E3S Web of Conferences 139, 01066 (2019) https://doi.org/10.1051/e3sconf/201913901066
RSES 2019

Topological algorithm for forming nodal stresses of complex networks energy systems

D.S. Akhmetbaev[1*] , *A.R.* Dzhandigulov[2] , *A.D.* Akhmetbaev[3]

[1]Seifullin Kazakh Agro Technical University, Nur-Sultan, Republic of Kazakhstan
[2]L.N. Gumilyov Eurasian National University, Nur-Sultan, Republic of Kazakhstan
[3]Information Systems Directorate, «Kazakhtelecom» JSC, Almaty, Republic of Kazakhsta

Abstract. The paper presents a new topological algorithm for the formation of nodal stresses of complex networks of power systems. The first step of the described algorithm is the search and determination of the values of all possible and specific trees of the graph corresponding to a given network of the power system. The well-known advantage of the topological approach compared to matrix methods, which allows one to obtain the final solution of non-linear equations of the steady state, has led to the development of many methods and corresponding software implementations. The paper provides a comparative analysis of the advantages and disadvantages of these methods. The main computational complexity of these methods is the search for 2 trees for each tree, which are obtained from it by dividing into two parts by removing any branch. A completely new topological approach that does not require finding 2 trees was proposed by one of the authors. The process of this method consists of the following steps: finding all possible graph trees, selecting specific graph trees, calculating the network node voltage. The work offers a unique algorithm for the implementation of these stages, which are implemented and tested at the software level. The result of the execution of the software package is the calculation of the steady state of a complex electric network using the distribution coefficients of the driving currents.

1 Introduction

The main task of the topological analysis of the electric network is to determine all of its possible trees [1-3]. From the theory of graphs [4] it is known that as the number of nodes increases, the number of trees in the graph of a real electric network increases quite quickly. Therefore, it is necessary to develop more efficient algorithms search all possible trees.

In [5, 6], three methods are realized for finding possible trees of complex electric networks. The essence of all these methods is to iterate over and study the subgraphs for whether they are trees or not. In this case, in one method, iteration is carried out along branches, the second - through nodes, and in the third - along contours. Another approach is implemented in the theory of structural numbers [7]. The disadvantages of these methods is the need to store a large data array in memory.

One of the reasons for the limited practical distribution of these methods is the need to find the so-called 2-trees for each tree found, which significantly increase the number of necessary operations [8-10].

In this paper, a new topological approach is implemented that does not require finding 2-trees. The first stage, as in existing methods, calculates all possible trees of the graph. The search algorithm is as follows: it is performed on a directed search of various samples of n-1 edges from all edges of the graph with simultaneous verification of the resulting subgraph for connectivity. The advantage of the proposed algorithm is the elimination of repetition due to the orientation of the sample.

Further, in the second stage, instead of finding 2 trees, specific graphs are distinguished from the number of all possible trees of the graph found in the first stage.

In fact, the trees themselves are not needed, but so-called tree sizes are needed, which are a product of the weights of the branches that make up the trees. Therefore, there is no need to remember these trees, it is enough to remember only their weights.

Knowing these weights is enough to calculate the current distribution coefficients, without solving any systems of equations, as is done in matrix methods. Current distribution coefficients are the initial data for the third stage - the calculation of the node-voltage of the network.

2 The first stage of the algorithm: determining the weights of all possible trees of the graph

* Corresponding author: axmetbaev46@mail.ru

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (http://creativecommons.org/licenses/by/4.0/).

The search algorithm for all possible trees of the graph with the calculation of their values.

Let m be the number of all branches of the original graph. n is the number of nodes (vertices) of the graph.
Step 1. We carry out directed enumeration of samples of n-1 elements from m elements.

Step 2. Checking the end of the search options: yes - complete the algorithm, no - continue the calculation.

Step 3. For the next sampling, we carry out a connection test.

Verification is carried out as follows. Starting from the first node (vertex), we begin the movement along the branches from the selection. Three options are possible: 1) either there are no new paths from the next vertex, 2) either the already selected vertex is returned (that is, a cycle has formed), 3) or all vertices are selected. In the first two cases, the check stops - the selection of branches does not form a tree.

Step 4. Calculation of tree sizes. Let G_k be the set of edge numbers of the kth tree. Y_i is the conductivity of the i-th branch, which is expressed from the given complex resistance of the i -th branch of Z_i according to the formula $Y_i = 1 / Z_i$.

Then the weight of the (kth) tree is calculated by the formula:

$$Q_k = \prod_{i \in G_k} Y_i \tag{1}$$

The resulting value of all possible trees is formed as the sum of the weights of the individual trees of the graph. According to this algorithm, a program is compiled that reads the source data from an external file and performs a preliminary check of the correctness of the source data.

As an illustration, we present the result of the program for determining all possible trees of a real 220 kV network, shown in Fig. 1 (Scheme with the parameters of the selected part of the network voltage of 220 kV of the current energy system of the Republic of Kazakhstan).

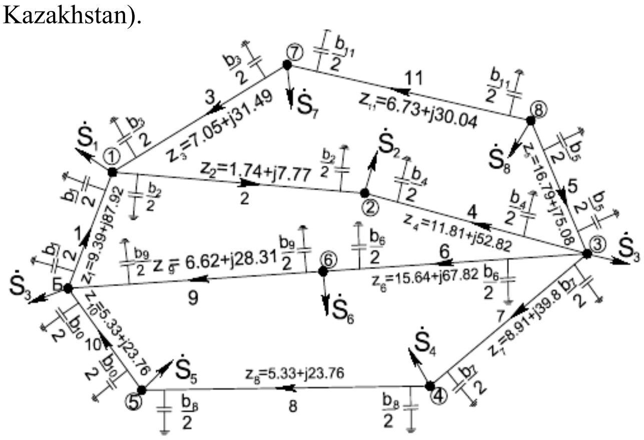

Fig. 1. The equivalent circuit

In the network under consideration, the number of all branches m=11 , the number of nodes n=9 .

The number of all possible trees is 85, and they are presented in table 1.

Table 1. All possible graph trees

tree №	List of branches	tree №	List of branches	tree №	List of branches
1	1 2 3 4 5 6 7 8	29	1 2 4 5 6 7 8 11	57	1 4 5 6 7 8 10 11
2	1 2 3 4 5 6 7 10	30	1 2 4 5 6 7 10 11	58	1 4 5 6 7 9 10 11
3	1 2 3 4 5 6 8 10	31	1 2 4 5 6 8 10 11	59	1 4 5 6 8 9 10 11
4	1 2 3 4 5 7 8 9	32	1 2 4 5 7 8 9 11	60	1 4 5 7 8 9 10 11
5	1 2 3 4 5 7 9 10	33	1 2 4 5 7 9 10 11	61	2 3 4 5 6 7 8 9
6	1 2 3 4 5 8 9 10	34	1 2 4 5 8 9 10 11	62	2 3 4 5 6 7 8 10
7	1 2 3 4 6 7 8 11	35	1 2 5 6 7 8 9 11	63	2 3 4 5 6 7 9 10
8	1 2 3 4 6 7 10 11	36	1 2 5 6 7 8 10 11	64	2 3 4 5 6 8 9 10
9	1 2 3 4 6 8 10 11	37	1 2 5 6 7 9 10 11	65	2 3 4 5 7 8 9 10
10	1 2 3 4 7 8 9 11	38	1 2 5 6 8 9 10 11	66	2 3 4 6 7 8 9 11
11	1 2 3 4 7 9 10 11	39	1 2 5 7 8 9 10 11	67	2 3 4 6 7 8 10 11
12	1 2 3 4 8 9 10 11	40	1 3 4 5 6 7 8 9	68	2 3 4 6 7 9 10 11
13	1 2 3 5 6 7 8 9	41	1 3 4 5 6 7 8 10	69	2 3 4 6 8 9 10 11
14	1 2 3 5 6 7 8 10	42	1 3 4 5 6 7 8 11	70	2 3 4 7 8 9 10 11
15	1 2 3 5 6 7 8 11	43	1 3 4 5 6 7 9 10	71	2 3 5 6 7 8 9 11
16	1 2 3 5 6 7 9 10	44	1 3 4 5 6 7 10 11	72	2 3 5 6 7 8 10 11
17	1 2 3 5 6 7 10 11	45	1 3 4 5 6 8 9 10	73	2 3 5 6 7 9 10 11
18	1 2 3 5 6 8 9 10	46	1 3 4 5 6 8 10 11	74	2 3 5 6 8 9 10 11
19	1 2 3 5 6 8 10 11	47	1 3 4 5 7 8 9 10	75	2 3 5 7 8 9 10 11
20	1 2 3 5 7 8 9 10	48	1 3 4 5 7 8 9 11	76	2 4 5 6 7 8 9 11
21	1 2 3 5 7 8 9 11	49	1 3 4 5 7 9 10 11	77	2 4 5 6 7 8 10 11
22	1 2 3 5 7 9 10 11	50	1 3 4 5 8 9 10 11	78	2 4 5 6 7 9 10 11
23	1 2 3 5 8 9 10 11	51	1 3 4 6 7 8 9 11	79	2 4 5 6 8 9 10 11
24	1 2 3 6 7 8 9 11	52	1 3 4 6 7 8 10 11	80	2 4 5 7 8 9 10 11
25	1 2 3 6 7 8 10 11	53	1 3 4 6 7 9 10 11	81	3 4 5 6 7 8 9 11
26	1 2 3 6 7 9 10 11	54	1 3 4 6 8 9 10 11	82	3 4 5 6 7 8 10 11
27	1 2 3 6 8 9 10 11	55	1 3 4 7 8 9 10 11	83	3 4 5 6 7 9 10 11
28	1 2 3 7 8 9 10 11	56	1 4 5 6 7 8 9 11	84	3 4 5 6 8 9 10 11
				85	3 4 5 7 8 9 10 11

3 Calculation of distribution coefficients of driving currents

The distribution coefficients of the driving currents are calculated on the basis of expressions obtained using matrix methods or network circuit topology methods. In [12], to determine the coefficients of current distribution by the matrix method, the following expression was obtained:

$$C = Z_B^{-1} M^T (M Z_B^{-1} M^T)^{-1} \quad (2)$$

where M is a rectangular matrix of the connection of nodes of dimension $m \times n$; Z_B^{-1} — diagonal branch conductivity matrix; T — sign transpose matrix. The elements of the current distribution matrix C are complex numbers.

The fundamental complexity of calculations using (1) lies in the need to perform the matrix inversion operation of a sufficiently large dimension. In addition, in some cases the problem is complicated by the poor conditionality of the matrix $M Z_B^{-1} M^T$. To increase the efficiency of computing this kind of matrix, the methods of factorization and matrix triangulation are used in practice [13].

If the matrix C is sought in the form of a solution to the matrix equation, then, in the general case, it can be found in the form

$$C = A/B \quad (3)$$

where B is the determinant of the matrix of the equation, A is the matrix of algebraic complements. The essence of the topological method is to present the expressions for A and B through the parameters of the network considered as a graph [5, 6].

In [8], the numerator of formula (3) is defined as the difference in the sums of weights of specific 2-trees, determined with respect to the index of the element of the desired matrix C, that is, the corresponding branch of the graph and with respect to the basis node. The main computational complexity of the method under consideration is finding 2 trees for each tree, which are obtained from it by dividing into two parts by removing any branch. Due to the need for cumbersome calculations, this method has not found wide practical application [9].

A completely new topological approach was proposed in the work of the first author [14–16]. Moreover, to calculate matrix A in formula (3), explicit expressions are obtained that are computed without the need to find 2-trees. Namely, its element A_{ij} is represented as the sum of the products of the conductivities of the branches of specific trees of the graph that contain the i-th branch in the path from the j-th node to the base node, taken taking into account the orientation of the i-th branch. The advantage of this approach is the absence of the need to calculate all 2-trees, which reduces the number of operations by n^2 times [17]. Due to the fact that we already calculated the module of specific trees at the first stage and it remains only to determine its sign, these two stages can be combined into one cycle, which further reduces the number of operations and the required memory size of the computing equipment.

To illustrate the operation of the program, we present in table 3 the results of calculating the current distribution coefficients in a real 220 kV network, shown in Fig. 1. Initial data - the matrix of complex resistances is presented in table 2.

Table 2. Complex resistance of branches

branch № j	1	2	3	4	5	6
ReZ_j	9.39	1.74	7.05	11.81	16.79	15.64
ImZ_j	87.92	7.77	31.49	52.82	75.08	67.82
branch № j	7	8	9	10	11	
ReZ_j	8.91	5.33	6.62	5.33	6.73	
ImZ_j	39.8	23.76	28.31	23.75	30.04	

The current distribution matrix has the dimension n x n. In our case, 9x9.

The accuracy of the calculations is estimated by the accuracy of the values of possible and specific trees, as well as the determination of modules and phase-node network voltages.

Table 3. Matrix of current distribution coefficients

-0.5043+0.0289*i	-0.4733+0.0272*i	-0.2631+0.0156*i	-0.1432+0.0085*i	-0.0716+0.0043*i	-0.0775+0.0048*i	-0.4487+0.0259*i	-0.3956+0.0229*i
0.3434+0.0200*i	-0.5957+0.0188*i	-0.1822+0.0108*i	-0.0992+0.0059*i	-0.0496+0.0030*i	-0.0537+0.0033*i	0.2222+0.0179*i	0.1066+0.0159*i
-0.1523-0.0089*i	-0.1224-0.0084*i	0.0808-0.0048*i	0.0440-0.0026*i	0.0220-0.0013*i	0.0238-0.0015*i	0.6709-0.0079*i	0.5023-0.0070*i
-0.3434-0.0200*i	-0.4043-0.0188*i	0.1822-0.0108*i	0.0992-0.0059*i	0.0496-0.0030*i	0.0537-0.0033*i	-0.2222-0.0179*i	-0.1066-0.0159*i
0.1523+0.0089*i	0.1224+0.0084*i	-0.0808+0.0048*i	-0.0440+0.0026*i	-0.0220+0.0013*i	-0.0238+0.0015*i	0.3291+0.0079*i	0.4977+0.0070*i
0.2357+0.0146*i	0.2504+0.0139*i	0.3504+0.0087*i	0.1907+0.0047*i	0.0953+0.0023*i	-0.1914+0.0030*i	0.2621+0.0133*i	0.2874+0.0120*i
0.2600+0.0143*i	0.2763+0.0133*i	0.3865+0.0069*i	-0.3339+0.0038*i	-0.1669+0.0019*i	0.1139+0.0018*i	0.2892+0.0126*i	0.3170+0.0109*i
0.2600+0.0143*i	0.2763+0.0133*i	0.3865+0.0069*i	0.6661+0.0038*i	-0.1669+0.0019*i	0.1139+0.0018*i	0.2892+0.0126*i	0.3170+0.0109*i
0.2357+0.0146*i	0.2504+0.0139*i	0.3504+0.0087*i	0.1907+0.0047*i	0.0953+0.0023*i	0.8086+0.0030*i	0.2621+0.0133*i	0.2874+0.0120*i
0.2600+0.0143*i	0.2763+0.0133*i	0.3865+0.0069*i	0.6661+0.0038*i	0.8331+0.0019*i	0.1139+0.0018*i	0.2892+0.0126*i	0.3170+0.0109*i
-0.1523-0.0089*i	-0.1224-0.0084*i	0.0808-0.0048*i	0.0440-0.0026*i	0.0220-0.0013*i	0.0238-0.0015*i	-0.3291-0.0079*i	0.5023-0.0070*i

4 Calculation of voltage nodes

Existing software for calculating and analyzing electric modes is based on classical mathematical models of the steady state of electric power systems; an analysis of their advantages and disadvantages can be found in [18]. In the work, the matrix equation developed in [19,20] is taken as the basis for calculating the steady states of complex networks

$$\dot{U} = U_0 + C^T Z_B C \hat{U}^{-1} \hat{S} \tag{4}$$

where C is the matrix of current distribution coefficients; Z_B is the diagonal matrix of branch resistances; $\hat{U}$– is the diagonal matrix of nodal conjugate voltages; $\hat{S}$ – is the column matrix of the conjugate powers of the nodal loads and generators.

The algorithms for implementing the calculation of node voltages depends on the form of representation of the nodal parameters, which are represented by constant active and reactive powers or active powers and voltages [21-23]. If the nodal parameters are given by powers, then the following expressions are valid for determining the node stresses:

$$U_k' = U_0 + \sum_{j=1}^{n} Z_{kj} U_j^{-1} \left(P_j \cos(\delta_k + \psi_{kj}) + Q_j \sin(\delta_k + \psi_{kj}) \right)$$

$$U''_k = \sum_{j=1}^{n} Z_{kj} U_j^{-1} \left(P_j \sin(\delta_k + \psi_{kj}) - Q_j \cos(\delta_k + \psi_{kj}) \right)$$

where $U_k = \sqrt{(U_k')^2 + (U_k'')^2}$ - voltage module of the k–th node;

$\delta_k = \operatorname{arctg} \dfrac{U_k''}{U_k'}$ - voltage phase of the k–th node;

$\underline{Z}_{kj} = \sqrt{(\operatorname{Re}\sum_{j=1}^{m} \underline{C}_{kj}{}^t \underline{Z}_j \underline{C}_{jk})^2 + (\operatorname{Im}\sum_{j=1}^{m} \underline{C}_{kj}{}^t \underline{Z}_j \underline{C}_{jk})^2}$ – mutual nodal resistance module;

$\psi_{kj} = \operatorname{arctg} \dfrac{\operatorname{Im}\sum_{j=1}^{m} \underline{C}_{kj}{}^t \underline{Z}_j \underline{C}_{jk}}{\operatorname{Re}\sum_{j=1}^{m} \underline{C}_{kj}{}^t \underline{Z}_j \underline{C}_{jk}}$ – phase of complex mutual nodal resistance.

If the nodal parameters are set by active powers and voltages, then the following expressions are valid for determining the node voltages:

$$Q_n^k = \frac{U_n^2 \cos\delta_n^{k-1} - U_0 U_n - Z_{nn} P_n \cos(\delta_n^{k-1} + \psi_{nn})}{Z_{nn} \sin(\delta_n^{k-1} + \psi_{nn})} - \frac{\sum_{j=1}^{n-1} Z_{nj} U_n U_j^{-1(k-1)} \left(P_j \cos(\delta_j^{k-1} + \psi_{nj}) + Q_j \sin(\delta_j^{k-1} + \psi_{nj}) \right)}{Z_{nn} \sin(\delta_n^{k-1} + \psi_{nn})}$$

The imaginary part of the voltage of the node under consideration, respectively, is equal to:

$$U_n'' = U_n \sin\delta_n^k = \sum_{j=1}^{n-1} Z_{nj} U_j^{-1(k-1)} \left(P_j \sin(\delta_j^{k-1} + \psi_{nj}) - Q_j \cos(\delta_j^{k-1} + \psi_{nj}) \right) + Z_{nn} U_n^{-1} \left(P_n \sin(\delta_n^{k-1} + \psi_{nn}) - Q_n^k \cos(\delta_n^{k-1} + \psi_{nn}) \right)$$

Hence, the value of the phase of the nodal stress for the kth iteration has the form:

As can be seen, from the above expressions, for calculating the voltage of the nodes it is enough to know the current distribution coefficients in the electric network. The algorithms developed above for the topological functions of a complex electric network are brought up to a calculation program for its steady state.

As an illustration, a comparative calculation of the circuit with the parameters of the selected part of the network with a voltage of 220 kV of the real power system of the Republic of Kazakhstan is made.

In order to verify the correctness of the results obtained, voltage calculations were performed at the nodes of the network at steady state using the proposed method and the industrial program RastrWin3.

The comparison results are presented in table 4.

Table 4. Results of comparative calculations of node voltages

	Calculation with applying a matrix C		Program verification calculation RASTRWin3		Deviations		
Nodal number	U, кV	δ, degree	U, кV	δ, degree	ΔU, кV	ΔU, %	Δδ, degree
1	241.43	-4.8315	240.65	-4.79	0,78	0,32	-0,04
2	241.53	-4.8433	240.78	-4.81	0,75	0,31	-0,03
3	240.89	-4.3086	240.36	-4.28	0,53	0,22	-0,03
4	237.38	-3.0902	237.1	-3.08	0,28	0,12	-0,01
5	235.60	-1.8466	235.45	-1.84	0,15	0,06	-0,01
6	236.27	-1.9230	236.12	-1.92	0,15	0,06	0,00
7	239.60	-6.2297	238.33	-6.17	1,27	0,53	-0,06
8	239.12	-6.5359	238.03	-6.49	1,09	0,46	-0,05

Deviations of the calculated values by the proposed method and using the industrial program RastrWin3 are less than 1%.

The execution time of the proposed program was 10-1 seconds. This means that it becomes possible to build a program complex for calculating the steady state of a complex electrical network of a power system in real time.

5 Conclusion

1. A program for calculating the possible trees of a complex directed graph has been developed.

2. A program for calculating current distribution coefficients based on the topology of a complex electric network has been developed.

3. A comprehensive program for calculating the steady-state regime of a complex electric network of a power system has been developed.

4. The software implementation of the algorithm for the formation of a steady-state regime significantly increases the efficiency of research as the number of nodes and branches of the electric network of the power system grows.

References

1. Reza F., Seeley S. Modern analysis of electrical circuits. Energiya. Moscow (1964).
2. Byerly R., Long R., King C. Logic for applying topological methods to electric networks. AIEE Transactions, Nov.1958, pt. I. v. 77.
3. Sato N. Digital calculation of network and mesh transformation matrices. AIEE Transactions, Oct. 1960, pt.III. v.79.
4. Berzh K. Graph theory and its applications. Publishing house of foreign literature, 1962. - 320 p..
5. Geraskin O. Finding a tree graph is an electric network using digital. Izvestiya AN USSR, Energetika i transport, №4, 53-60 (1965).
6. Geraskin O. Topological content of nodes and circuits of the electrical network and the calculation of their size with the help of a digital computer // Izvestia ANSSR. Energy and transport, 1966. №2. - pp. 59-70.
7. Bellert S., Wozniacki G. Analysis and synthesis of electrical circuits by the method of structural numbers. Mir. Moscow (1972).
8. Geraskin O. Methods for the determination of 2 - trees of the graph in the topology of electrical networks. Izvestiya AN USSR, Energetika i transport, №5, 106-112 (1967).
9. Geraskin O. Topological analysis of current distribution coefficients in electric networks. Izvestiya vuzov. Energetika, No.5, p. 20–28 (1967)
10. Christofides N. Graph Theory. Algorithmic approach. Mir. Moscow (1978).
11. Dzhandigulov A., Akhmetbayev D. Finding all spanning graphs of a given graph. The program for the computer. Certificate Republic of Kazakhstan on entering information into the state register of rights to objects protected by copyright (Republic of Kazakhstan). No. 1551 dated January 31, 2019
12. Melnikov N. Matrix method of analysis of electric circuits. Energiya. Moscow (1972).
13. Brameller A., Allan R., Hammam Y. Weakly filled matrices. Energiya. Moscow (1979).
14. Akhmetbayev D., Aubakir D., Sarsikeyev Y., Bainiyazov B., Surkov M., Rozhkov V., Ansabekova G., Yerbolova A., Suleimenov A., Tokasheva M. Development of Topological Method for Calculating Current Distribution Coefficients in Complex Power// Networks, Results in Physics, 2017, 7, pp. 1644–1649.
15. Akhmetbayev D., Akhmetbayev A., Zhumazhanov S., Zhakishev B. New Modeling of Steady-State Modes of Complex Electrical Grids of Power Systems MATEC Web of Conferences 155, 01043 (2018).
16. Akhmetbayev D., Akhmetbayev A., Aidarova A. Determination of rational transformation coefficients transformers distribution networks//E3S Web of conferences 25.04003 (2018)
17. Dzhandigulov A., Akhmetbaev D. Topological implementation of a new algorithm to calculate the coefficients of the current distribution in complex electrical networks. computer program. Certificate of entry into the state register of rights to objects protected by copyright. (Republic of Kazakhstan). No. 1552 dated January 31, 2019
18. Nepsha F., Otdelnova G., Savinkina O. Comparison of the functionality of existing software tools for calculating and analyzing electrical modes. // Vestnik of KuzGTU. Electrical complexes and systems (series), 2013 (2) p. 116–118.
19. Akhmetbaev D. The method of calculating the steady-state modes of electrical networks based on current distribution coefficients. Elektrichestvo. №11. 23-27(2010).
20. Akhmetbayev D., Akhmetbayev A., Suleymenov A., Kolcun M. Modeling the set mode of complex power grid, based on infeed coefficients// Proceedings of the 9th International Scientific Symposium Elektroenergetika – 2017, 12–14. 9. 2017, Stará Lesná, Slovak Republic.
21. Akhmetbayev D., Akhmetbayev A., Zhantlessova A. Formation of the Z-Form of Equations of Steady-State Modes of Energy Systems' Complex Electric Networks. Web of Conferences58, 02021 (2018)
22. Akhmetbaev D., Akhmetbaev A., Berdygozhin A. A topological method for the formation of nodal equations in the inverse form for electric power systems // Elektrichestvo. Moscow. No. 5 (2018) pp. 18–27.
23. Akhmetbayev D., Dzhandigulov A. Development of algorithms for a new topological method for calculating current distribution coefficients in complex electrical networks//Eurasian Journal of Mathematical and Computer Applications ISSN 2306–6172 Volume 7, Issue 3 (2019) 4–12.