

# Sensitivity Analysis Of Influencing Factors On Tunnel Stability In Bad Geological Slope Sections

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**Abstract:** Based on a tunnel through slope of Yongji highway project, relying on ABAQUS geotechnical numerical analysis software, using the appearance of plastic penetration area for instability criterion, choosing 11 typical influential factors as independent variables to tunnel stability, the orthogonal test array  $L_{50}(5^{11})$  is used to analyze sensitivity of influence factors under evaluating indexes of safety factor to the tunnel stability and the maximum principal tensile on the inner surface of surrounding rock. Research results show that: there are some certain differences between calculation results of safety factor and maximum principal tensile, but overall presentation represents favorable consistency, and the sensitivity of influence factors to the stability of tunnel through unfavorable geological slope can be preferably analyzed by orthogonal test. The influence of the cohesion of rock  $c$ , the horizontal distance  $d$  between tunnel centerline and slope toe, the inclination angle of slope  $\alpha$  are significant. Comprehensive consideration of safety factor and maximum principal tensile of tunnel circumference is more perfect than one-sided safety factor when performing the analysis of tunnel stability. The research results of this paper can provide reference to design, construction of similar tunnels.

## 1 INTRODUCTION

China is a mountainous country, with mountainous areas accounting for about two thirds of the land area<sup>[1]</sup>. Especially in the central and western regions, there are many mountains, rugged terrain, few surface attachments, exposed rocks, severe weathering, and extensive distribution of adverse geological mass slopes. With the rapid development of China's economy, the investment in infrastructure construction is increasing year by year, and the number of tunnels is increasing day by day. Especially after the implementation of the western development strategy, a large number of railways and highways were built in the mountainous areas in the central and western regions, and some lines were restricted by plane lines, natural geographical environment and other factors, so they often had to pass through the bad geological slopes in the form of tunnels. Therefore, it is common and unavoidable to build the tunnel into the mountain slope in the bad geological area.

Domestic and overseas scholars have conducted in-depth and systematic studies on the stability of surrounding rock in tunnel engineering and achieved fruitful results<sup>[2-5]</sup>. At the same time, the research results and a large number of engineering practice showed that: the stability of surrounding rock in tunnel engineering is influenced by many factors, not only including the properties of surrounding rock, rock-soil structure, initial

in-situ stress, geological structure, groundwater and other engineering geological conditions, but also by factors such as tunnel depth, section size, excavation method and support method. Adverse geological engineering geological conditions of slope area are much more complicated, the shear strength parameters (Cohesion and Internal friction angle), bulk density, poisson's ratio and elastic modulus, tunnel central line and the slope toe distance, the tunnel bottom section size and height of slope toe, Tunnel section size and Tunnel support force factors such as the sensitivity of the influence on the stability of surrounding rock of tunnel engineering is unclear. These factors are random and uncertain, sensitivity to the effect on the stability of the tunnel are also different, and the orthogonal experiment method is a more efficient processing level of multi-factor optimization experiment design method, orthogonal experiment design in multiple factors level sensitivity analysis can be less desired, scientific experiments and statistical effect<sup>[6]</sup>. Therefore, it is of great significance for the tunnel design and construction to grasp the sensitivity of influencing factors of tunnel stability in bad geological slope area.

In conclusion, based on the geotechnical numerical analysis software of ABAQUS, take the strength reduction technique, with the aid of relying on the engineering example, which transfixion of plastic zone for instability criterion, respectively, with the maximum

principal tensile stress safety coefficient and tunnel weeks as the dependent variable, using the principle of the orthogonal experiment research of various influencing factors of significance level, analysis the influence factors and the correlation between the stability of surrounding rock, and identify impact adverse geological section of tunnel surrounding rock slope stability of main control factors, determine the significance level of all the factors, in order to adverse geological section of tunnel slope design and construction to provide the reference.

## 2 PROJECT SUMMARY

Longtangping tunnel on YongJi expressway in hunan province is a separated single-hole tunnel on the left side, with the mileage pile NO. K37+531~K37+726 on the left side. The tunnel is 195m in length, and the maximum buried depth is 24.64m. The tunnel is located in the northern mountainous area of Guzhang county, Xiangxi autonomous prefecture, Hunan province. The site of the tunnel is a less mountainous area. It is located on the top of the ridge in the direction of a vertical line. The natural angle of the mountain is about 20°~65°, and the inlet and outlet sections of the tunnel are all steep slopes with a slope of about 50°, with local scarps and vegetation development. According to the geological drilling data, the surrounding rock of the tunnel is mainly slate of Banxi group Madiyi formation, the rock mass is broken, the degree of weathering is serious, and the thin layer of sandstone is locally mixed.

Through the laboratory test of core uniaxial saturated compressive strength, the value of uniaxial saturated compressive strength was 19.5~68.5MPa, and the average value was 42.5MPa. The point load laboratory test on the fractured rock block shows that the compressive strength of the rock sample point load test  $I_{s(50)}$  1.275~2.629MPa, which is calculated by substituting into the empirical formula for calculating the uniaxial saturated compressive strength of the core recommended in the "Detailed Rules For The Design Of Highway Tunnel" (JTGD/T d70-2010)<sup>[7-8]</sup> :

$$R_C = 22.82I_{s(50)}^{0.75} \quad (1)$$

Wherein,  $I_{s(50)}^{0.75}$  is core point load strength index,

$R_C$  is uniaxial saturated compressive strength of core.

Through calculation, the uniaxial saturated compressive strength of the core after conversion is 27.38~47.11MPa, which is close to the test data, verifying the rationality and validity of the test data. The uniaxial saturated compressive strength test values of the core were 19.5~68.5MPa, and were substituted into equation (2) to calculate the  $BQ$  value of the basic mass index of surrounding rock:

$$BQ = 90 + R_C + 250K_V \quad (2)$$

Wherein,  $K_V$  is rock mass integrity coefficient,

According to the actual engineering situation,  $K_V$  is set at 0.2.

The calculated  $BQ$  value is 198.5-345.5, and the mean value is 267.5. Then substitute equation (3) to calculate the modified value of the basic mass index of surrounding rock of  $BQ$  value  $[BQ]$ :

$$[BQ] = BQ - 100(K_1 + K_2 + K_3) \quad (3)$$

Wherein,  $K_1$  is the correction coefficient of groundwater influence, according to the actual engineering situation,  $K_1=0.5$ ;  $K_2$  is the correction coefficient of the influence of attitude of the main weak structure surface. According to the actual engineering situation,  $K_2=0.3$  is taken;  $K_3$  is the influence correction coefficient of the initial stress state. According to the actual engineering situation, take  $K_3=0$ .

The modified value of  $[BQ]$  was 118.5~265.5, with an average value of 187.5. Considering the structural integrity, weathering degree of rock mass, the engineering geological environment, embedded depth of the tunnel roof and  $BQ$  value and  $[BQ]$  factors, such as longtan ping level of surrounding rock for V level, to refer to the "Detailed Rules For The Design Of Highway Tunnel" (JTGD70-2010 / T) where the 6.4.2-1 draw longtan on basic physical and mechanical parameters of surrounding rock values range as shown in table 1.

**Table 1** The basic physical-mechanical parameters of Longtanping tunnel rock

Paramete	$\gamma(\text{kN/m}^3)$	$c(\text{kPa})$	$\varphi(^{\circ})$	$E(\text{GPa})$	$\mu$
<b>Value Range</b>	17~20	50~200	20~27	1~2	0.35~0.45

## 3 TUNNEL STABILITY ANALYSIS AND ABAQUS REALIZATION

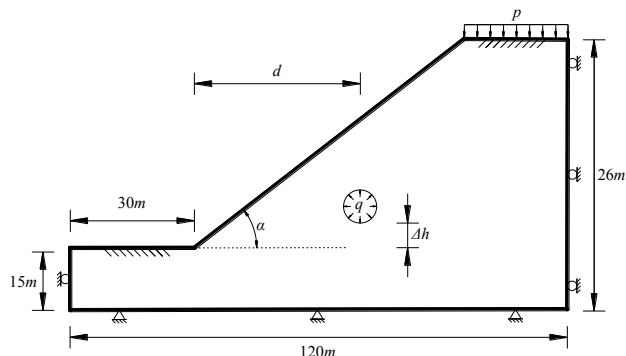
Proposed by O. C. Zienkiewicz et al.<sup>[9]</sup> in 1975, strength reduction theory has been widely applied in stability analysis of geotechnical and underground engineering. With the development of finite element and computer technology, its successful combination with numerical analysis method of elastic-plastic finite element has achieved fruitful scientific research results<sup>[10-12]</sup>.

The basic principle of strength reduction technology is to divide the shear strength indexes  $c$  and  $\tan \varphi$  of rock and soil mass by a reduction coefficient  $K$ , so as to get a new set of  $c'$  and  $\varphi'$  values as the strength parameters of rock mass into the finite element analysis of slope stability. By constantly changing the reduction coefficient  $K$  and repeatedly analyzing until the critical state is reached, the reduction coefficient  $K$  at this moment is also known as the strength reserve safety factor, which is consistent with the slope stability safety factor  $K$  of rigid body limit equilibrium method in concept, namely the slope stability safety factor  $K$ . Wherein, the material strength parameters  $c'$  and  $\varphi'$  can be obtained from equations (4) and (5) respectively:

$$c' = \frac{c}{K} \quad (4)$$

$$\varphi' = \arctan\left(\frac{\tan \varphi}{K}\right) \quad (5)$$

Wherein,  $c$  and  $\varphi$  are shear strength parameters that can be provided by rock mass;  $c'$  and  $\varphi'$  are the shear strength parameters of rock mass required to maintain slope balance.



**Fig.1** Simplified numerical model for the stability of tunnel through slope

ABAQUS software is used to establish a numerical analysis model for tunnel stability in slope sections, whose geometric elements and boundary conditions are shown in Fig 1. ABAQUS software can easily realize the process of modeling and strength reduction calculation. Please refer to "application of ABAQUS in geotechnical engineering"<sup>[13]</sup> for specific operation steps, which will not be described in this paper.

In the existing tunnel stability analysis studies, the instability failure criteria widely used mainly include the equivalent plastic zone penetration criterion, the displacement mutation criterion of feature points and the convergence criterion of calculation based on force and displacement criteria. Relatively speaking, the plastic zone penetration is more widely used as the tunnel stability analysis criterion<sup>[14-15]</sup>. However, the loose broken rock and soil mass in the slope area has a low tensile strength, and the surrounding rock of the tunnel is prone to tensile failure. In particular, once the tensile crack element appears on the surface of the tunnel, the tensile stress will transfer to the surrounding element, so that the local failure evolves into a larger vault or cave wall collapse. Therefore, this paper, based on the finite element strength reduction method, applies the principle of orthogonal test, combines the safety factor, and considers the maximum principal tensile stress around the tunnel, to analyze the sensitivity of factors influencing the tunnel stability in the slope section.

## 4 ORTHOGONAL TEST ANALYSIS OF SENSITIVITY OF FACTORS INFLUENCING TUNNEL STABILITY

A large number of engineering practice and research results showed that the stability of the slope section of tunnel of many influence factors are divided into two categories: engineering geological environment in the class of objective factors and subjective factors of human engineering action classes, specific include: the physical

and mechanical properties of surrounding rock, slope, slope Angle, groundwater conditions, tunnel position and section size, slope overloading and tunnel week surface supporting force, etc. Due to groundwater occurrence mode are extremely complex, there are many uncertainties, precise calculation of groundwater pressure size is very difficult to analyze the stability of the tunnel, mostly not considering the influence of the groundwater effect directly, but according to the lithology and the volume of groundwater, the influence of groundwater can be converted to the change of the physical and mechanical parameters of surrounding rock, the groundwater influence on adverse geological section of tunnel slope stability can be expressed by the change of the physical and mechanical parameters of surrounding rock<sup>[16]</sup>.

Therefore, this paper select several parameters: Unit Weight( $\gamma$ ), cohesion( $c$ ), Internal Friction Angle( $\varphi$ ), Poisson's Ratio( $\mu$ ) and Elastic Modulus( $E$ ), Slope Angle( $\alpha$ ), Tunnel Bottom And Slope Toe Elevation Difference( $\Delta h$ ), The Distance Between The Middle Line And The Toe Of The Tunnel( $d$ ), Tunnel Section Size( $r$ ), Tunnel Surrounding Surface Support Force( $q$ ), The Overload Value Of The Slope Top( $p$ ), the 11 kinds of factors in bad geological slope section of tunnel stability influence factors sensitivity analysis.

### 4.1 Basic Principles Of Orthogonal Experimental Design

Orthogonal test design is a method of scientific arrangement and analysis of multi-factor test by using "orthogonal table". Secondly, the level of influencing factors is selected and the factor level table is made. Then, select the appropriate orthogonal table for the orthogonal test table head design; Furthermore, the test scheme is defined, the test is carried out according to the prescribed scheme, and the test results are determined; Finally, the range analysis and variance analysis were carried out. The main advantage of orthogonal test is to obtain the test data that can best reflect the change law of objective things through the least number of tests. Through the statistical analysis of the test results, the primary and secondary order and significance level of each influencing factor can be obtained, so as to determine the best or better engineering scheme.

### 4.2 Design Of Orthogonal Experiment

According to the value range of basic physical and mechanical parameters of surrounding rock of longtanping tunnel (table 1), the selected influencing factors are divided into five levels, as shown in table 2.

**Table 2** Levels of influence factors for the stability of Longtanping tunnel

Level	$\gamma(\text{kN/m}^3)$	$c(\text{kPa})$	$\varphi(^{\circ})$	$\mu$	$E(\text{GPa})$	$\alpha(^{\circ})$	$\Delta h(\text{m})$	$d(\text{m})$	$r(\text{m})$	$q(\text{kPa})$	$p(\text{kPa})$
1	17	50	20	0.33	1	45	0	20	3	100	50
2	18	100	22.5	0.36	1.3	47.5	2	25	3.5	150	75
3	19	150	25	0.39	1.6	50	4	30	4	200	100
4	20	200	27.5	0.42	1.9	52.5	6	35	4.5	250	125
5	21	250	30	0.45	2.2	55	8	40	5	300	150

### 4.3 Orthogonal Test Results And Analysis

The test scheme does not consider the possible interaction among the factors, that is, each factor is considered to be independent of the other. According to the textbook of mathematical statistics,  $L_{50}(5^{11})$  orthogonal test table was used to arrange the numerical simulation calculation of 50 working conditions at the level of 11 factors and 5 levels. The calculation results are shown in table 3

#### 4.3.1 Range Analysis

Range analysis is a statistical analysis method to find out the factors that contribute the most by comparing the effects of various factors at different levels. When the factor level changes, the influence on the test results is different. The greater the range, the greater the influence of this factor on the test results. By comparing the horizontal effects of the same influence factor one by one, the best and worst levels can be found and the range can be calculated by subtraction. The calculation formula and calculation process are detailed in the reference<sup>[17]</sup>. The range analysis results are shown in table 4.

**Table 3** Results of orthogonal test for the stability of Longtanping tunnel

No.	$K$	$\sigma(\text{kPa})$	No.	$K$	$\sigma(\text{kPa})$	No.	$K$	$\sigma(\text{kPa})$	No.	$K$	$\sigma(\text{kPa})$
1	0.8344	34.11	14	1.656	44.98	27	1.069	137.1	40	2.147	88.84
2	1.175	51.56	15	1.808	331.1	28	1.614	163.5	41	1.007	211.5
3	1.496	66.35	16	0.8331	182	29	1.774	126.9	42	1.041	47.31
4	1.744	212.6	17	1.231	207.9	30	2.333	85.05	43	1.177	540.1
5	1.967	302.6	18	1.258	209.5	31	0.7682	264.3	44	1.511	134.2
6	0.7275	279.7	19	1.1	35.9	32	1.259	193.5	45	1.881	258.5
7	1.145	77.59	20	1.946	103.9	33	1.413	76.62	46	0.8856	246.4
8	1.575	110.8	21	0.9463	182	34	2.191	87.38	47	0.9542	214.9
9	1.938	140.2	22	0.9199	199.5	34	2.107	17.42	48	1.401	31.6
10	1.965	86.09	23	1.224	257.8	36	0.8815	222.2	49	1.685	54.19
11	0.6882	165.2	24	1.381	209	37	1.03	90.57	50	1.909	124.6
12	1.064	228.2	25	2.100	116.4	38	1.428	141.3			
13	1.591	136.4	26	0.6592	216	39	1.394	74.23			

**Table 4** Results of range analysis for the stability of longtanping tunnel

Factor	$\gamma(\text{kN/m}^3)$	$c(\text{kPa})$	$\varphi(^{\circ})$	$E(\text{GPa})$	$\mu$	$\alpha(^{\circ})$	$\Delta h(\text{m})$	$d(\text{m})$	$r(\text{m})$	$p(\text{kPa})$	$q(\text{kPa})$
$k_{1j}$	1.467	0.823	1.292	1.373	1.406	1.468	1.408	1.319	1.440	1.461	1.366
$k_{2j}$	1.509	1.089	1.293	1.419	1.431	1.450	1.406	1.415	1.424	1.347	1.425
$k_{3j}$	1.369	1.418	1.398	1.430	1.338	1.399	1.332	1.400	1.415	1.447	1.443
$K_{4j}$	1.299	1.637	1.445	1.406	1.432	1.362	1.402	1.442	1.366	1.364	1.410
$K_{5j}$	1.341	2.016	1.555	1.355	1.377	1.304	1.435	1.407	1.338	1.365	1.339
$R_j$	0.210	1.193	0.263	0.075	0.094	0.164	0.103	0.123	0.102	0.114	0.104
Range Analysis Results						Priorities: $c > \varphi > \gamma > \alpha > d > p > q > \Delta h > r > \mu > E$					
$k_{1j}$	139.577	200.341	163.683	143.815	200.429	116.652	146.349	86.145	127.969	132.039	147.162
$k_{2j}$	133.360	144.813	161.210	186.079	133.495	93.867	141.938	181.873	143.869	146.148	148.742
$k_{3j}$	152.302	173.397	151.393	144.830	160.444	195.336	143.799	113.189	139.528	133.207	141.701
$K_{4j}$	193.081	111.958	148.019	169.684	152.798	173.919	186.146	179.732	192.326	179.883	148.609
$K_{5j}$	163.639	151.450	157.654	137.551	134.793	202.185	163.727	221.020	178.267	190.682	195.745

$R_j$	59.721	88.383	15.664	48.528	66.934	108.318	44.208	134.875	64.357	58.643	54.044
Range Analysis Results						Priorities: $d > a > c > \mu > r > \gamma > p > q > E > \Delta h > \varphi$					

As can be seen from table 4, continuous penetration in the plastic zone is taken as the tunnel instability criterion, and the ranges of the influencing factors with the safety factor as the test evaluation index are in turn as follows:  $c > \varphi > \gamma > a > d > p > q > \Delta h > r > \mu > E$ . With the maximum principal tensile stress around the tunnel as the test evaluation index, the ranges of the influencing factors are in turn as follows:  $d > a > c > \mu > r > \gamma > p > q > E > \Delta h > \varphi$ . And the analysis results of  $K$  and  $\sigma$  were different to some extent, but showed good consistency on the whole. Based on the range analysis results of the two test evaluation indexes, it was concluded that the range difference of physical and mechanical parameters was greater than other factors, indicating that physical and mechanical parameters were the main control factors affecting tunnel stability in the slope section of bad geological body,  $d$  and  $a$  have a significant effect. At the same time, the tunnel supporting force of extremely small hole week, week of tunnel hole

supporting force for slope poor geological section of tunnel stability is not too big effect, namely the hole week supporting force influence on the stability of the tunnel is not obvious, this view and the second lining the security reserve engineering measures, that are not obvious effect on the stability of surrounding rock control consistent conclusion, confirmed the results in this paper, the correctness and effectiveness.

### 4.3.2 Variance Analysis

Testing scheme does not consider possible interaction effects between various factors, the factors that are independent of each other. Test reference statistics teaching material, choose  $L_{50}(5^{11})$  orthogonal test table for 50 set of conditions of numerical simulation, the calculation formula and calculation process can be found in the references<sup>[17]</sup>, the variance analysis results as shown in table 5.

**Table 5** Results of variance analysis for the stability of longtanping tunnel

Source Of Variation	$f$	K			$\sigma$		
		S	F	Significance	S	F	Significance
$\gamma$	4	0.310	0.336		22285.471	0.579	
$c$	4	8.661	9.388	**	43535.616	1.131	
$\varphi$	4	0.490	0.531		1730.616	0.045	
$E$	4	0.040	0.043		17048.393	0.443	
$\mu$	4	0.064	0.069		29593.823	0.769	
$a$	4	0.178	0.193		94094.729	2.443	(*)
$\Delta h$	4	0.059	0.064		14074.664	0.365	
$d$	4	0.086	0.093		121719.556	3.161	**
$r$	4	0.073	0.079		30188.549	0.784	
$p$	4	0.113	0.122		29631.836	0.769	
$q$	4	0.074	0.080		19687.746	0.511	
error	44	10.15	44		423591.00		

Coming From The Table:  $F_{1-0.01}(4, 44)=3.74$ ,  $F_{1-0.05}(4, 44)=2.58$ ,  $F_{1-0.1}(4, 44)=2.08$

Comprehensive evaluation index variance analysis results, two kinds of test by available: table 5  $c$  for the effect on the stability of the slope section of tunnel poor geological highly significant, the influence of  $a$ ,  $d$  significantly,  $\gamma$ ,  $p$ ,  $r$ ,  $\mu$ ,  $q$ ,  $\varphi$ ,  $E$ ,  $\Delta h$  is relatively weak, the influence of the variance analysis result tallies with the poor results.

## 5 CONCLUSION

(1) transfixion of plastic zone was regarded as the instability criterion, safety coefficient and tunnel weeks maximum principal tensile stress analysis results have certain difference, but the overall show good consistency, which showed that the maximum principal tensile stress safety coefficient and tunnel weeks for test evaluation index of the orthogonal experiment design are suitable for the adverse geological section of tunnel slope

stability influence factors sensitivity analysis;

(2) the cohesion of rock and soil mass, the horizontal distance between the middle line of the tunnel and the toe of the slope, and the inclination Angle of the slope have significant effects on the tunnel stability in the bad geological slope section, which should be paid special attention to in practical projects. The support force on the surrounding surface of the tunnel has less significant influence, which means that the second lining does not improve the stability of surrounding rock significantly as a safety reserve engineering measure;

(3) taking the safety factor alone as the test evaluation index, it unilaterally emphasizes the cohesion of rock and soil mass, and weakens the influence of other factors. It is relatively perfect to conduct tunnel engineering stability analysis with consideration to the maximum principal tensile stress around the tunnel. Therefore, it is suggested to use the safety factor and the maximum principal tensile stress around the tunnel as the

evaluation index of tunnel stability in the bad geological body slope sections, and the research results can provide reference for the design and construction of related projects.

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