

# Exponential Functions for Modelling Hysteresis of Soil-Water Characteristic Curves

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**Abstract.** Soil – water characteristic curve (SWCC) is an important property of unsaturated soils that can be used to estimate various parameters to describe unsaturated soil behavior. SWCC is reported to be hysteretic because the water content at a given suction in the wetting process is less than that in the drying process. In order to simulate the hysteretic characteristics of SWCC, many models have been proposed by different researchers. However, majority of the existing models are complex and their parameters are not related to the physical significances of SWCC variables. In this study, the new equations are developed to model drying and wetting SWCC. In addition, some indexes are proposed to estimate the wetting SWCC from drying SWCC. The new equations for SWCCs were evaluated with the laboratory data from published literatures. The results showed that the proposed equations performed well in modelling drying and wetting SWCC. The new equation has less parameters than the existing published equation.

## 1 Introduction

Soil – water characteristic curve (SWCC) is an important property of unsaturated soils which can be used to estimate the permeability and shear strength of the unsaturated soil. These unsaturated soil properties are required to address many geotechnical problems. Since SWCC is used as the basis for the estimation of other unsaturated soil parameters, it is important to have a mathematical model which fitting parameters can be used to represent the variables of SWCC. Numerous equations have been suggested to model the experimental data of SWCC [1, 2, 3, 4, 5, 6]. However, these equations were limited for the drying SWCC.

The hysteretic nature of SWCC has been known for a long time and it can be attributed to multiple factors such as “ink-bottle” effect, contact angle difference during the drying and wetting processes, entrapped air and thixotropic regain due to drying and wetting history of the soil [7]. Bear [8] demonstrated that the hysteresis of SWCC is resulted from “ink-bottle” effect and “rain-drop” effect, which means that the contact angle at an advancing interface in the wetting process is different from that at a receding interface in the drying process. Assuming the soil pores are connecting to each other, the capillary water rise height is shorter in the wetting process than that in the drying process. This phenomenon is called “ink-bottle” effect.

Many models have been proposed by different researchers to simulate the drying and wetting curve of SWCC [9, 10, 11, 12, 13, 14]. However, their mathematical equations are complex and their parameters are not related to the physical significances of SWCC variables. Therefore, in this paper, a new equation to model drying and wetting SWCC is proposed. In addition, the new method to predict the wetting SWCC from the drying SWCC using the proposed equation is also provided in this paper.

## 2 Theory

SWCC is defined as the relationship between water content and suction of a soil [15, 16]. The water content defines the amount of water contained within the pores of the soil. Volumetric water content,  $\theta_w$  (volume of water in soil divided by total volume of soil, or  $V_w/V$ ) is most commonly used. The suction used is matric suction,  $\Psi$  (i.e.  $u_a - u_w$ , where  $u_a$  is the pore air pressure and  $u_w$  is the pore water pressure). SWCC is usually plotted in a semi-logarithmic scale, i.e. volumetric water content is in arithmetic scale, and matric suction is in logarithmic scale.

The air-entry value,  $\Psi_{aev}$ , of a soil is the matric suction where air first enters the largest pores in the soil. The residual water content,  $\theta_r$ , is the water content where the slope of the SWCC decreases considerably and a

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large suction change is required to remove additional water from the soil. The suction corresponding to zero water content appears to be the same for all types of soil. For a wide range of soil, a value of  $10^6$  kPa has been experimentally supported [15]. In other words, the maximum matric suction to be considered in the SWCC is  $10^6$  kPa for all types of soil.

Many empirical equations have been proposed to best fit the SWCC. One of the earliest equations was proposed by Brooks and Corey [1]. Their equation was found to perform quite well for suction ranges greater than the air-entry value of the soil. However, it did not fit well under a fully saturated condition or near maximum desaturation. Another frequently used equation for SWCC was given by van Genuchten [2]. In an attempt to obtain a closed-form equation for permeability, van Genuchten [2] suggested the relationship  $m = 1 - 1/n$ . However, this restricts the flexibility of the equation. The equation would perform better without  $m$  and  $n$  having any fixed relationship. Fredlund and Xing [6] proposed a four-parameter equation to best fit SWCC. Their equation has been proven to provide better fit than other equations mentioned above [17]. However, the parameters of their equation are not related to the variables of drying and wetting SWCC.

Pham et al. [12] proposed a simple scaling method to predict the boundary wetting curve entirely from the boundary drying curve. The curve-fitting parameters of the boundary wetting curve can be calculated using fitting parameters of the boundary drying curve, together with the distance, and slope ratio between the two boundary curves. Pham et al. [12] suggested several values for the slope ratio and the distance between the two boundary curves for different types of soil. It has been shown that the scaling method provides acceptable prediction of the boundary wetting curve. However, this method still uses the equation which parameters do not take into account the physical significance of its parameters.

### 3 Proposed Equation

The proposed equations for modelling SWCC in this study have the form of exponential functions, and each fitting parameter has a physical meaning related to the fitting curve. The equations were derived based on the consideration that the shape of SWCC is similar to the graph of the exponential function (Figure 1). It can be seen that the values of the exponential function in Figure 1 are within the range of 0 to 1. Therefore, the equation for SWCC can be expressed as:

$$\Theta = \exp[-F(\psi)] \quad (1)$$

where :

$$\theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \text{normalized volumetric water content} \quad (2)$$

$F(\psi)$  = a function of matric suction, whose parameters are properties of the SWCC such as the air-entry value,  $\Psi_{aev}$ , and residual matric suction,  $\Psi_r$ .

$$F(\psi) = \begin{cases} 0 & \text{for } \psi < \psi_{aev} \\ 5 \left( \frac{\psi - \psi_{aev}}{\psi_r - \psi_{aev}} \right) & \text{for } \psi \geq \psi_{aev} \end{cases} \quad (3)$$

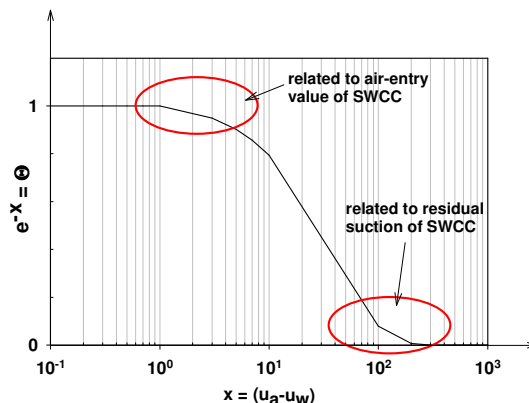


Fig. 1. Graph of exponential function  $f(x) = \exp(-x)$

The Fredlund and Xing [6] correction factor was applied to the equation. The volumetric water content at any matric suction is calculated from the equation 4. The initial value of each fitting parameter can be approximated from the variables of the SWCCs as obtained from laboratory tests and illustrated in Figure 2. An iterative non-linear regression procedure can be used to obtain the final values of each fitting parameters. Regardless of the approximations of the initial values, the final values of each fitting parameter will be the same, demonstrating the uniqueness of the solution to this equation.

$$\theta(\psi) = \left[ 1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{10^6}{\psi_r}\right)} \right] \left\{ (\theta_s - \theta_r) \exp\left[-5 \left( \frac{\psi - \psi_{aev}}{\psi_r - \psi_{aev}} \right)\right] + \theta_r \right\} \quad (4)$$

where:

$\theta_s$  = saturated volumetric water contents

$\theta_r$  = residual volumetric water contents

$\Psi_{aev}$  = air-entry value of soil

$\Psi_r$  = residual matric suction

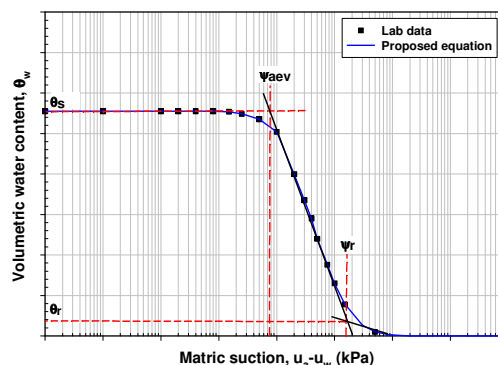


Fig. 2. Approximation of initial values for best fit parameters of SWCC

For wetting SWCC, the equation has the following form:

$$\theta(\psi) = \left[ 1 - \frac{\ln\left(1 + \frac{\psi}{\psi_w}\right)}{\ln\left(1 + \frac{10^6}{\psi_{rw}}\right)} \right] \left\{ (\theta_s - \theta_r) \exp\left[-5 \left(\frac{\psi - \psi_{bw}}{\psi_w - \psi_{bw}}\right)\right] + \theta_r \right\} \quad (5)$$

where:

$\psi_{bw}$  = wetting saturation point (expulsion value) of SWCC

$\psi_w$  = water-entry value of SWCC

The wetting saturation point and water-entry value of SWCC are related to the air-entry value and residual matric suction of the SWCC by the following expressions:

$$\psi_{bw} = \chi_{aev} \psi_{aev} \quad (6)$$

$$\psi_w = \chi_r \psi_r \quad (7)$$

where:

$\chi_{aev}$  and  $\chi_r$  = parameters related to the hysteresis in SWCC

In Equations 5, 6 and 7, the exponential functions are to take value of 1 if the value of the exponent is positive.

#### 4 Evaluation of the proposed equation and estimation of wetting SWCC

A total of 12 soil data were used for evaluation of fitting drying and wetting SWCCs which were taken from different published literatures [11, 18, 19, 20, 21]. Basic properties of those soils are summarized in Table 1.

**Table 1.** Basic properties of the data used in this study

Soil type	Source	Void ratio, e	Dry density, $\rho_d$ (Mg/m <sup>3</sup> )	Specific gravity, G <sub>s</sub>
Beaver Creek Sand	Pham et al. (2003)	0.563	1.70	2.65
Processed silt		0.689	1.58	2.67
Soil mixture	Indrawan et al. (2007)	0.510	1.74	2.63
Gravelly sand I		0.640	1.62	2.66
Gravelly sand II	Yang et al. (2004)	0.617	1.62	2.62
Compacted silt	Trinh et al. (2006)	1.083	1.35	2.65
Loose silty sand		0.677	1.61	2.70
Dense silty sand	Pham et al. (2003)	0.424	1.88	2.68
Caribou silt loam		0.655	1.61	2.67
Medium sand	Yang et al. (2004)	0.538	1.69	2.60
Fine sand		0.699	1.56	2.65
Coarse kaolin	Meilani et al. (2005)	-	-	-

For verification of the accuracies of the proposed equations, a nonlinear least squares optimization procedure was used to analyze the laboratory data. The procedure was performed using an iterative nonlinear regression procedure that is provided in the Microsoft Excel software [22]. The coefficient of determination ( $R^2$ ) of the fitted curves versus the laboratory data were used to evaluate the accuracies of the proposed equations.

The coefficient of determination ( $R^2$ ) and values of fitting parameters of each best fit curve are presented in

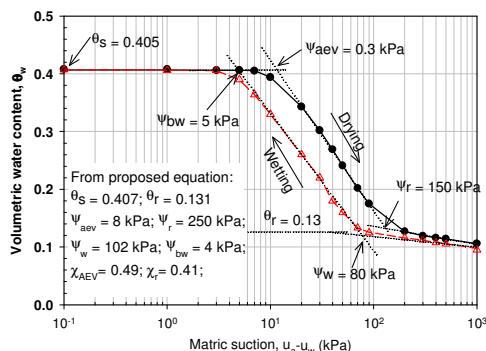
Tables 2 and 3. Tables 2 and 3 show that the coefficients of determination,  $R^2$ , of all best fit curves are very close to unity, with the lowest  $R^2$  being above 0.96. The average values of  $R^2$  are 0.997 for drying SWCC and 0.991 for wetting SWCC. Therefore, both equations performed well in fitting data of SWCC. Moreover, the fitting parameters are closely related to the variables of SWCC. The fitted values of  $\psi_{bw}$  and  $\psi_w$  are approximately equal to the wetting saturated suction and water-entry value of the SWCC. This can be verified from the graphs of the fitted curves. Figures 3 to 5 show the best fit curves for the soils: processed silt, gravelly sand I, and dense silty sand, respectively. However, the values of two parameters ( $\chi_{bw}$  and  $\chi_w$ ) in each best fit curve were not predicted from the drying SWCC, but they were fitting parameters. Therefore, it is necessary to have some kind of expressions to calculate  $\chi_{bw}$  and  $\chi_w$  or some proposed values of  $\chi_{bw}$  and  $\chi_w$  for different soil types for complete prediction of boundary wetting SWCC from boundary drying SWCC. For this purpose, the two parameters ( $\chi_{bw}$  and  $\chi_w$ ) have been related to two parameters: the slope ratio, RSL, and distance between the two boundary curves, DSL, as mentioned in Pham et al. (2005). A schematic illustration of the relationship between the parameters of the two curves is shown in Figure 6.

**Table 2.** Values of Fitting Parameters and Coefficient of Determination ( $R^2$ ) of drying SWCC

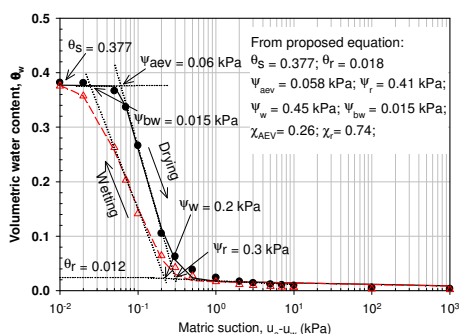
Soil	Drying SWCC (Equation 4)				
	$\psi_{aev}$ (kPa)	$\psi_r$ (kPa)	$\theta_s$	$\theta_r$	$R^2$
Beaver Creek Sand	1.6	18	0.361	0.119	0.992
Processed silt	6.2	269	0.407	0.131	0.998
Soil mixture	0.10	13	0.334	0.185	0.992
Gravelly sand I	0.06	0.61	0.377	0.018	0.998
Gravelly sand II	0.12	0.94	0.380	0.024	0.999
Compacted silt	36	583	0.520	0.129	0.999
Loose silty sand	2.5	7.9	0.408	0.033	0.999
Dense silty sand	2.8	8.9	0.467	0.052	0.999
Caribou silt loam	7.6	41	0.398	0.306	0.999
Medium sand	0.81	9.1	0.350	0.074	0.996
Fine sand	1.4	11	0.411	0.025	0.996
Coarse kaolin	42	617	0.519	0.073	0.999

**Table 3.** Values of Fitting Parameters and Coefficient of Determination ( $R^2$ ) of wetting SWCC

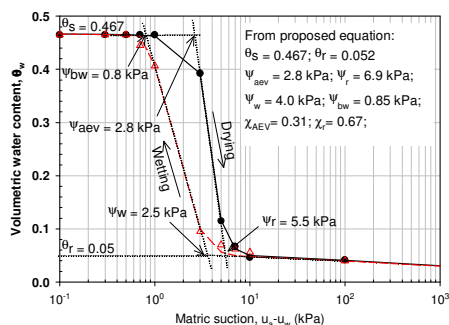
Soil	Wetting SWCC (Equation 5)				
	$\psi_{bw}$ (kPa)	$\psi_w$ (kPa)	$\chi_{aev}$	$\chi_r$	$R^2$
Beaver Creek Sand	0.88	9.4	0.55	0.52	0.998
Processed silt	3.04	110	0.49	0.41	0.997
Soil mixture	0.09	1.6	0.90	0.12	0.964
Gravelly sand I	0.016	0.45	0.26	0.74	0.997
Gravelly sand II	0.001	0.46	0.01	0.49	0.991
Compacted silt	14	315	0.40	0.54	0.998
Loose silty sand	0.85	4.98	0.34	0.63	0.998
Dense silty sand	0.868	6.0	0.31	0.67	0.998
Caribou silt loam	2.356	27	0.31	0.66	0.981
Medium sand	0.1458	4.2	0.18	0.46	0.980
Fine sand	0.87	4.5	0.62	0.41	0.996
Coarse kaolin	11	413	0.25	0.67	0.998



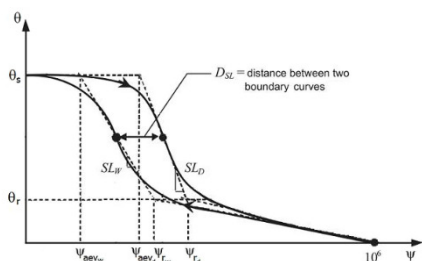
**Fig. 3.** Best fit curves of drying and wetting SWCC for the soil processed silt



**Fig. 4.** Best fit curves of drying and wetting SWCC for the soil gravelly sand I



**Fig. 5.** Best fit curves of drying and wetting SWCC for the soil dense silty sand



**Fig. 6.** A schematic illustration of the slope and distance between the two boundary curves (modified after Pham et al. [12])

The slopes of the boundary drying SWCC and boundary wetting SWCC can be calculated as:

$$SL_D = \frac{\theta_s - \theta_r}{\log \psi_r - \log \psi_{aev}} = \frac{\theta_s - \theta_r}{\log \frac{\psi_r}{\psi_{aev}}} \quad (8)$$

$$SL_W = \frac{\theta_s - \theta_r}{\log \psi_w - \log \psi_{bw}} = \frac{\theta_s - \theta_r}{\log \frac{\psi_w}{\psi_{bw}}} \quad (9)$$

The slope ratio  $R_{SL}$  can be expressed as:

$$R_{SL} = \frac{SL_D}{SL_W} = \frac{\log \frac{\psi_w}{\psi_{bw}}}{\log \frac{\psi_r}{\psi_{aev}}} = \frac{\log \frac{\chi_r \psi_r}{\chi_{aev} \psi_{aev}}}{\log \frac{\psi_r}{\psi_{aev}}} = 1 + \frac{\log \frac{\chi_r}{\chi_{aev}}}{\log \frac{\psi_r}{\psi_{aev}}} \quad (10)$$

Let  $\lambda$  be the logarithm of the ratio between residual matric suction and air-entry value of the boundary drying SWCC:

$$\lambda = \log \frac{\psi_r}{\psi_{aev}} \quad (11)$$

Equation 11 can be rewritten to obtain:

$$\log \frac{\chi_r}{\chi_{aev}} = \lambda(R_{SL} - 1) \quad (12)$$

The ratio between  $\chi_r$  and  $\chi_{aev}$  can now be written as:

$$\frac{\chi_r}{\chi_{aev}} = 10^{\lambda(R_{SL}-1)} \quad (13)$$

It is reasonable to assume that the distance between the two boundary curves can be estimated as the average of the distance between the air-entry values to the wetting saturated point ( $D_{aev}$ ) and the distance between the residual matric suctions to the water-entry value ( $D_r$ ) of the drying and wetting curves:

$$D_{SL} = \frac{1}{2}(D_{aev} + D_r) \quad (14)$$

$D_{aev}$  and  $D_r$  can be calculated as:

$$D_{aev} = \log \psi_{aev} - \log \psi_{bw} = \log \frac{\psi_{aev}}{\psi_{bw}} = \log \frac{1}{\chi_{aev}} \quad (15)$$

$$D_r = \log \psi_r - \log \psi_w = \log \frac{\psi_r}{\psi_w} = \log \frac{1}{\chi_r} \quad (16)$$

Equation 14 can be rewritten as:

$$D_{SL} = \frac{1}{2} \left( \log \frac{1}{\chi_{aev}} + \log \frac{1}{\chi_r} \right) = \frac{1}{2} \log \left( \frac{1}{\chi_{aev} \chi_r} \right) \quad (17)$$

Equation 17 can be rearranged as follows:

$$\chi_{aev} \chi_r = 10^{-2D_{SL}} \quad (18)$$

Combining Equation 14 and 18,  $\chi_{aev}$  and  $\chi_r$  can be calculated as follows:

$$\chi_{aev} = 10 \left[ -D_{SL} - \frac{1}{2} \lambda(R_{SL} - 1) \right] \quad (19)$$

$$\chi_r = 10 \left[ -D_{SL} + \frac{1}{2} \lambda(R_{SL} - 1) \right] \quad (20)$$

From Equations 18 and 19, two parameters ( $\chi_{aev}$  and  $\chi_r$ ) can be calculated from the slope ratio, RSL, distance between the two curves, DSL, and the ratio between residual matric suction and air-entry value of the boundary drying curve,  $\lambda$ . Pham et al. (2005) suggested different values of DSL and RSL for different types of

soil, as summarized in Table 6. These values can be used to calculate  $\chi_{aev}$  and  $\chi_r$ , and therefore the boundary wetting SWCC can be completely predicted from the boundary drying SWCC. Comparisons of the  $R^2$  of the predicted wetting curves between this method and Pham et al. [12] scaling method are presented in Tables 4 and 5, respectively.

**Table 4.**  $R^2$  of the proposed method in predicting of wetting SWCC

Soil	DSL	RSL	Proposed method		
			$\chi_{aev}$	$\chi_r$	$R^2$
Beaver Creek Sand	0.35	1.0	0.45	0.45	0.988
Processed silt	0.35	1.0	0.45	0.45	0.996
Soil mixture	0.35	1.0	0.45	0.45	0.865
Gravelly sand I	0.20	2.0	0.19	1.0	0.983
Gravelly sand II	0.20	2.0	0.22	1.0	0.725
Compacted silt	0.35	1.0	0.45	0.45	0.993
Loose silty sand	0.20	2.0	0.35	1.0	0.969
Dense silty sand	0.20	2.0	0.35	1.0	0.978
Caribou silt loam	0.50	1.5	0.21	0.48	0.873
Medium sand	0.20	2.0	0.19	1.0	0.926
Fine sand	0.20	2.0	0.22	1.0	0.959
Coarse kaolin	0.50	1.5	0.16	0.62	0.992

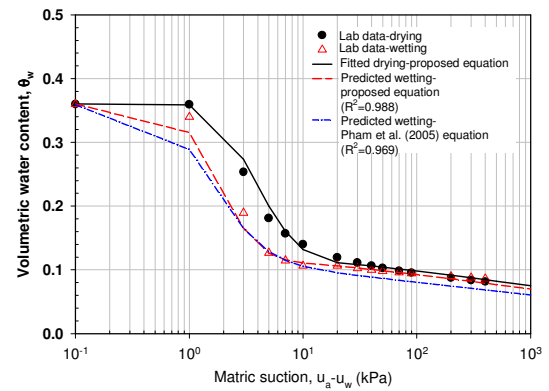
**Table 5.**  $R^2$  of Pham et al. [12] scaling method in predicting wetting SWCC

Soil	DSL	RSL	Proposed method			
			$b_w$	$c_w$	$d_w$	$R^2$
Beaver Creek Sand	0.35	1.0	2.82	0.112	1.94	0.969
Processed silt	0.35	1.0	131.8	0.134	1.65	0.996
Soil mixture	0.35	1.0	0.49	0.202	1.19	0.868
Gravelly sand I	0.20	2.0	0.06	0.015	1.17	0.967
Gravelly sand II	0.20	2.0	0.07	0.015	1.39	0.751
Compacted silt	0.35	1.0	9746	0.121	2.36	0.989
Loose silty sand	0.20	2.0	11.9	0.040	3.46	0.954
Dense silty sand	0.20	2.0	20.1	0.056	3.32	0.973
Caribou silt loam	0.50	1.5	132.9	0.328	3.67	0.828
Medium sand	0.20	2.0	1.37	0.077	1.30	0.920
Fine sand	0.20	2.0	2.63	0.051	1.96	0.949
Coarse kaolin	0.50	1.5	1870	0.084	2.10	0.865

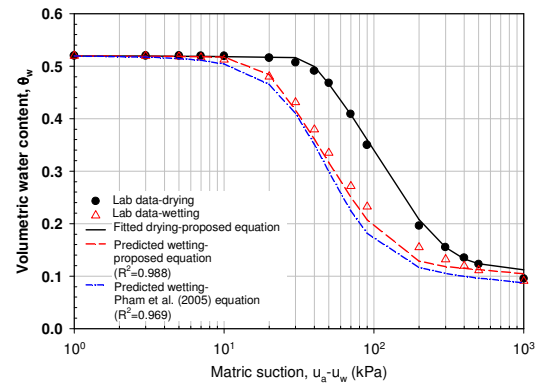
From Tables 4 and 5, it can be observed that the coefficients of determination,  $R^2$ , of most of the predicted curves are quite close to unity for both methods. Thus, the proposed method performed well in predicting wetting SWCC from drying SWCC. Figures 7 to 9 show the graphs of predicted wetting SWCCs for the soils: Beaver Creek sand, compacted silt, and coarse kaolin, respectively. In addition, the fitting parameters of the proposed equation for drying and wetting SWCC are closely related to the variables of the SWCC, such as air-entry value, and residual matric suction. The hysteresis effect was characterized by two parameters,  $\chi_{ave}$  and  $\chi_r$ , which are the ratio of the air-entry values of SWCC to the wetting saturation point of SWCC, and ratio of the residual matric suctions of SWCC to the water-entry value of SWCC, respectively. For simplicity, values of  $\chi_{ave}$  and  $\chi_r$  are suggested for several types of soil, as shown in Table 6.

**Table 6.** Suggested value of  $\chi_{ave}$  and  $\chi_r$  for several types of soil

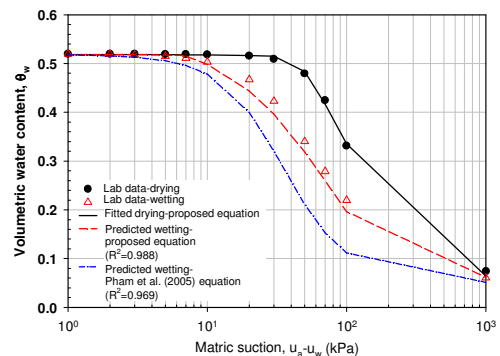
Soil type	$\chi_{ave}$		$\chi_r$	
	Range	Typical value	Range	Typical value
Graded sand	0.10-0.30	0.20	0.40-0.80	0.60
Silty sand and clayey sand	0.20-0.50	0.35	0.60-0.80	0.70
Silt loam and clay loam	0.20-0.40	0.30	0.40-0.70	0.55
Compacted silt	0.40-0.60	0.50	0.40-0.60	0.50



**Fig. 7.** Experimental data and predicted wetting SWCC for the soil Beaver Creek sand



**Fig. 8.** Experimental data and predicted wetting SWCC for the soil compacted silt



**Fig. 9.** Experimental data and predicted wetting SWCC for the soil coarse kaolin

## 6. Conclusions

A mathematical model of SWCC has been developed and verified with experimental data. The equations were proposed for modelling drying and wetting SWCC. In addition, a new method was also proposed to predict wetting SWCC from drying SWCC. By comparison with the published experimental data, the proposed equations have been proved to provide a good fit for drying and wetting SWCC. The proposed method for predicting wetting SWCC has been also proven to provide good prediction of wetting SWCC for different types of soils. The fitting parameters of the proposed equations closely represent the physical properties of the fitting curves,

which can be used to describe the entire best fit curve of SWCC. The procedure of best fitting SWCC can also be shortened since the fitting parameters can be readily approximated from experimental data.

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