Calculation of the transformation plasticity strain in the shape memory cylinder

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Abstract. A connected thermomechanical boundary problem for an infinite shape memory alloy cylinder loaded by an axial force and subjected to heating or cooling from the surface is solved. The evolution of stress-strain state is calculated for a process of the transformation plasticity. The mechanical properties of a material point are given by a micromechanical model accounting for the deformations due to elasticity, thermal expansion and phase transformation. The obtained problem was solved numerically using the iterative procedure with a variable iterative parameter. The influence of the surface temperature rate and radius of cylinder on the transformation plasticity is investigated. It is shown, that the elongation due to the transformation plasticity effect decreases with the increasing temperature rate. The elongation also decreases with the increasing cylinder diameter. This phenomenon can be explained by an inhomogeneity of the temperature and stress fields causing different conditions for the phase transformation in different points of the body. Stress in the local region can overtop more than twice the mean value.

1 Introduction

Shape memory materials are functional materials that can undergo large reversible strains with a change in temperature. Due to their physical and mechanical properties, shape memory alloys (SMA) find application in various branches of technology and medicine. They can be used as heat-sensitive and control elements in different applications, for example, in thermomechanical couplings, presses and actuators [1-7]. To effectively design elements from SMA, it is not enough to know only the properties of a material but also to take into account the effect of sizes and shape of an object on the deformation effects. Therefore, it is necessary to develop an effective method for calculating the stress–strain state in SMA objects.

The martensitic phase transformation, which underlies functional behavior of shape memory materials, is thermoelastic and has a latent heat, so that it depends on by both the temperature and stresses arising in an object from such a material. The change in temperature can cause phase composition change, which in turn leads to stress and temperature changes. Temperature significantly affects the modulus of elasticity, yield

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strength, internal friction, and other mechanical and physical properties. The problem on determining stresses and strains in shape memory bodies should be generally solved jointly with the heat conduction problem. Thus, we have completely connected problem including the constitutive relations and the equilibrium and thermal conductivity equations with the corresponding boundary and initial conditions.

Some works was devoted to the solution of boundary-value problems for SMA bodies, where the mechanical and thermal conductivity problems were solved in different character of connectivity, and the properties of SMA were described in terms of the macroscopic or microstructural models. In the papers [8, 9] authors used macroscopic phenomenological model for constitutive equations to describe the strain accumulation on cooling and its recovery on heating. The solution of boundary problem for SMA hollow cylinder and sphere, which was obtained in [9] was not taken into account nonuniform radial distributions of temperature and volume fraction of martensite.

The heterogeneity of the temperature and martensitic phase fields can influence on the inelastic phase deformation and on the size of transformation plasticity effect or shape memory effect.

The problems of beam bending and pure torsion of shape memory alloy circular bars were solved in [10 – 13].

2 Formulation of the connected thermomechanical boundary-value problem for SMA cylinder

In this study, we solved a connected thermomechanical axisymmetric problem for an infinite circular SMA cylinder with radius b, loaded by an external axial force F_z and cooled from the surface. The forces on the lateral surface are zero. The effect of stress on the martensitic transformation is taken into account. The cylindrical coordinates r, θ, z are used, were axis z is the axis of the cylinder. The stress and strain tensors depend only on radius r. Displacement vector components are $u_r = u_r(r)$, $u_\theta = 0$, $u_z = \text{const} \cdot z$. Consequently diagonal components of strain tensor are $\varepsilon_r = du_r/dr$, $\varepsilon_\theta = u_r/r$, $\varepsilon_z = \text{const}$, other components are zero. Total strain consists of the elastic (ε^e) and inelastic (e) strains (Eq. 1). Inelastic strain in turn consists of the phase strain (e^Φ) and the thermal expansion strain (e^{Te}):

$$\varepsilon = \varepsilon^e + e, \quad e = e^\Phi + e^{Te}$$

(1)

The thermal expansion strain is calculated by formula (2)

$$e_r^{Te} = e_\theta^{Te} = e_z^{Te} = \alpha\left(T - T_0\right),$$

(2)

where $e_r^{Te}, e_\theta^{Te}, e_z^{Te}$ – diagonal components of the strain tensor e^{Te}, α – heat expansion coefficient, T_0 – initial temperature, T – current temperature.

The mechanical equilibrium equation with the boundary conditions is following:

$$\frac{d\sigma_r}{dr} + \frac{1}{r}\left(\sigma_r - \sigma_\theta\right) = 0,$$

(3)

$$\sigma_r\big|_{r=b} = 0, \quad 2\pi \int_0^b \sigma_z \, dr = F_z, \quad \varepsilon_z = \text{const}$$

(4)

where σ_r, σ_θ, σ_z – diagonal components of the stress tensor.

Hook's law for the elastic strains:

$$\sigma = \frac{E}{1+\nu}(\varepsilon - e) + \frac{\nu E}{(1+\nu)(1-2\nu)} \vartheta I, \quad \vartheta = \varepsilon_r + \varepsilon_\theta + \varepsilon_z - e_r - e_\theta - e_z$$

(5)

In Eq. 5 E is Young's modulus, ν is Poisson's ratio, I – unit tensor.

When solving the heat conduction equation (6) we take into account the transformation latent heat release (W).

$$c\rho \frac{\partial T}{\partial t} = \lambda \nabla^2 T + W, \quad W = \rho q_0 \dot{\Phi}_M$$

(6)

In Eq. 6 T is the temperature, c is the specific heat capacity, ρ is density, λ is the heat conductivity W is the volume power of the heat sources, q_0 is the specific latent heat of the transformation, Φ_M – volume fraction of martensite. Dot indicates derivatives with time.

The initial and boundary conditions for heat conduction equation are:

$$T\big|_{r=b} = T_{\text{amb}}, \quad T\big|_{t=0} = T_0(r)$$

(7)

where T_{amb} – ambient temperature, T_0 – initial temperature.

The constitutive relations for the medium, describing the evolution of the volume fraction of martensite and phase deformation, are given by the microstructural model [14], which can be schematically written as

$$\dot{e} = F_1\left(T, \dot{T}, \sigma, \dot{\sigma}, X\right)$$

(8)

$$\dot{X} = F_2\left(T, \dot{T}, \sigma, \dot{\sigma}, X\right)$$

(9)

$$X = \left(\Phi_1(1), ..., \Phi_N(1), ..., \Phi_1(N_{Gr}), ... \Phi_N(N_{Gr})\right)$$

(10)

$$\Phi_M = \frac{1}{N_{Gr}} \sum_{\omega=1}^{N_{Gr}} \frac{1}{N} \sum_{i=1}^{N} \Phi_i(\omega)$$

(11)

where X are the internal parameters, indicating the volume fraction $\Phi_i(\omega)$ of Bain's martensite variants in each of 1, …, ω, …, N_{gr} grains, F_1, F_2 – functions given by the microstructural model [14].

The obtained twice connected thermomechanical problem was solved numerically using the iterative procedure with a variable iterative parameter. The problem was divided into three subproblems: determination of the stresses and strains on the assumption that the inelastic strain is known, calculation of the temperature field with a known heat source, and

obtainment of the inelastic strains and heat release at known stresses and change in temperature.

Since the temperature distribution affects the inelastic strains and stresses, and the stress, in turn, affects the martensitic transformation and, therefore, the field of heat sources and the change in the temperature field. So that, the mechanical and heat problems cannot be solved independently. To solve the heat-conduction equation, we used the finite-difference implicit scheme, and an analytical solution was found for the mechanical problem. The connected problem was solved stepwise, with increment in time, axial forces and ambient temperature at each step.

In previously work [15] it was shown that the problem in such formulation can be successfully solved, but the Newton condition for the heat exchange with a medium for heat – conduction equation was used.

3 Results of numerical experiment

The calculation was carried out for material with the following characteristics: Young's modulus $E = 78$ GPa; Poisson's ratio $\nu = 0.33$, specific heat $c = 4.7 \cdot 10^2$ J·kg^{-1}·K^{-1}, heat conductivity $\lambda = 10$ W·m^{-1}·K^{-1}, density $\rho = 6.5 \cdot 10^3$ kg·m^{-3}; specific latent heat $q_0 = -150$ J·g^{-1}; transformation temperatures $M_s = 315$ K, $M_f = 300$ K, $A_s = 350$ K, $A_f = 365$ K; heat expansion coefficients for martensite and austenite $\alpha_A = 14 \cdot 10^{-6}$ K^{-1}, $\alpha_M = 6 \cdot 10^{-6}$ K^{-1}. Number of grains in the representative volume is 100. These characteristics correspond to TiNi.

We modeled loading of cylinder by an axial force corresponding to the initial stress $\sigma_z = 100$ MPa at a temperature of 350 K (austenitic state), subsequent cooling through the range of direct martensitic transformation. Upon cooling, we set the change in the surface temperature from 350 K to 290 K with four values of rate (0.01 K s^{-1}, 1 K s^{-1}, 10 K s^{-1}, 100 K s^{-1}) and exposure at 290 K until the temperature became homogeneous. The evolution of stress-strain state was calculated for a process of the transformation plasticity. The influence of the surface temperature rate and radius of SMA cylinder on the transformation plasticity is investigated.

The calculations performed indicate that, even at the low cooling rate or at the small radius, the temperature distribution over the cylinder radius is nonuniform. For example, for surface temperature rate - 1 K s^{-1} the maximum difference in the temperatures in the bulk of the cylinder and on its surface is 6 K for cylinder with a radius of 5 mm and 60 K for cylinder with a radius of 50 mm. The larger the radius of the cylinder, the larger time needed for temperature equalization.

The table 1 contains the maximum difference in the temperatures in the bulk of the cylinder and on its surface for cylinder with radius 10 mm for four surface temperature rates.

Table 1. The maximum difference in the temperatures in the bulk of the cylinder and on its surface for cylinder with radius 10 mm.

- dT_b/dz, [K/s]	0.01	1	10	100
max| $T(0) - T(b)$|, [K]	0	18	55	60

Figure 1 show the radial distribution of temperature in cylinder with a radius 10 mm at surface temperature rate - 10 K s^{-1}. The curves were built for several consecutive points of time. In this case, maximum difference between internal and surface temperatures is 55 K.
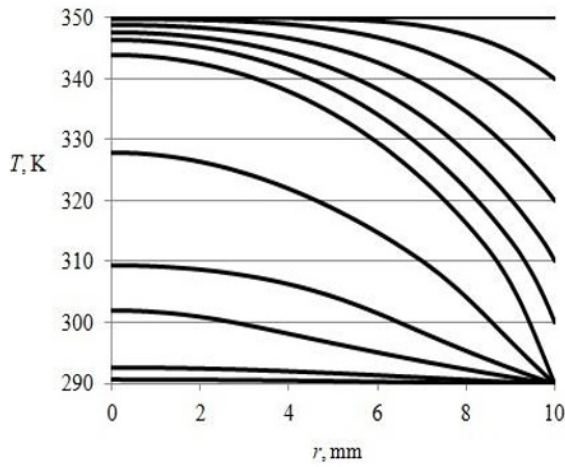
Fig. 1. Radial distribution of temperature in cylinder with a radius 10 mm at surface temperature rate - 10 K s^{-1}

Nonuniform temperature distribution causes inhomogeneity of the phase composition fields and stress (Fig. 2, 3).
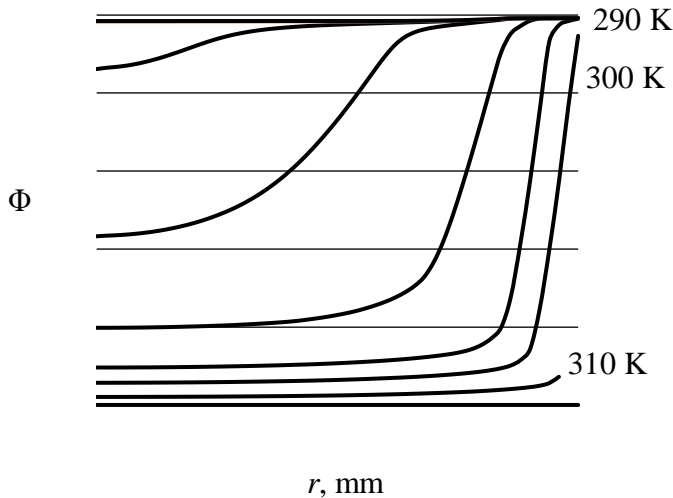

r, mm

Fig. 2. Radial distribution of volume fraction of martensite in cylinder with a radius 10 mm at surface temperature rate - 10 K s^{-1}. Numbers near the curves indicate the values of the surface temperature

The normal axial stress in the outer layers of cylinder can overtop more than twice the initial stress $\sigma_z = 100$ MPa. The stress changes the phase transformation temperatures. A direct phase transformation begins at the temperature 340 K. After the start of direct martensitic transformation axial stress in subsurface layers decreases, but the stress inside of the cylinder begins to grow. After the finish of martensitic transformation in subsurface layers axial stress is rising again.

If the radial distribution of temperature is uniform, that the radial distribution of volume fraction of martensite is uniform too, and axial stress equals approximately initial stress $\sigma_z = 100$ MPa (Fig. 4).

Because of the temperature and stress inhomogeneity the strain accumulated on cooling decreases with increasing surface temperature rate. Analogous, it can be seen that, the larger the radius, the smaller the summary relative elongation of the cylinder. Figure 5 shows accumulation of axial strain in cylinders with different radii during the realization of the transformation plasticity effect. Dependence of the transformation plasticity deformation on the cylinder radius and surface temperature rate on a cooling are shown on the Fig. 6, 7.
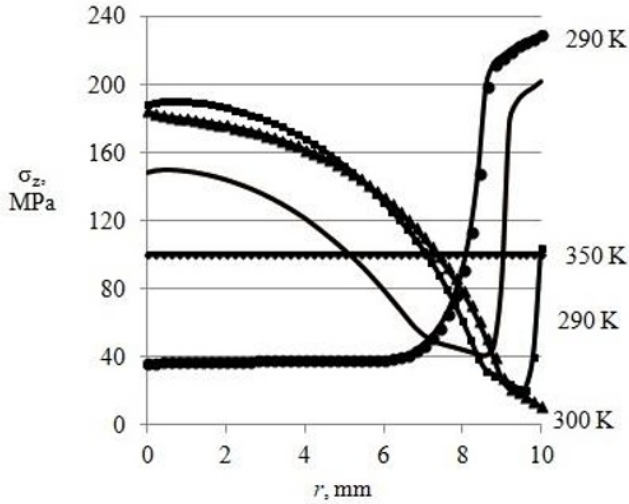
Fig. 3. Radial distribution of normal axial stress in cylinder with a radius 10 mm at surface temperature rate - 10 K s^{-1}. Numbers near the curves indicate the values of the surface temperature
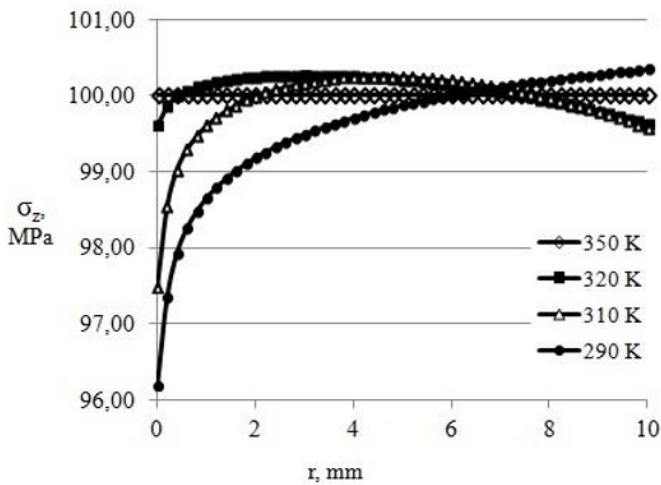

Fig. 4. Radial distribution of normal axial stress in cylinder with a radius 10 mm at surface temperature rate – 0.01 K s^{-1}. Numbers to the right indicate the values of the surface temperature
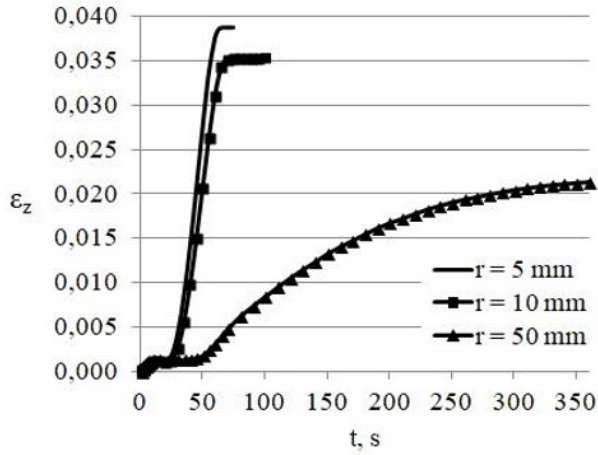
Fig. 5. Accumulation of axial strain in cylinders with radii 5, 10, 50 mm during the realization of the transformation plasticity effect (surface temperature rate - 10 K s^{-1})
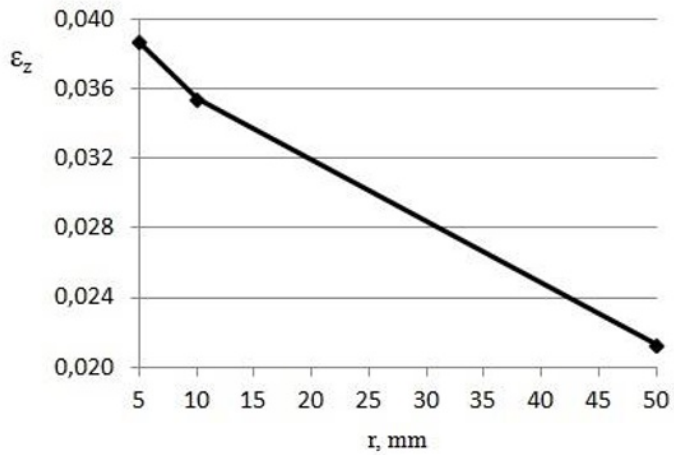

Fig. 6. Dependence of the summary relative elongation of cylinder ε_z on the radius (surface temperature rate - 10 K s^{-1})
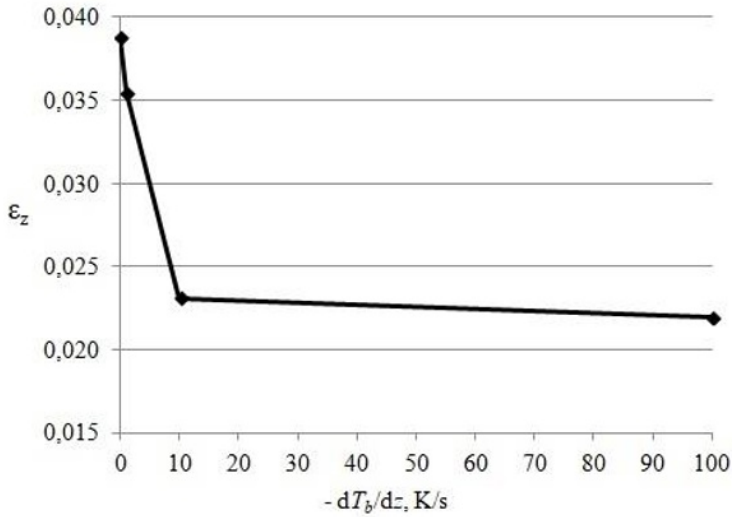
Fig. 7. Dependence of the summary relative elongation of the cylinder ε_z on the surface temperature rate (radius of cylinder is 10 mm)

3 Conclusions

The possibility of solving the twice connected thermomechanical boundary problem for an infinite SMA cylinder loaded by an axial force and subjected to heating or cooling from the surface was demonstrated in the case the constitutive relations set by the microstructural model [14]. The process of transformation plasticity was modeled.

It was studied how sizes of the body and surface temperature rate influence on transformation plasticity deformation. It was shown, that the larger the radius, the smaller the summary relative elongation of the cylinder. Analogous, with increasing surface temperature rate the summary relative elongation of the cylinder decreases. This phenomenon can be explained by an inhomogeneity of the temperature and stress fields causing different conditions for the phase transformation in different points of the body.

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