

# Monitoring the deformation of the earth's surface in the zone of influence construction

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**Abstract.** The issues of ensuring the safe construction and operation of buildings and structures in a metropolis are considered. A description of the monitoring technique is given, which is to control the deformation process during new construction near existing buildings. The necessity of performing geodetic observations of deformations of the earth's surface is emphasized. A precalculation of the accuracy of determining the position of deformation grades located in the influence zone of an object under construction is given. Based on the simulation results, control zones are identified in which it is proposed to monitor the process of deformation of the soil mass at the boundary of the pit. The advantages of the integrated deformation monitoring technique over traditional observation methods are presented.

## 1 Geodetic monitoring of deformations of the object under construction

Geodetic monitoring of deformations of the object under construction is a complex of geodetic works that is carried out during the building and is intended to ensure the reliability, preserve the technical condition of the surrounding buildings, as well as preserve the environment.

The objects of geodetic monitoring are:

- Structures under construction;
- Existing buildings, structures, construction in progress, which fall into the zone of influence of construction;
- Other objects provided by the technical specifications.

Currently, in the Russian Federation, the main regulatory document that defines issues related to monitoring the deformation of buildings and structures is GOST 24846-2012. In the given document, it is recommended to use geometric leveling as the main method for observing vertical movements. The method of axial observations, individual directions, triangulation, photogrammetry, or combinations of the above methods are for observing horizontal movements.

Recommended in GOST methods provide only determining the position of control points located on the observed object. With this approach, the measurement results only indicate a negative impact [1, 2]. The fact is that the construction of facilities is associated

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with the occurrence of open pit mine - a foundation pit. Its creation leads to a redistribution of forces acting in the soil mass and, as a result, deformation of saturated soil. As a rule, existing objects can undergo deformations - cracks appear in their walls or there is a breakdown of the serviceability of individual structural elements.

Thus, a well-grounded decision is the organization and monitoring of the process of deformation of the earth's surface near the object under construction. Such observations should be supplemented by traditional geodetic methods for determining the position of control points located on the observed object. Carrying out such complex observations will allow:

- Improve the reliability of security monitoring of existing facilities;
- Highlight cases when the construction of objects affects the technical condition of existing ones, and when does not;
- Take the necessary protective measures in a timely manner.

To maintain the technical condition of existing facilities during the construction of new ones, it is necessary to determine the zone of influence of new construction, i.e. the area where deformation processes can occur due to the influence of the pit. The results of studies to determine the zone of influence of new construction are given in the publication [3]. The obtained information allows predicting the zones in which it is necessary to monitor the deformation processes occurring on the earth's surface.

When organizing observations of the deformation process in new construction, it is necessary in each case to determine potentially dangerous areas. Further, if potentially dangerous areas reach the protected object, then an observation station consisting of deformation marks is laid in it. Points of the planning and high-altitude reference geodetic grid are laid outside the zone of influence of construction and at the same time in places convenient for conducting observations on deformation marks. Strongholds are fixed with planning and high-altitude geodetic signs no later than 2 months before the start of observations.

A deformation grid is a collection of deformation marks. Surface soil grades are used to determine the displacement of the soil mass in the zone of influence of open excavation on the surrounding buildings. To make observations of horizontal movements of the earth's surface, it is proposed to use tubular grades with a screw anchor. When constructing objects under conditions of infill development, the distance between the soil marks is proposed to be taken no more than 5 m. At the same time, they should be laid at a distance of 5 m from the edge of the pit, evenly with a step of 5 m. The laying of soil marks is done on profile lines.

The number and location of profile lines is proposed to be selected depending on the ratio of the length of the pit to its width. With the square configuration of the pit  $L1 = L2$ , the laying of soil grades is proposed to be done on profile lines located perpendicular to the objects in potentially dangerous areas. With the rectangular configuration of the pit  $L1 > 2L2$ , it is proposed to lay soil grades on the profile, located along and across the strike of the pit perpendicular to the objects located in.

It is proposed to use film reflectors mounted on the walls of buildings and structures to monitor their deformations located in potentially dangerous areas [3, 4, 5, 6, 7, 8].

To justify the use of a comprehensive deformation monitoring technique, it is necessary to pre-calculate the accuracy of determining the coordinates of deformation grades.

The horizontal displacements of the deformation marks are defined as the difference between the coordinates of the marks obtained from the observations from the initial (0) and current (i) observation cycles.

The horizontal displacement of the mark is determined by the formula:

$$\delta = \sqrt{(\delta_x^2 + \delta_y^2)} = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}, \quad (1)$$

where  $\delta_x, \delta_y$  - components of horizontal displacement along the coordinate axes,  
 $x_i, y_i$  u  $x_0, y_0$  – coordinates of the mark in the  $i$ -th and zero observation cycle.

RMS error determining the horizontal displacement  $m_\delta$  is calculated by the formula:

$$m_\delta = m_{\delta_x}^2 + m_{\delta_y}^2, \quad (2)$$

where  $m_{\delta_x}, m_{\delta_y}$  – RMS error determining the components of horizontal displacement along the coordinate axes.

$$\begin{aligned} m_{\delta_x}^2 &= m_{x_0}^2 + m_{x_i}^2 = 2m_x^2 \\ m_{\delta_y}^2 &= m_{y_0}^2 + m_{y_i}^2 = 2m_y^2. \end{aligned} \quad (3)$$

Hence

$$m_\delta^2 = 2m_x^2 + 2m_y^2. \quad (4)$$

If  $m_x = m_y$ , then

$$m_\delta^2 = 4m_x^2. \quad (5)$$

In consequence

$$m_\delta = 2m_x = 2m_y. \quad (6)$$

While maintaining the observation pattern of RMS error, the determination of the position of the deformation mark in two observation cycles can be considered the same, then:

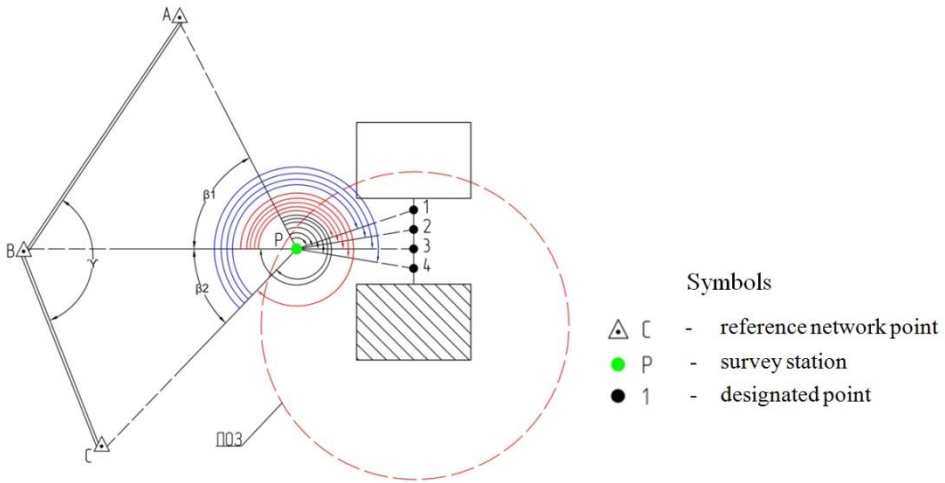
$$m_\delta = 2m_{x,y}. \quad (7)$$

According to Russian State Instruction RD 07-166-97 the limit value of horizontal displacements of deformation marks can be set using the observation methodology adopted for the protection of buildings and structures during the operation of mining facilities (quarries, mines). According to research by All-Russian Research Institute of Mining Geomechanics and Survey, one of the indicators characterizing the dangerous effect on existing objects is the horizontal extension of the earth's surface. The critical value of the indicator is  $\varepsilon=0,5 \cdot 10^{-3}$ . Then, by analogy with the assessment of the influence of underground mining, the critical horizontal deformations of the earth's surface at a distance of 5 m between the deformation marks will be equal to 2.5 mm, and the maximum horizontal displacement of the deformation marks will be  $\Delta=1.25$  mm. According to Building regulations 27751-2014 the values determined by the rule  $2m$  and  $3m$  are taken as the limiting error  $\Delta$ . With these values of RMS error, the reliability of measurements will be 0.954 and 0.997, respectively.

Then, taking the reliability of measurements  $2m$  (95%), RMS error determining the position of the deformation points will be:

$$m = \frac{\Delta}{2} = \frac{1.25}{2} \approx 0.6 \text{ mm.} \quad (8)$$

In conditions of infill development, visibility may be limited, i.e. it will not always be possible to take measurements directly from the points of the basic network, in which case it is proposed to perform observations from auxiliary points (survey stations). The position of the survey station is determined by the method of "resection". In this approach, an electronic tachymeter is used, which is installed in a convenient place for measuring, and its position is determined by the backsight from the points of the reference planned network (Fig. 1). The horizontal deformations of the earth's surface are determined by the difference in the distances between the deformation marks obtained from two observation cycles by the polar method.



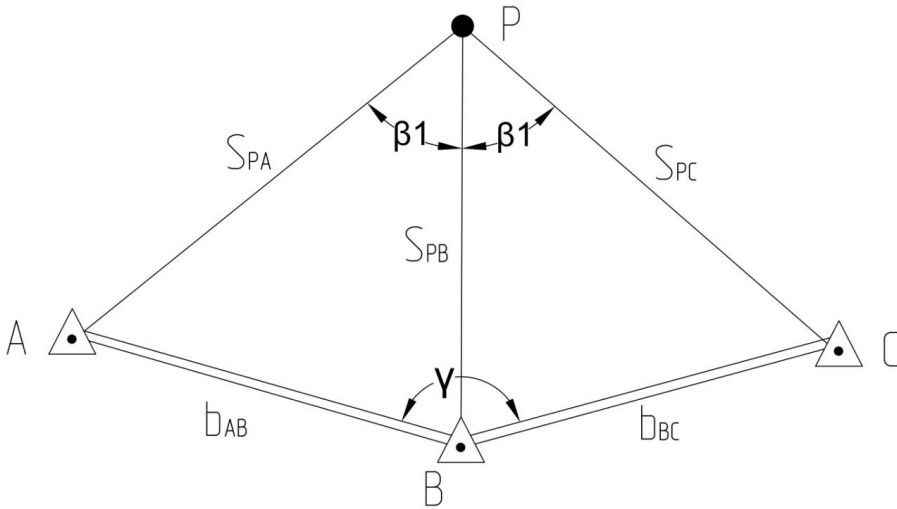
**Fig. 1.** Earth surface deformation observation scheme .

Linear-angle measurements will be performed with the following accuracy characteristics: distance measurement error by electronic tachymeter  $m_s = 2\text{mm} \pm 2 \cdot 10^{-6} \text{mm/km} \cdot L$ , where L – side length in km; RMS error horizontal angle measurement is  $m_\beta = 2''$ .

The error in determining the coordinates of the deformation point  $m_{x,y}$  is calculated by the formula:

$$m_{x,y}^2 = m_{back}^2 + m_{pol}^2, \quad (9)$$

where  $m_{back}$  – the error of determining station coordinates by backsight;  $m_{pol}$  - the error in determining the coordinates of deformation points by the polar method.



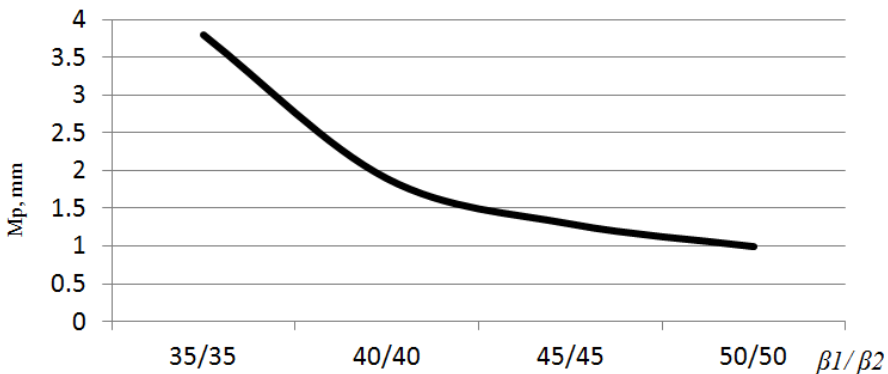
**Fig. 2.** Backsight scheme.

RMS error determining the position backsight is calculated by the formula:

$$m_{back} = \frac{m_{\rho} \cdot S_{PB}}{\rho \sin(\beta_1 + \beta_2 + \gamma)} \sqrt{\left(\frac{S_{PA}}{b_{AB}}\right)^2 + \left(\frac{S_{PC}}{b_{BC}}\right)^2}, \quad (10)$$

where  $\gamma$  - angle between the original sides;  $\beta_1, \beta_2$  - point angles;  $b_1, b_2$  – original sides;  $\rho = 206265''$ .

Since the observations are proposed to be carried out in urban areas, we assume that the distance from the starting points to the survey station will be no more than 100 m. We take the angle between the original sides  $\gamma$  equal to  $120^\circ$ . The dependence of RMS error determining the position of the survey station by backsight for a different combination of the angles  $\beta_1$  and  $\beta_2$  at the determined point P is shown in Fig. 3.



**Fig. 3.** The dependence of RMS error determining the position of the survey station by backsight for a different combination of the angles  $\beta_1$  and  $\beta_2$  at the determined point P.

RMS error determining the position of the deformation point by the polar method is calculated by the formula:

$$m_1 = \sqrt{m_s^2 + S^2 \left(\frac{m_\beta}{\rho}\right)^2}, \quad (11)$$

where  $S$  – measured distance from the survey station to the deformation point.

According to formula (11), the error in determining the position of the deformation point depends on the magnitude of the measured distance and the errors in measuring the distance and horizontal angle by an electronic tachymeter.

According to the dependence presented in Figure 3 and the calculation by formula (11), the observation of the proposed methods using an electronic tachymeter an accuracy of measuring distances  $m_s = 2\text{mm} \pm 2 \cdot 10^{-6}\text{mm/km} \cdot L$ , horizontal angle  $m_\beta = 2''$  does not provide accuracy in determining the position of deformation points of 0.6 mm in plan. Therefore, it is necessary to develop such an observation technique that will improve the accuracy of measurements. In this regard, a precalculation of the accuracy of linear-angular measurements with redundant measurements in the network and its subsequent adjustment by a strict method was performed.

## 2 Modeling the error in determining the position of deformation marks

Evaluation of the accuracy of points can be performed by a rigorous method. We assume that the guarded building is located in the potentially dangerous area under construction. Since the observations are proposed to be carried out in the conditions of sealing development, in accordance with RD 07-166-97, the distance between the long sides of the existing building and the object under construction should be taken at least 20 m. Therefore, the minimum length of the profile line will be 20 m. Figure 1 shows the scheme of fixing deformation marks on the profile line of the observation station located between the object under construction and the guarded building.

To find the expected RMS error to determine the coordinates of the deformation marks, we perform a precalculation of linear-angular measurements. To do this, a planned geodetic grid was constructed, which was equalized in a parametric way using the LS method. The grid measured the distances and directions from the survey station  $P$  to all other points of it including directions to the points of the basic network and deformation points - a total of 25 values, of which 10 are necessary and 15 are redundant.

In the parametric method, some parameters are accepted as equalized quantities that most fully reflect the characteristics of a physical object or phenomenon. In this case, the adjusted parameters are the coordinates of the defined points -  $x_i, y_i$ .

Let  $n$  measured values of  $u$  be given, of which  $t$  values are necessary for a one-time obtaining of the desired parameters. The adjustment problem arises under condition  $n > t$ . The difference  $r = n - t$  is equal to the number of redundant measurements.

The equation of connection between the measured elements  $u$  and the coordinates of the points  $x, y$  in general can be written:

$$u = f(x_1, y_1, x_2, y_2, \dots). \quad (12)$$

The nonlinear model represented by the equations of constraints is linearized to simplify it — the equations of constraints are replaced by linear equations. The linear dependence of one quantity on another means the proportionality of their increments. In linear equations, all variables are involved in the first degree, and solving systems of linear equations is much simpler than nonlinear.

We set the approximate coordinates of  $x_1^0, y_1^0, x_2^0, y_2^0, \dots$  points and take the right side of the constraint equation (12) in a Taylor series expansion:

$$u = u^0 + \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial y_1} dy_1 + \frac{\partial f}{\partial x_2} dx_2 + \frac{\partial f}{\partial y_2} dy_2 + \dots \quad (13)$$

As a result, we obtain a linear equation, which we rewrite, denoting the partial derivatives by the letters  $a, b, \dots$ . We pass from the differentials  $dx, dy$  to the corrections  $\delta x, \delta y$ .

$$a\delta x_1 + b\delta y_1 + \dots + (u^0 - u) = 0, \quad (14)$$

where  $u = u^{meas} + \delta u$ ,  $u^{meas}$  – measured element value  $u$ ,  $\delta u$  – correction to the measurement result.

The equation of corrections will have the following form:

$$a\delta x_1 + b\delta y_1 + \dots + (u^0 - u^{meas}) = \delta u. \quad (15)$$

Rewrite it

$$a\delta x_1 + b\delta y_1 + \dots + l = v. \quad (16)$$

The presence of  $n$  measured elements in the network leads to a system of  $n$  correction equations:

$$\left. \begin{aligned} a_1\delta x_1 + b_1\delta y_1 + \dots + l_1 &= v_1 \\ a_2\delta x_1 + b_2\delta y_1 + \dots + l_2 &= v_2 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ a_n\delta x_1 + b_n\delta y_1 + \dots + l_n &= v_n \end{aligned} \right\} \quad (17)$$

The system of equations (17) is a linear mathematical model of a geodesic network.

To obtain estimates with minimal variances, system (17) is solved by LS method. That is, they are looking for a solution that provides a minimum of the sum of the squared corrections:  $[v^2] = \min$ .

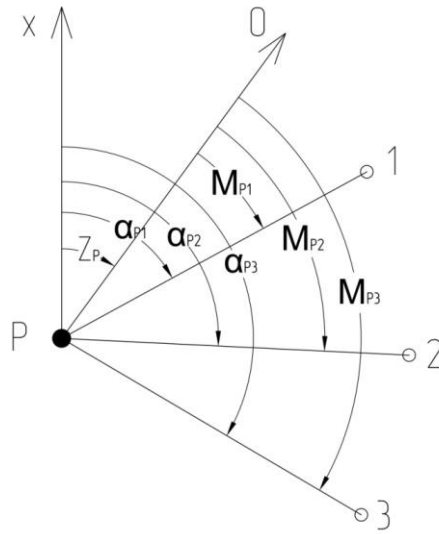
In unequal measurements of the weight  $p_i$ , the equations are not the same, and the sum function is  $[pv^2] = \min$ .

The derivatives  $a_1 = \frac{\partial f}{\partial x_1}, b_1 = \frac{\partial f}{\partial y_1}, \dots$  can be calculated numerically, but more often they are determined by well-known formulas.

### 3 Correction equation for measured horizontal directions

Using these equations, a relationship is established between the corrections  $v_i$  in the measured directions and the corrections  $\delta_{xi}$  and  $\delta_{yi}$  to the preliminary values of the adjusted parameters — the coordinates of the points being determined. Here  $i = 1, n$ , and  $j = 1, t$ , where  $n$  is the number of quantities measured in the network;  $t$  - equalized parameters.

Suppose that at point P (Fig. 4) a group of directions to points 1, 2, 3 is measured. In this case, the zero direction takes the position P-0.



**Fig. 4.** Scheme of directions measured at the point where  $M'_{p1}, M'_{p2}, M'_{p3}$  - measured directions;  $M_{p1}, M_{p2}, M_{p3}$  - equalized values of the measured directions;  $\alpha_{p1}^0, \alpha_{p2}^0, \alpha_{p3}^0$  - preliminary values of the directional angles of the sides on which the directions are measured;  $\alpha_{p1}, \alpha_{p2}, \alpha_{p3}$  - the equalized values of these directional angles;  $Z_p^0$  и  $Z_p$  - preliminary and equalized directional angle value of zero direction.

The directional angle of the zero direction  $Z_p$  is usually called the reference angle, and the correction to this angle, obtained from the equalization, - the reference correction.

We obtain the following equalities:

$$\begin{aligned} \alpha_{p1} - Z_p &= M_{p1}, \\ \alpha_{p2} - Z_p &= M_{p2}, \\ \alpha_{p3} - Z_p &= M_{p3}, \end{aligned} \tag{18}$$

which are valid for the equalized values included in them quantities. Instead of the equalized values, we substitute the following expressions into equality (18):

$$\begin{aligned} \alpha_{p1} &= \alpha_{p1}^0 + \delta\alpha_{p1}, \\ Z_p &= Z_p^0 + \delta z_p, \\ M_{p1} &= M'_{p1} + v_{p1}, \end{aligned} \tag{19}$$

where  $\delta\alpha_{p1}, \delta z_p, v_{p1}$  - corrections to preliminary and measured values of the corresponding values obtained after equalization.

For the first equation (18) we obtain:

$$\alpha_{p1}^0 + \delta\alpha_{p1} - Z_p^0 - \delta z_p = M'_{p1} + v_{p1} \tag{20}$$

After regrouping:



$$-\delta z_p + \delta \alpha_{p1} + (\alpha_{p1}^0 - M'_{p1} - Z_p^0) = v_{p1} \quad (21)$$

Values enclosed in round brackets are known before equalization. Therefore, they can be considered free terms of the obtained equations, denoting

$$l_{p1} = \alpha_{p1}^0 - Z_p^0 - M'_{p1} \quad (22)$$

Similarly, we can transform other equalities from system (18). As a result, we obtain the following system of correction equations corresponding to the group of directions measured at step p:

$$\begin{aligned} -\delta z_p + \delta \alpha_{p1} + l_{p1} &= v_{p1}; \\ -\delta z_p + \delta \alpha_{p2} + l_{p2} &= v_{p2}; \\ -\delta z_p + \delta \alpha_{p3} + l_{p3} &= v_{p3}. \end{aligned} \quad (23)$$

If we take directional angles in the network as equalized parameters, then equations (23) will be the final expression of the parametric corrections equations. However, if the distances between points are measured in the network, then the expressions for corrections to these distances through corrections to the directional angles will have a complex form. When adjusting planned networks, it is more convenient to take the coordinates of the defined points as equalized parameters and find, based on the results of the adjustment, corrections to the preliminary coordinates of these points, the values of which can be calculated before adjustment using the measured values.

To go from the corrections  $\delta \alpha$  of equations (23) to the corrections to the coordinates  $\delta x$  and  $\delta y$ , we find the total differential of the function:

$$\alpha_{p1} = \arctg \frac{y_1 - y_p}{x_1 - x_p} = \arctg \frac{\Delta y}{\Delta x}, \quad (24)$$

then

$$\delta \alpha_{p1} = \rho \frac{\sin \alpha_{p1}}{s_{p1}} \delta x_p - \rho \frac{\cos \alpha_{p1}}{s_{p1}} \delta y_p - \rho \frac{\sin \alpha_{p1}}{s_{p1}} \delta x_1 + \rho \frac{\cos \alpha_{p1}}{s_{p1}} \delta y_1. \quad (25)$$

We make a replacement in this expression:

$$a_{p1} = \rho \frac{\sin \alpha_{p1}}{s_{p1}}; \quad b_{p1} = -\rho \frac{\cos \alpha_{p1}}{s_{p1}}. \quad (26)$$

We substitute the obtained expressions in the first equation of system (23). As a result, we find the parametric equation of corrections for the direction measured between points p and 1. After similar transformations of the remaining equations (23), we obtain a system of parametric equations for the corrections of the group of directions measured at point i:

$$\begin{aligned} -\delta z_p + a_{p1} \delta x_p + b_{p1} \delta y_p - a_{p1} \delta x_1 - b_{p1} \delta y_1 + l_{p1} &= v_{p1}; \\ -\delta z_p + a_{p2} \delta x_p + b_{p2} \delta y_p - a_{p2} \delta x_2 - b_{p2} \delta y_2 + l_{p2} &= v_{p2}; \\ -\delta z_p + a_{p3} \delta x_p + b_{p3} \delta y_p - a_{p3} \delta x_3 - b_{p3} \delta y_3 + l_{p3} &= v_{p3}. \end{aligned} \quad (27)$$

Note that each group of jointly measured directions has its own specific correction  $\delta z$  (unknown to the coordinate corrections). To exclude the approximate correction  $\delta z$  from

the system of parametric equations of corrections (27), it is sufficient to compose an equation of the form:

$$[a]\delta x_p + [b]\delta y_p - a_{p,1}\delta x_1 - b_{p,1}\delta y_1 - a_{p,2}\delta x_2 - b_{p,2}\delta y_2 - a_{p,3}\delta x_3 - b_{p,3}\delta y_3 + [l] = [v]$$

with weight  $p/m$ . (28)

where  $[a] = a_{p,1} + a_{p,2} + a_{p,3}$ . The sums  $[b]$ ,  $[l]$  are calculated similarly;  $m$  – the number of directions measured in a group.

## 4 Correction equations for measured distances

To obtain the parametric equations, we establish the relationship between the corrections to the preliminary values of the coordinates of the end points of the side which length is measured and the correction to the measured value of this length. For this, we introduce the notation:  $S'_{ij}$  - measured length;  $S_{ij}$  - equal value of this length,  $S^0_{ij}$  – length value calculated from the preliminary coordinates of the end points.

The equalities are written:

$$S_{p1} = S^0_{p1} + \delta S_{p1},$$

$$S_{p1} = S'_{p1} + v_{S,p1},$$
(29)

where  $\delta S_{p1}$  – correction to preliminary length value  $S^0_{p1}$ , found from adjustment,  $v_{S,p1}$  – similar correction to measured value.

Subtracting the second from the first equality, we obtain:

$$\delta S_{p1} + (S^0_{p1} - S'_{p1}) = v_{S,p1}. \tag{30}$$

The expression in round brackets is the free term of the resulting corrections equation. Let it by the letter  $l_{S,p1}$ .

To express the correction  $\delta S_{p1}$  in terms of corrections to the preliminary values of the coordinates of the end points, we obtain the total differential of the function:

$$S^2_{p1} = (X_1 - X_p)^2 + (Y_2 - Y_p)^2 = \Delta x^2 + \Delta y^2, \tag{31}$$

Let

$$a_{S,p1} = -\frac{\Delta x}{S_{p1}} = -\cos\alpha_{p1}; \quad b_{S,p1} = -\frac{\Delta y}{S_{p1}} = -\sin\alpha_{p1}. \tag{32}$$

Then equation (30), taking into account expressions (31) and (32), is transformed into a parametric equation of corrections for the distance measured between points p and 1:

$$a_{S,p1}\delta x_p + b_{S,p1}\delta y_p - a_{S,p1}\delta x_1 - b_{S,p1}\delta y_1 + l_{S,p1} = v_{S,p1}. \tag{33}$$

If one of the points between which the distance is measured is the source, then the corrections  $\delta x$  and  $\delta y$  corresponding to it are equal to zero.

We write the system of parametric equations of corrections (3.17) in matrix form:

$$V=AT+L, \tag{34}$$

where  $A$  – coefficient matrix of the parametric corrections equations  $a_i, b_i$  ( $i = 1, 2, \dots, n$ ),  $L$  – free term vector  $l_i$ ,  $V$  – correction vector  $v_i$ ,  $T$  – correction vector to the preliminary value of the parameters.

To determine the vector of free terms from preliminary values of the coordinates of points, we calculate the network elements (horizontal directions and distances) and find their differences with directly measured values.

The matrix  $N$  of the coefficients of the normal equations is:

$$N = A^T P A \tag{35}$$

To find the correction vector in the preliminary value of the parameters, it suffices to multiply the matrix equation by the left by the inverse matrix  $N^{-1}$ :

$$T = -N^{-1} A^T P L, \tag{36}$$

Substituting the vector  $T$  into equation (3.4.25), we obtain the correction vector  $V$  to the measured values:

$$V = AT + L \tag{37}$$

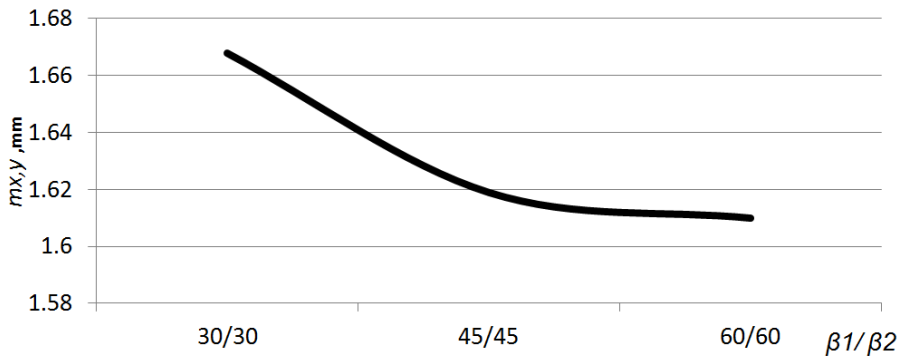
The posterior mean square error of a unit of weight  $\mu$  is calculated by the formula:

$$\mu = \sqrt{\frac{|V^T P V|}{n-k}}, \tag{38}$$

where  $k$  – number of parameters (dimension of vector  $T$ ).

The accuracy precalculation of determining the elements of the planned network is performed using the software package LabVIEW.

According to the accuracy precalculating results of the RMS error, the determination of the planned position of the deformation grades was 1.67 mm at backsight angles  $\beta_1/\beta_2 = 30^\circ$ . Similarly, RMS errors were obtained for determining the planned position of deformation grades at the backsight angles  $\beta_1/\beta_2$  equal to  $45^\circ$  and  $60^\circ$ , which amounted to 1.62 mm and 1.61 mm, respectively. The dependence of RMS error determine the planned position of the deformation grades for various angles  $\beta_1$  and  $\beta_2$  at the survey station P is shown in Fig. 5.

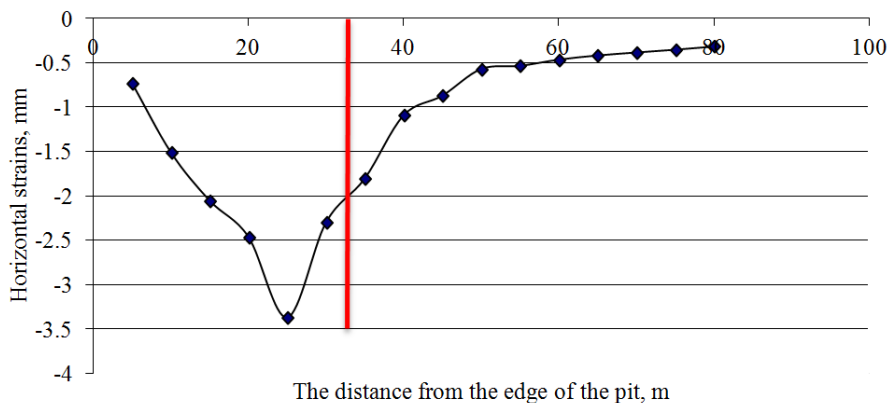


**Fig. 5.** The dependence of RMS error determine the planned position of the deformation grades for various angles  $\beta_1$  and  $\beta_2$  at the survey station P.

According to the dependence presented in Figure 5, the use of an electronic tachymeter with the above accuracy characteristics and the observation of the proposed method does not allow the accuracy of determining the position of deformation points of 0.6 mm in plan.

Generally, the necessary accuracy can be achieved using expensive equipment, such as a laser tracker, or by complicating the network circuit and increasing the number of redundant measurements. In turn, this can complicate the observation process. Therefore, it is necessary to consider in detail the process of deformation of the earth's surface and establish the displacements of the earth's surface that occur at the boundary of the pit and the displacements of the earth's surface, which cause critical horizontal deformations at the boundary of potentially dangerous areas. In order to determine how the process of deformation of the earth's surface actually occurs, we perform mathematical modeling of the deformation process in the Plaxis 3D program.

Based on the simulation results, a graph of the distribution of horizontal strains in the potentially dangerous area is obtained. Figure 6 shows the graphs of the distribution of horizontal strains in the potentially dangerous area for a foundation pit with geometric parameters:  $L1 = 50$  m,  $L2 = 50$  m,  $h = 5$  m, mechanical properties of rocks in the model: the elastic modulus is 2 MPa, the Poisson's ratio is 0.3, the adhesion is 1 kPa, the angle of internal friction is  $30^\circ$ , the specific gravity for the model was 1.7 t / m.



**Fig. 6.** The distribution of horizontal strains in the potentially dangerous area.

As shown by studies of the deformation process, deformations in the case of the formation of potentially dangerous areas can vary in a wide range of values: from critical values at the boundary of potentially dangerous areas ( $0,5 \cdot 10^{-3}$ ) to tens of mm at the boundary of the pit. Critical deformations at the boundary of the potentially dangerous area are caused by a significant displacement of the earth's surface near the boundary of the pit due to inelastic deformation of the soil mass. Consequently, it is not necessary to determine critical deformations by high-precision methods, but it is necessary to monitor the process of deformation of the soil mass at the boundary of the pit. Therefore, a specific section (control zone) is identified within the potentially dangerous area, in which horizontal deformations exceeding  $2 \cdot 10^{-3}$  arise.

Having allocated a control zone within the potentially dangerous area, observations can be performed with accuracy corresponding to the process of deformation of the earth's surface at the boundary of the pit. Then, using the indicator  $2 \cdot 10^{-3}$  as a critical value to determine the horizontal deformations of the earth's surface at distances of 5 m between the deformation marks, the limiting value of the horizontal displacements of the deformation marks will be  $\Delta = 5$  mm. According to the formula (7), assuming the reliability of the

measurements is 2m (95%), the RMS error for determining the position of the deformation marks will be  $m = \frac{\Delta}{2} = 2.5$  mm.

The results of mathematical modeling of the error in determining the position of deformation marks showed that the developed observation technique allows to achieve the required accuracy, but observations can only be performed in certain areas where deformations exceeding  $2 \cdot 10^{-3}$  - control zones.

It should be noted that the proposed methodology for complex strain monitoring has advantages over traditional methods of observation. Since the technique allows, in addition to monitoring horizontal deformations occurring along the profile line, to control the deviations of the deformation marks from the alignment by changing the coordinates of these marks obtained from different observation cycles.

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