer and its transparency, we basically form a generalized mathematical model which we shall further modify for concrete tasks considered within the framework of this paper. We turn to the expressions for reflection coefficient V and transmission coefficient W.

Address to the theory of wave reflection from the interface of homogeneous half-spaces, a plane layer and a layer system [8, 9]. As long as this theory is widely known, it is unreasonable to describe it. We consider only the basic principle, interactions and resulting formulas which are applied in this paper to make numerical experiments.

If we follow the problem, formulated in the previous section, the generalized physical model has the form. The radiator position medium, the layer and the medium, where a wave should arrive, are assigned the numbers 3, 2, 1, respectively. The angles of incidence, reflection and passage are denoted by θ_3, θ_2, θ_1.

To find the coefficient of reflection from a layer, we need to find the impedance of the layer under consideration. The coefficient of reflection from layer 2 though impedance at the boundary 3–2 (Z_{IN}) has the form

$$V = \frac{Z_{IN}\cos\theta_3 - \rho_3 c_3}{Z_{IN}\cos\theta_3 + \rho_3 c_3} = \frac{Z_{IN} - Z_3}{Z_{IN} + Z_3}, \tag{7}$$

where $Z_3 = \dfrac{\rho_3 c_3}{\cos\theta_3}$ is a plane wave impedance in medium 3.

As the result of rereflection at the boundaries, two waves with different propagation directions, symmetrical to plane 2, are formed inside the layer. Owing to that, the pressure inside layer 2 has the form

$$p_2 = (Ae^{-ik_{2z}z} + Be^{-ik_{2z}z})e^{-ik_{2x}x}, \tag{8}$$

for $k_{2x}^2 + k_{2z}^2 = k_2^2$, $k_2 = \dfrac{\omega}{c_2}$, where A and B are undetermined constants.

It is known that in case of harmonic wave propagation in a homogeneous medium, particle oscillating velocity takes the form

$$v = \frac{\operatorname{grad} p}{i\omega\rho}, \tag{9}$$

where $p = p(x, y, z, t)$ is the pressure; ω is the frequency; ρ is the medium density.

Expression for z-component of velocity v_2 is obtained by the substitution of (8) into (9):

$$v_{2z} = \frac{1}{i\omega\rho_2}\frac{\partial p_2}{\partial z} = \frac{k_{2z}}{\omega\rho_2}\left(Be^{-ik_{2z}z} - Ae^{-ik_{2z}z}\right)e^{-ik_{2x}x}, \tag{10}$$

we denote

$$-\left(\frac{p_2}{v_{2z}}\right)_{z=0} = Z_1, \tag{11}$$

here

$$Z_1 = -i\omega\rho_1 p_1 \left(\frac{\partial p_1}{\partial z}\right)^{-1} = \frac{\rho_1 c_1}{\cos\theta_1}. \tag{12}$$

Substituting (8) and (10) into expression (11), we obtain:

$$\frac{B}{A} = \frac{Z_1 - Z_2}{Z_1 + Z_2}. \quad (13)$$

Plane wave impedance in layer 2 is

$$Z_2 = \frac{\omega c_2}{k_{2z}} = \frac{\rho_2 c_2}{\cos\theta_2}. \quad (14)$$

Input impedance at the upper boundary of layer $z = d$ is

$$Z_{IN} = -\left(\frac{p_2}{v_{2z}}\right)_{z=d}. \quad (15)$$

Substituting (8) and (10) into (15), we obtain the expression for the layer input impedance

$$Z_{IN} = \frac{Z_1 - iZ_2\ \mathrm{tg}\ k_{2z}d}{Z_2 - iZ_1\ \mathrm{tg}\ k_{2z}d} Z_2. \quad (16)$$

Now, substituting expressions (16) into (7), we obtain a formula for the coefficient of reflection from layer 2

$$V = \frac{(Z_1 + Z_2)(Z_2 - Z_3)e^{-ik_{2z}d} + (Z_1 - Z_2)(Z_2 + Z_3)e^{ik_{2z}d}}{(Z_1 + Z_2)(Z_2 + Z_3)e^{-ik_{2z}d} + (Z_1 - Z_2)(Z_2 - Z_3)e^{ik_{2z}d}}. \quad (17)$$

In the case when impedances of edge mediums are similar, the expression of the coefficient of reflection from the layer has the form

$$V = \frac{Z_2^2 - Z_1^2}{Z_1^2 + Z_2^2 + 2iZ_1Z_2\ \mathrm{ctg}\ k_{2z}d}. \quad (18)$$

If d layer thickness tends to zero, reflection coefficient is given by the formula

$$V = \frac{Z_1 - Z_3}{Z_1 + Z_3}. \quad (19)$$

From the described expression of pressure continuity when passing the boundary $z = 0$, the following expression is fair:

$$(p_2 - p_1)_{z=0} = 0; \quad (20)$$

$$A + B = W. \quad (21)$$

The condition of sound pressure continuity at the boundary $z = d$ can be written as

$$1 + V = (p_2)_{z=d} = Ae^{-ik_{2z}d} + Be^{ik_{2z}d}, \quad (22)$$

where $1 + V$ is the sum of pressures created by incident and reflected waves.

Dividing (21) by (22) and applying (13), we obtain the expression for transparency coefficient

$$W = \frac{1+V}{\cos k_{2z}d - i\left(\frac{Z_2}{Z_1}\right)\sin k_{2z}d}. \quad (23)$$

Substituting (17) into (23) we obtain the final expression for the coefficient of transmission into the layer

$$W = \frac{4Z_1Z_2}{(Z_1-Z_2)(Z_2-Z_3)e^{ik_{2z}d} + (Z_1+Z_2)(Z_2+Z_3)e^{-ik_{2z}d}}. \quad (24)$$

If d layer thickness tends to zero, the transmission coefficient is given by the formula

$$W = \frac{2Z_1}{Z_1+Z_3}. \quad (25)$$

3 Results of numerical studies

Fig. 3-8 show the results of calculations based on the proposed theory.

In free space, Green's function at the frequency f=1 kHz behaves as follows:

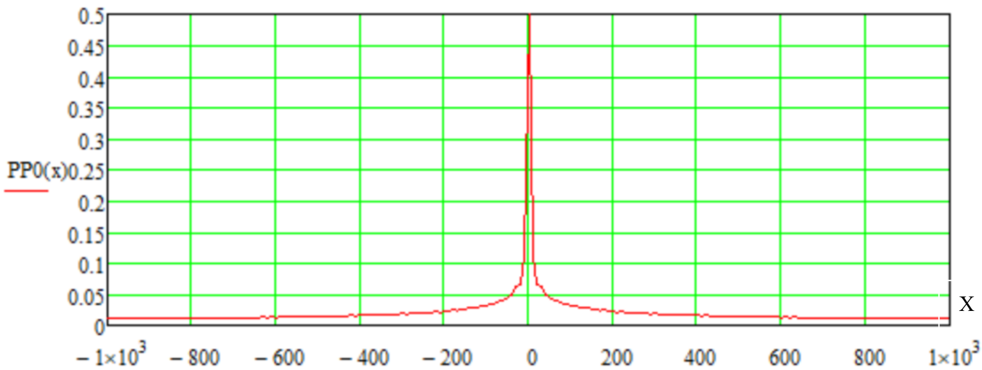

Fig. 3. Pressure changes in free space.

Initial data of medium characteristics are:

- frequency is f=1kHz;
- ultrasound wave propagation velocity in water is 1500 m/s;
- ultrasound wave propagation velocity in air is 331 m/s;
- distance between the radiator and the hydrophone is 1000 m.

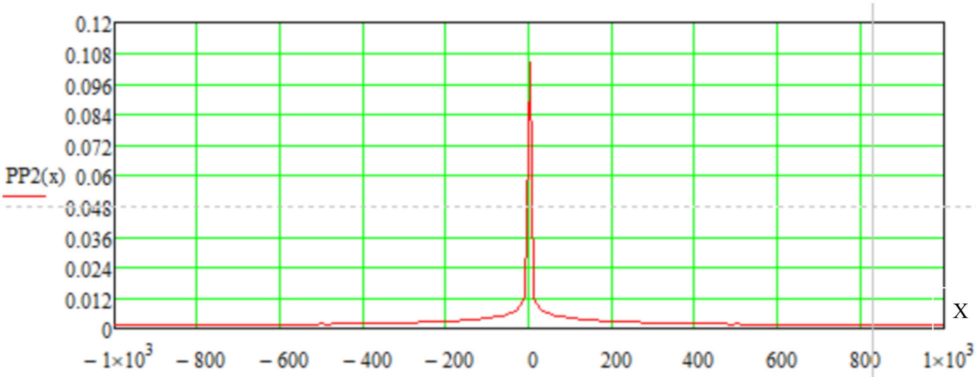

Fig. 4. Pressure changes in ice-water half-space.

Initial data of medium characteristics are:

- frequency is *f*=1кГц;
- ultrasound wave propagation velocity in water is 1500 m/s;
- ultrasound wave propagation velocity in air is 331 m/s;
- ultrasound wave propagation velocity in ice is 3980 m/s;
- distance between the radiator and the hydrophone is 1000 m.

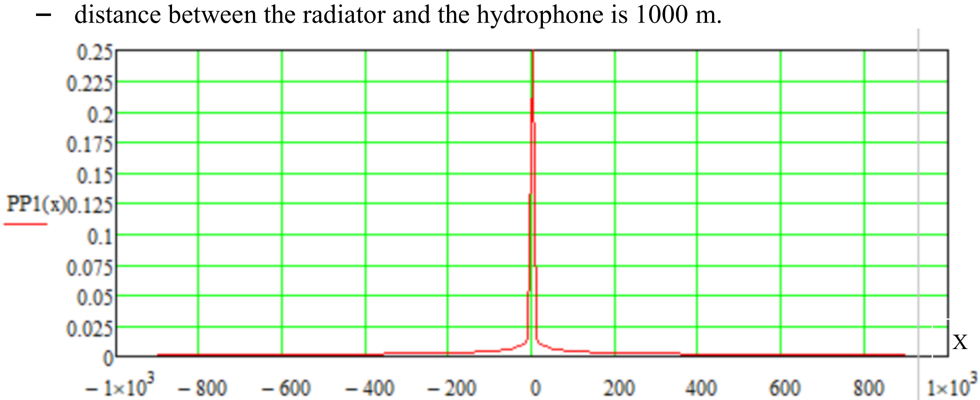

Fig. 5. Pressure changes in air-ice half-space.

Initial data of medium characteristics are:

- frequency is *f*=1кГц;
- ultrasound wave propagation velocity in water is 1500 m/s;
- ultrasound wave propagation velocity in air is 331 m/s;
- ultrasound wave propagation velocity in ice is 3980 m/s;
- distance between the radiator and the hydrophone is 1000 m.

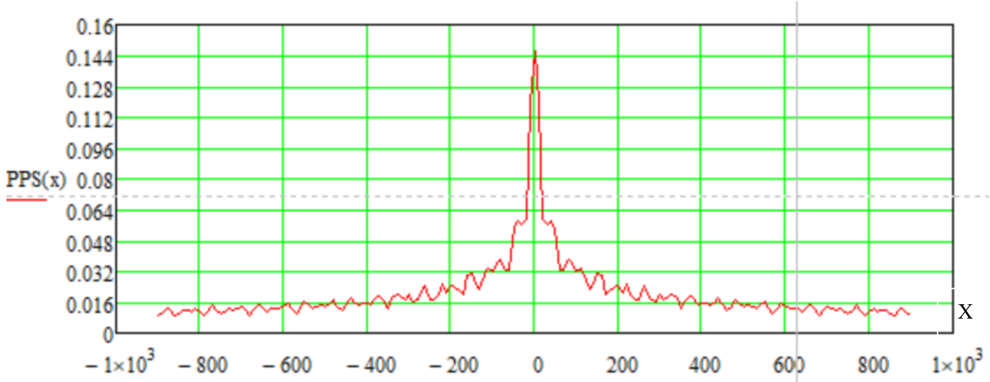

Fig. 6. Pressure distribution in ice layer.

Initial data of medium characteristics are:

– frequency is *f*=1кГц;
– ultrasound wave propagation velocity in water is 1500 m/s;
– ultrasound wave propagation velocity in air is 331 m/s;
– ultrasound wave propagation velocity in ice is 3980 m/s;
– distance between the radiator and the hydrophone is 1000 m.

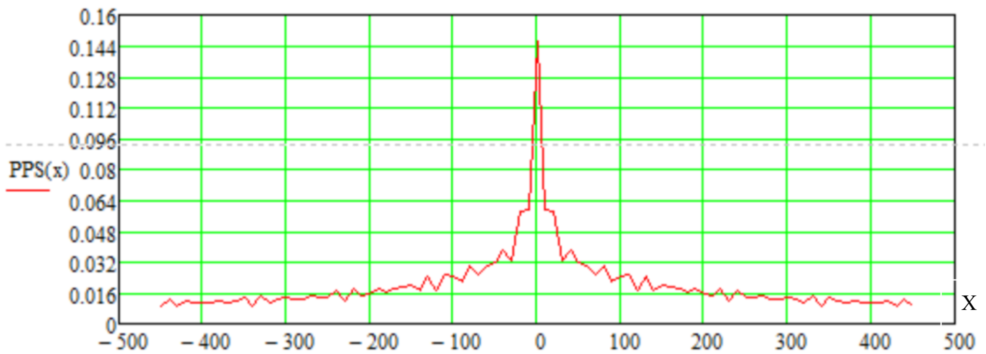

Fig. 7. Pressure distribution in ice layer.

Initial data of medium characteristics are:

– frequency is *f*=1кГц;
– ultrasound wave propagation velocity in water is 1500 m/s;
– ultrasound wave propagation velocity in air is 331 m/s;
– ultrasound wave propagation velocity in ice is 3980 m/s;
– distance between the radiator and the hydrophone is 1000 m.

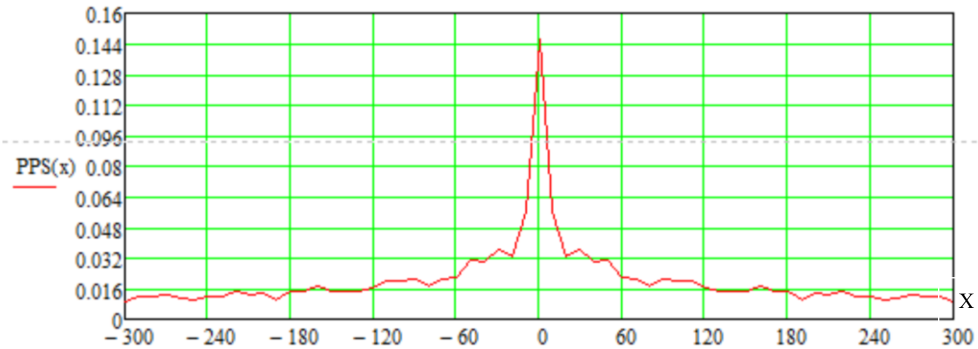

Fig. 8. Pressure distribution in ice layer.

Initial data of medium characteristics are:

- frequency is *f*=3кГц;
- ultrasound wave propagation velocity in water is 1500 m/s;
- ultrasound wave propagation velocity in air is 331 m/s;
- ultrasound wave propagation velocity in ice is 3980 m/s;
- distance between the radiator and the hydrophone is 1000 m.

4 Conclusions

1. Applying an ice layer as a path for wave propagation, we can extend the communication system distance by about 1.5-2 times since the sound wave $P(x)$ decreases according to the cylindrical law (Fig. 6,7,8)).
2. The time for calculation of one variant of a field does not exceed one minute on medium-powered computers.
3. The developed theory may be applied in online mode for theoretical analysis of sound fields accompanying earthquakes.

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