Real-Time Determination of Overall Heat Transfer Coefficient from the Seebeck Effect by Using Adaptive Learning-Rate Optimization

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Abstract. It is challenging to determine the actual overall heat transfer coefficient under thermal conditions during processes. In a conventional approach, they are obtained as a constant with the empirical formula for the given conditions. In this study, the Adaptive moment estimation (Adam) technique is investigated for adaptive learning-rate optimization in the real-time determination of the overall heat transfer coefficient via the Seebeck effect in the thermoelectric modules. Two thermoelectric modules detect heat transfer as solid surfaces exposed to the outdoor air. The principle of energy balance and the Seebeck effect determine the overall heat transfer coefficients over time. The heating/cooling process of a copper plate is considered with exposure to the outdoor air. The overall heat transfer coefficient is determined with the proposed methodology over time. The temperature of the copper plate is numerically determined by the mathematical models with the obtained values of the overall heat transfer coefficient. It is confirmed that the calculated values of temperature are close to the measured values, with RMSE = 0.07 °C.

1 Introduction

In thermal analysis, the overall heat transfer coefficient is an important parameter of heat transfer in various engineering applications. Several research works have experimentally investigated the overall heat transfer coefficient, to improve the performance and efficiency of certain processes. For example, an empirical formula was used to increase the thermal efficiency and reduce energy consumption in thermal processes such as fin-type heat exchanges in electronic devices [1] and the heat loss of building facades [2], respectively. Recently, the determination of the convective heat transfer coefficient under actual convection processes over time has been proposed, using thermoelectric modules based on the Seebeck effect [3]. It was found that the convective heat transfer coefficient was dependent on time-variant properties of air and thermal surfaces. To determine the convective heat transfer coefficient, the recursive optimization with the gradient descent technique is used to search for the actual values over time [4]. However, the difficulty in employing the gradient descent approach is that a reasonable step size is required to be adjusted in real time. To overcome this problem, the Adaptive moment estimation (Adam) is proposed to determine the overall heat transfer coefficient in the optimization. Presently, the Adam technique is one of the most effective adaptive learning-rate optimization in deep learning models. The Adam technique provided a fast converge rate with accurate results [5]. Also, a majority of well-known deep learning libraries supported the Adam technique, such as Tensorflow, Keras, Caffe, Torch, etc. [6]. Therefore, the Adam technique is adapted with optimization for the real-time determination of the overall heat transfer coefficient via the Seebeck effect in this work. This study is an experimental investigation of a heating/cooling process with dynamic exposure to the outdoor air.

This paper is organized as follows. In Section 2, the Adam technique is presented for adaptive learning-rate optimization in the real-time determination of the overall heat transfer coefficients. In Section 3, the values of the overall heat transfer coefficient in the heating/cooling process of a copper plate in the outdoor air are determined over time by the proposed technique, as a case study. To validate the effectiveness, the Adam technique is applied to predict the temperatures of the copper plate during the heating/cooling process in outdoor air with mathematical models. A comparison between the measured temperature and the calculated temperature, which are obtained with the overall heat transfer coefficient, is shown. The conclusion about the viability of the proposed technique in the real-time determination of the overall heat transfer coefficient via the Seebeck effect is stated in Section 4.

2 Real-time determination of overall heat transfer coefficients by using adaptive learning-rate optimization

In thermal analysis, the overall heat transfer coefficient of a solid surface exposed to the outdoor air is comprised

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of a convective heat transfer coefficient and a radiant heat transfer coefficient [7], as expressed by:

$$h_T = h_c + h_r \tag{1}$$

where h_T is the overall heat transfer coefficient, h_c is the convective heat transfer coefficient, and h_r is the radiant heat transfer coefficient.

To determine the overall heat transfer coefficients over time, the Seebeck effect of a thermoelectric module is applied when the heat transfer takes place, as shown in Figure 1. One surface of the thermoelectric module is faced to an air and sunlight while another surface is attached with a heat sink. The principle of energy balance is used to determine the temperature of the ceramic substrate at the hot side:

$$m_{cs} c_{cs} \frac{dT_{ph,s}}{dt} = I - h_T A_T (T_{ph,s} - T_a) - Q_{T,s} \tag{2}$$

where m_{cs} is the mass of ceramic substrate, c_{cs} is the specific heat of ceramic substrate, $T_{ph,s}$ is the temperature of ceramic substrate at the hot side, I is the incident irradiance of sunlight, A_T is the surface area of the thermoelectric module, $Q_{T,s}$ is the heat transfer through the thermoelectric module when the cold side is induced by a heat sink, and t is the time.

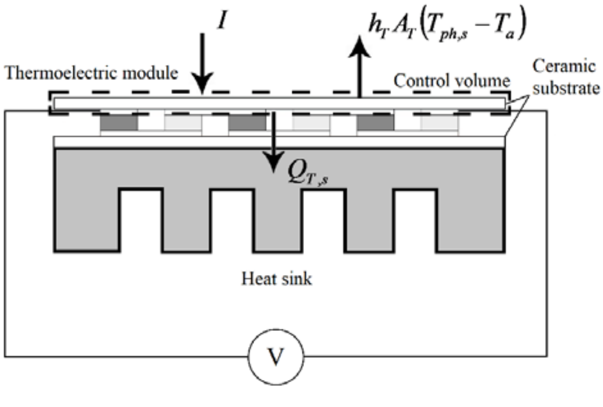

Fig. 1. Real-time determination of the overall heat transfer coefficients via Seebeck effect.

According to the Seebeck effect [3], the Seebeck voltage of the thermoelectric module can be generated when there are temperature differences between the two ceramic substrates of the thermoelectric module. The amount of heat transfer through the thermoelectric module is determined by:

$$Q_{T,s} = \frac{k_T}{\beta} V \tag{3}$$

where V is the Seebeck voltage across the thermoelectric module, k_T is the thermal conductivity coefficient of a thermoelectric module, and β is the Seebeck effect coefficient.

The rate of heat flowing through the thermoelectric module can be determined by the Seebeck voltage across the thermoelectric module. The direction of the heat flow through the thermoelectric module is indicated by the polarity of the Seebeck voltage across the thermoelectric module.

According to equation (2), the values of the incident irradiance of sunlight and the overall heat transfer coefficient are unknown while the temperatures and the Seebeck voltage are measured. To determine the overall heat transfer coefficient, another thermoelectric module is attached in parallel, as shown in Figure 2. However, the heat sink is changed to an insulated wall. This modification results in the temperature difference between the two thermoelectric modules. With the same consideration, the temperature of the ceramic substrate of the thermoelectric module coupled with an insulator at the hot side can be described as:

$$m_{cs} c_{cs} \frac{dT_{ph,i}}{dt} = I - h_T A_T (T_{ph,i} - T_a) - Q_{T,i} \tag{4}$$

where $T_{ph,i}$ is the temperature of the ceramic substrate at the hot side when an insulator is installed at the cold side, and $Q_{T,i}$ is the heat transfer through the thermoelectric module when the cold side is attached by an insulator.

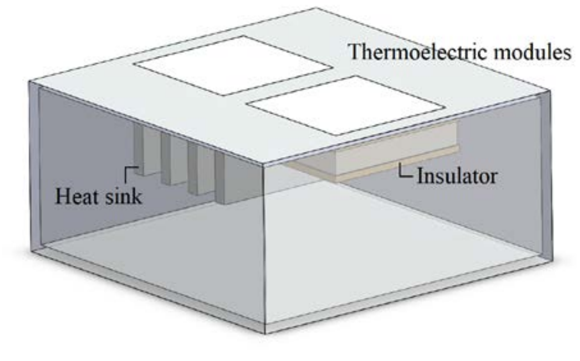

Fig. 2. Two thermoelectric modules coupled with heat sink and insulator.

Equation (2) is subtracted by equation (4). The incident irradiance of sunlight is eliminated. The temperatures of the ceramic substrates at the hot side are written as:

$$m_{cs} c_{cs} \frac{d(T_{ph,s} - T_{ph,i})}{dt} = -h_T A_T (T_{ph,s} - T_{ph,i}) - (Q_{T,s} - Q_{T,i}) \tag{5}$$

The temperature difference between the two thermoelectric modules at the hot sides is defined as $Y(t) := T_{ph,s}(t) - T_{ph,i}(t)$ with:

$$m_{cs}c_{cs}\frac{dY}{dt}=-h_T A_T Y-(Q_{T,s}-Q_{T,i}) \qquad (6)$$

The overall heat transfer coefficient can be determined when the temperature difference between the two thermoelectric modules at the hot side in equation (6) is equal to the temperature difference that is obtained from the measurement. The error function is defined as:

$$E=\frac{1}{2}\left(Y(t)-Y_m(t)\right)^2 \qquad (7)$$

where Y_m is the measured value of the temperature difference between the two thermoelectric modules. The backward finite difference approximation is applied to equation (6).

$$\frac{Y(t)-Y(t-\Delta t)}{\Delta t}=-\frac{h_T(t)A_T}{m_{cs}c_{cs}}Y(t)-\frac{1}{m_{cs}c_{cs}}\left(Q_{T,s}(t)-Q_{T,i}(t)\right) \qquad (8)$$

where Δt is the time step.

According to equation (9), the current temperature difference can be determined when the previous temperature difference is known.

$$Y(t)=\frac{Y(t-\Delta t)-\frac{\Delta t}{m_{cs}c_{cs}}\left(Q_{T,s}(t)-Q_{T,i}(t)\right)}{1+\frac{h_T(t)A_T\Delta t}{m_{cs}c_{cs}}} \qquad (9)$$

By taking the derivative of equation (8), the derivative of the temperature difference with respect to the overall heat transfer coefficient can be obtained as:

$$\frac{\partial Y(t)}{\partial h_T}=\frac{\frac{\partial Y(t-\Delta t)}{\partial h_T}-\frac{A_T\Delta t}{m_{cs}c_{cs}}Y(t)}{1+\frac{h_T(t)A_T\Delta t}{m_{cs}c_{cs}}} \qquad (10)$$

The derivative of the error function with respect to the overall heat transfer coefficient can be determined by:

$$\frac{\partial E}{\partial h_T}=\frac{\partial E}{\partial Y}\frac{\partial Y}{\partial h_T}=\left(Y(t)-Y_m(t)\right)\left(\frac{\frac{\partial Y(t-\Delta t)}{\partial h_T}-\frac{A_T\Delta t}{m_{cs}c_{cs}}Y(t)}{1+\frac{h_T(t)A_T\Delta t}{m_{cs}c_{cs}}}\right) \qquad (11)$$

In this work, the adaptive moment estimation (Adam) algorithm is applied, to minimize the error function in equation (7):

$$h_T(t+\Delta t)=h_T(t)-\eta\frac{\hat{m}(t+\Delta t)}{\sqrt{\hat{v}(t+\Delta t)}+\varepsilon} \qquad (12)$$

With the bias corrected estimators for the first and second moments,

$$\hat{m}(t+\Delta t)=\frac{m(t+\Delta t)}{1-\beta_1} \qquad (13)$$

and

$$\hat{v}(t+\Delta t)=\frac{v(t+\Delta t)}{1-\beta_2} \qquad (14)$$

The moving averages of gradient and the moving averages of squared gradient are defined as:

$$m(t+\Delta t)=\beta_1 m(t)+(1-\beta_1)\frac{\partial E}{\partial h_T} \qquad (15)$$

and

$$v(t+\Delta t)=\beta_2 v(t)+(1-\beta_2)\frac{\partial E}{\partial h_T} \qquad (16)$$

where $h_T\left(t+\Delta t\right)$ is the updated value of the overall heat transfer coefficient at the next time, $h_T\left(t\right)$ is the value of the overall heat transfer coefficient at the current time, η is the learning rate or step size, and ε is the empirical constant used to prevent division by zero in the implementation. β_1 and β_2 are the hyper parameters of the algorithm, which are the exponential decay rate for the first moment estimates and the exponential decay rate for the second-moment estimates, respectively.

The initial numerical values of the parameters are in Table 1. The derivative of the error function with respect to the overall heat transfer coefficient in equations (15) and (16) is determined by equation (11).

Table 1. Numerical values of parameters.

Parameters	**Numerical values**
Learning rate or step size, η	0.09
Empirical constants, ε	10^{-8}
Running average of the gradient, β_1	0.9
Running average of the squared gradient, β_2	0.999
Initial value of overall heat transfer coefficient, h_T	20
Initial value of moving average of gradient, m	0
Initial value of moving averages of squared gradient, v	0

3 Results and Discussion

To apply the proposed methodology in real time, an experiment for temperature prediction of a copper plate is set up, as shown in Figure 3.

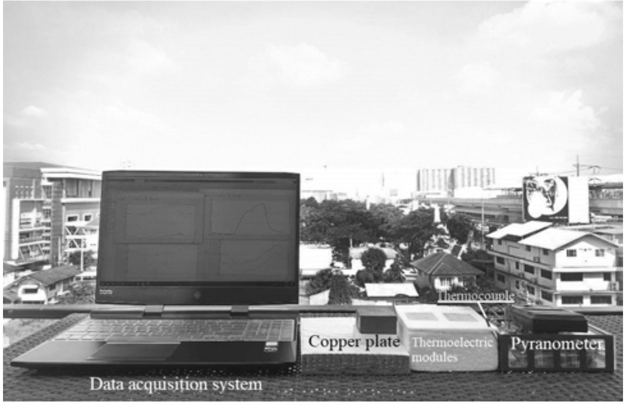

Fig. 3. Experiment of copper plate in outdoor air.

The copper plate is placed in an insulated box where a single surface is exposed to atmosphere during the daytime. The temperature of the copper plate is governed by the energy balance equation, equation (17).

$$m_c c_c \frac{dT}{dt} = I - h_T A_c \left(T - T_a\right) \tag{17}$$

where T is the temperature of copper plate, m_c is the mass of copper plate, c_c is the specific heat of the copper plate, and A_c is the transferred area of the copper plate.

The physical properties of the thermoelectric modules and the copper plate are listed in Table 2. To solve equation (17), the overall heat transfer coefficient, incident irradiance, and air temperature must be known over time. The proposed thermoelectric modules are located near the copper plate for determining the unknown overall heat transfer of the copper plate to air. A pyranometer (SolData98HP) with an accuracy of ± 3%FS, and a K-type thermocouple are used for measuring solar irradiance on a copper plate and air temperature, respectively

Table 2. Physical properties and parameters.

Parameters	**Numerical values**
Mass of ceramic substrate, m_{cs} (g)	4.0456
Specific heat of ceramic substrate, c_{cs} (J/g°C)	0.88
Seebeck effect coefficient, β (V/ °C)	0.0424
Thermal conductivity coefficient, k_T (W/ °C)	0.1639
Mass of copper plate, m_c (g)	28.672
Specific heat of copper plate, c_c (J/g°C)	0.385
Area of copper plate, A_c (m^2)	0.0016

The solar radiation flux density and air temperature are observed as illustrated in Figure 4(a). The proposed methodology in Section 2 is implemented for determining the values of the overall heat transfer coefficient over time. The values of the error function for computation performance are plotted against time in Figure 4 (b), as they are acceptably small. This result indicates that the calculated temperature differences are in good agreement with the measurements. As reported in Figure 4 (c), the values of the overall heat transfer coefficient are obtained over time. Together with the incident irradiance and air temperature, the calculated values of the overall heat transfer coefficient are inputted into the mathematical model of the copper plate in equation (17). Figure 4 (d) shows a comparison of the simulated results of copper plate temperature with measured values. It is observed that the simulated results of copper plate temperature closely track the measured values with RMSE = 0.07 °C. Furthermore, the simulated results with an estimated overall heat transfer coefficient of 20 W /m^2K are comparatively presented with higher deviations as a conventional approach.

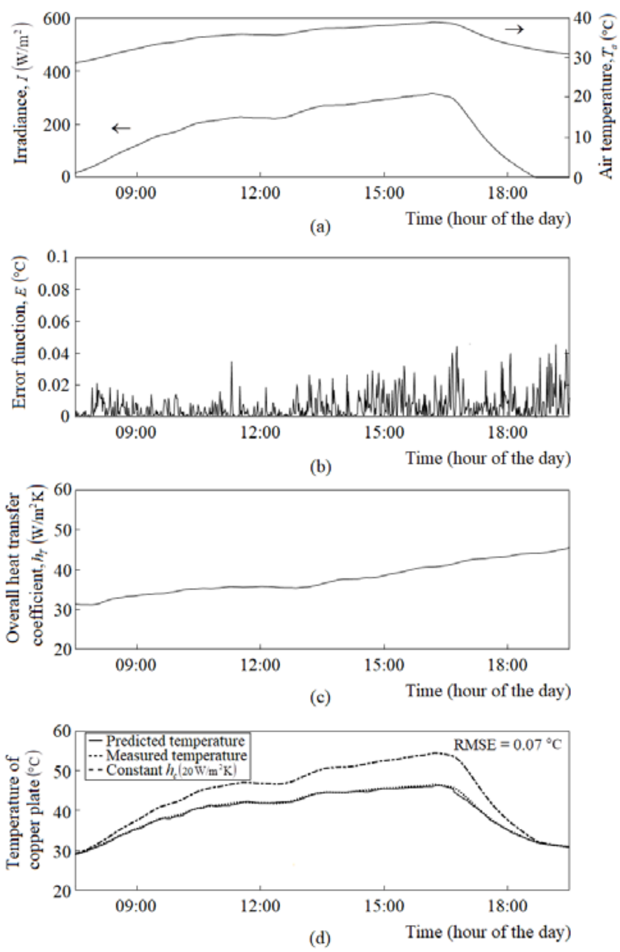

Fig. 4. Experimental results of temperature prediction for the copper plate over time: (a) irradiance and air temperature, (b) error function, (c) calculated overall heat transfer coefficient, (d) predicted and measured temperature of the copper plate.

4 Conclusion

The overall heat transfer coefficient is a crucial parameter in determining the heat transferred between a surface and the outdoor air at different temperatures. In this study, the Adam technique is investigated for adaptive learning-rate optimization. This is used for the real-time determination of the overall heat transfer coefficients via the Seebeck effect in thermoelectric modules. Two thermoelectric modules detect heat transfer as solid surfaces are exposed to the outdoor air. The principle of energy balance and the Seebeck effect are used to determine the overall heat transfer coefficient over time. To demonstrate the viability of the proposed methodology in a real-time implementation, the values of the overall heat transfer coefficient of a copper plate in ambient conditions during the daytime are determined, to predict the copper plate temperatures. The temperatures are found to be in good agreement with the measured values, with RMSE = 0.07 ^{o}C. However, the predicted temperatures from the estimated overall heat transfer coefficient of 20 W/m^{2}K do not fit measured temperatures.

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