

Optimization of Short-term Modes of Hydrothermal Power System

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Abstract. Optimal planning of short-term modes of power systems is a complex nonlinear programming problem with many simple, functional and integral constraints in the form of equalities and inequalities. Especially, the presence of integral constraints causes significant difficulties in solving of such problem. Since, under such constraints, the modes of power system in separate time intervals of the considered planning period become dependent on the values of the parameters in other intervals. Accordingly, it becomes impossible to obtain the optimal mode plan as the results of separate optimization for individual time intervals of the period under consideration. And the simultaneous solution of the problem for all time intervals of the planning period in the conditions of large power systems is associated with additional difficulties in ensuring the reliability of convergence of the iterative computational process. In this regard, the issues of improving the methods and algorithms for optimization of short-term modes of power systems containing thermal and large hydroelectric power plants with reservoirs, in which water consumption is regulated in the short-term planning period, remains as an important task. In this paper, we propose the effective algorithm for solving the problem under consideration, which makes it possible to quickly and reliably determine the optimal operating modes of the power system for the planned period. The results of research of effectiveness of this algorithm are presented on the example of optimal planning of daily mode of the power system, which contains two thermal and three hydraulic power plants..

1 Introduction

Considering hydroelectric power plants (HPPs) in the tasks of optimization the modes of power system, we mean the presence of a regulated reservoir with it, which allows it to work with a variable mode of power output in the context of the planned period. Hydroelectric power plants operating on the domestic flow of rivers, the capacities of which are unambiguously determined by the current inflows of water, are usually replaced by negative loads and do not participate in optimization. In the presence of large hydroelectric power plants with reservoirs and, accordingly, with given water flow rates, the modes of power systems for the time interval of the planning period become dependent and the procedure for optimization of their daily mode becomes more complicated.

Hydroelectric power plants play a large role in the operation of modern EPS: by removing the peak part of their loads, they allow reducing the installed capacity of TPPs. In addition, the presence of HPPs in the power system is essential for maintaining the balance of power in them in emergency modes, since HPPs are more mobile than thermal power plants (TPPs).

Usually, for hydroelectric power plants participating in the optimal planning of short-term modes of power systems, a limitation on water consumption associated with the need to maintain the required water level in reservoirs for irrigation or other purposes is established,. Thus, the problem of optimal planning of short-term mode of power system containing HPPs with reservoirs is to determine such load schedules for all TPPs and HPPs, which ensure the minimum total fuel costs in TPPs, required water levels in HPP reservoirs, minimal impact on the environment and all other established operating and technological restrictions [1-4]. Accordingly, the problem under consideration is mathematically a complex nonlinear programming problem. It is characterized by the presence, in the general case, of many simple, functional and integral constraints [5-7]. Integral constraints exclude the use of short-term planning methods based on individual solutions to optimization problems for individual time intervals of the period under consideration. Since, due to integral limitations, the power system modes in certain time intervals become dependent on the parameters of other intervals. On the other hand, the simultaneous solution of such problem for all time intervals of the planning period is associated with special difficulties in ensuring the reliable convergence of iterative

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calculation process to the solution with sufficient accuracy.

Many works of authors from all over the world are devoted to the issue of optimal planning of short-term modes of hydrothermal power system, who have made a significant contribution to the development of the theory and methods for solving the problem under consideration. These include a number of algorithms that implement various optimization methods such as Lagrange multipliers [8, 9], gradient search [10, 11, 16], evolutionary programming [12, 13], genetic [14, 15], etc. etc. At the same time, the algorithms for solving the problem under consideration cannot be considered perfect. For example, the algorithms mentioned above based on the methods of Lagrange multipliers and gradient search are characterized by slow, and sometimes not reliable, convergence of the iterative calculation process. Evolutionary and genetic algorithms also suffer from poorer search performance. Thus, the issues of improving the methods and algorithms for optimization of short-term modes of power systems with large hydroelectric power plants participating in the optimization and regulation of reservoir modes during the planned periods remains an urgent task.

In this paper, an effective algorithm for solving the described problem, which is distinguished by a sufficiently fast and reliable convergence of iterative calculation process, is presented.

2 The method and algorithm of optimization

The problem of optimization of short-term, for example daily, mode of an energy system containing n TPPs and m HPPs with water flow control during the planning period, in the general case, can be represented as follows:

minimize the function of total fuel costs in TPPs per day

$$B = \sum_{i=1}^n \sum_{t=1}^{24} B_i(P_{it}) \quad (1)$$

subject to constraints:

by balance of active power in power system –

$$\sum_{i=1}^n P_{it} + \sum_{j=1}^m P_{jt} = P_{Lt}, \quad t = 1, 2, \dots, 24; \quad (2)$$

by available capacity of power plants –

$$P_{it}^{min} \leq P_{it} \leq P_{it}^{max}, \quad i = 1, 2, \dots, n, \quad t = 1, 2, \dots, 24; \quad (3)$$

$$P_{jt}^{min} \leq P_{jt} \leq P_{jt}^{max}, \quad j = 1, 2, \dots, m, \quad t = 1, 2, \dots, 24; \quad (4)$$

on flows of active power in controlled power lines –

$$P_{lt}^{min} \leq P_{lt} \leq P_{lt}^{max}, \quad l = 1, 2, \dots, L_p, \quad t = 1, 2, \dots, 24; \quad (5)$$

by water consumption in HPPs per day

$$\Delta Q_j = \sum_{t=1}^{24} Q_{jt}(H_{jt}, P_{jt}) - Q_{j\beta} = 0, \quad j = 1, 2, \dots, m. \quad (6)$$

In (1) - (6) B is total fuel costs in n TPPs per day; P_{it} , P_{jt} are active loads of i th TPP and j th HPP at the t th hour of the day; P_{Lt} is total load of power system consumers at the t th hour of the day; P_{it}^{min} , P_{it}^{max} , P_{jt}^{min} , P_{jt}^{max} are the

minimum and maximum possible loads of the i th TPP and the j th HPP at the t -th hour of the day (taking into account the optimal composition of operating units in power plants); P_{lt} is the flow of active power into the l -th controlled power transmission line (PTL) at the t -th hour of the day; P_{lt}^{min} , P_{lt}^{max} are the minimum and maximum possible flows of active power in the l th controlled transmission line at the t th hour of the day; L_p is the number of transmission lines in which the power flows are controlled; H_{jt} , Q_{jt} are head and water flow rate in the j th hydroelectric power station at the t th hour of the day; $Q_{j\beta}$ is a given volume of water in the j th hydroelectric power plant, which should be optimally consumed during the day.

In the proposed algorithm, the solution to problem (1) - (6) is reduced to minimizing the Lagrange function

$$L = \sum_{i=1}^n B_i(P_{it}) + \sum_{j=1}^m \lambda_j \Delta Q_j \quad (7)$$

subject to constraints (2) - (5). The indefinite factor λ_j in (7) is physically the costly equivalent of the water consumption in the j th hydroelectric power plant and is numerically equal to the amount of saved fuel costs at the thermal power plant with an increase in the daily water consumption in the j th hydroelectric power plant by 1 m^3 . It remains constant during the regulation cycle [1].

At the known values of the factors λ_j , the solution of the problem is carried out as an optimization of mode of power system containing only the calculated TPPs, since multiplying the energy characteristics of each HPP by the corresponding factors λ_j it is reduced, in the calculated sense, into the category of TPPs. In this case, the problem of optimal coverage of the daily load schedule of power system is reduced to sequential optimal distribution of power system load, for each interval of regulation cycle, between the calculated power plants, taking into account constraints (3) - (5). In this case, the objective function is represented in the form (7).

Thus, in the setting adopted here, the main problem is related to the determination of the values of the Lagrange multipliers λ_j . The presence of kinks and vertical-horizontal sections in the $\Delta Q_j(\lambda_j)$ dependences

complicates the use of methods using derivatives $\frac{\partial Q}{\partial \lambda}$

for this purpose.

The described algorithm for optimal planning of short-term mode of power system provides for the determination of values of these factors λ_j by a combined algorithm based on the use of chord methods and simple selection in the process of iterative calculation. Below we describe the essence of the algorithm.

The choice of the initial values of the factors $\lambda_j^{(0)}$ ($j = 1, 2, \dots, m$) is carried out by the method of average values as in [1]. Refinement of values of factors after the first iteration $\lambda_j^{(1)}$, if the condition for the convergence of the iterative calculation process is not met

$$\left| \sum_{i=1}^{24} Q_{j_i}(P_{j_i}) - Q_j \right| \leq \varepsilon_{\Delta Q}, \quad j = 1, 2, \dots, m \quad (8)$$

is carried out by the following expression

$$\lambda_j^{(1)} = \begin{cases} \lambda_j^{(0)} + \gamma_j^{(0)} \Delta \lambda_j & \text{if } \Delta Q_j^{(0)} > 0, \\ \lambda_j^{(0)} - \gamma_j^{(0)} \Delta \lambda_j & \text{if } \Delta Q_j^{(0)} < 0, \end{cases} \quad (9)$$

where $\Delta \lambda_j$ is a small positive number - the increment of the factor; $\gamma_j^{(0)} = 1$ is the scale factor, which is found in subsequent iterations as $\gamma_j^{(k)} = 2\gamma_j^{(k-1)}$.

In subsequent t th iterations for those HPPs, in which the change in the sign of the residual ΔQ_j has not yet been observed in the previous iterations, is found by expression (9), where instead of $\lambda_j^{(0)}$, $\gamma_j^{(0)}$ uses $\lambda_j^{(t-1)}$ and $\gamma_j^{(t-1)}$, appropriately, and HPP, in which a change in the sign of the residual ΔQ_j has already been observed, they are calculated by the chord method using the expression

$$\lambda_j^{(t)} = \lambda_j^* - \frac{\lambda_j^* - \lambda_j'}{\Delta Q_j^* - \Delta Q_j'} \cdot \Delta Q_j' \quad (10)$$

In (10), individual parameters are determined according to the following conditions:

if the first change in the sign of the residual ΔQ_j occurred in the t - l th iteration, then $\Delta Q_j' = \Delta Q_j^{(t-2)}$, $\Delta Q_j^* = \Delta Q_j^{(t-1)}$, $\lambda_j' = \lambda_j^{(t-2)}$, $\lambda_j^* = \lambda_j^{(t-1)}$;

if the first change in the sign of the residual ΔQ_j occurred earlier - before the t - l th iteration, where the residual and the factor had the values of $\Delta Q_j^{(l)}$ and $\lambda_j^{(l)}$, then in the case of $\Delta Q_j^{(t-1)} \cdot \Delta Q_j^{(l)} < 0$ it is taken $\Delta Q_j' = \Delta Q_j^{(l)}$, $\Delta Q_j^* = \Delta Q_j^{(t-1)}$, $\lambda_j' = \lambda_j^{(l)}$, $\lambda_j^* = \lambda_j^{(t-1)}$;

Table 1. Daily load schedule of power system.

<i>t, h.</i>	1	2	3	4	5	6	7	8	9	10	11	12
<i>P_L, MW</i>	2850	2750	2700	2750	2850	2900	3000	3100	3150	3200	3250	3200
<i>t, h.</i>	13	14	15	16	17	18	19	20	21	22	23	24
<i>P_L, MW</i>	3000	2950	3050	3100	3250	3400	3500	3600	3550	3400	3250	3000

Table 2. CRI of equivalent fuel consumption at TPP.

ТЭС-1 <i>B₁^{min} = 250,5</i> <i>t.f.e./h.</i>	<i>P_{T1}, MW</i>	750	1200	1350	1355	1500				
	<i>b₁, $\frac{t.f.e.}{MW \cdot h}$</i>	0.295	0.296	0.297	0.3	3.3001				
ТЭС-2 <i>B₂^{max} = 114,8</i> <i>t.f.e./h.</i>	<i>P_{T2}, MW</i>	400	565	583	595	647	715	759	790	
	<i>b₂, $\frac{t.f.e.}{MW \cdot h}$</i>	0.2705	0.2836	0.2967	0.3098	0.3229	0.336	0.3491	0.3622	

Table 3. Characteristics of the relative sprouting of water discharge of HPP.

HPP-1, <i>Q₁^{min} =</i> <i>104 m³/s.</i> <i>Q_{1av.day} =</i> <i>570 m³/s.</i>	<i>P_{T1}, MW</i>	0	323	579	582	586	589	592	595	598
	<i>q₁, $\frac{m^3}{MW \cdot h}$</i>	1.128	1.1368	1.1456	1.1544	1.1632	1.172	1.1808	1.1896	1.1984
HPP-2, <i>Q₂^{min} =</i> <i>20 m³/s.</i> <i>Q_{2av.day} =</i> <i>200 m³/s.</i>	<i>P_{T2}, MW</i>	0	100	200	300	400	500	600	800	1000
	<i>q₂, $\frac{m^3}{MW \cdot h}$</i>	0.2	0.3	0.4	0.5	0.7	1.0	2.0	4.3	8.0
HPP-3, <i>Q₃^{min} =</i> <i>30 m³/s.</i> <i>Q_{3av.day} =</i> <i>160 m³/s.</i>	<i>P_{T3}, MW</i>	0	160	180	196	390	400	440	480	
	<i>q₃, $\frac{m^3}{MW \cdot h}$</i>	0.1	0.14	0.21	0.42	0.47	0.55	1.27	4.03	

in the case of $\Delta Q_j^{(t-1)} \cdot \Delta Q_j^{(l)} > 0$ it is taken $\Delta Q_j^* = \Delta Q_j^{(t-1)}$, $\lambda_j^* = \lambda_j^{(t-1)}$ is taken, and the values of $\Delta Q_j'$, λ_j' remain unchanged - such as when performing previous iteration.

The minimization of function (7) subject to constraints (2) - (5) is carried out by the methods proposed in [17, 18].

3 Results and discussion

The efficiency of the described algorithm was researched, in particular, on the example of the optimal coverage of the load graph of the power system, given in Table 1, by two TPPs and three HPPs. The characteristics of relative increments (CRI) of the equivalent fuel consumption in TPPs and water consumption in HPPs are given in Tables 2 and 3, respectively. In these tables B_i^{min} is the consumption of equivalent fuel in the i th TPP at its minimum load; Q_j^{min} is a water consumption in the j th HPP at its minimum load; $Q_{j,av.day}$ is the given average daily water consumption in the j th HPP. It is assumed that the given CRIs correspond to the optimal compositions of the operating units in the corresponding plants during the day.

To compare the optimization results according to the proposed algorithm, Table 4 shows the optimal result obtained by the classical method based on the equality of the relative increases in fuel costs, with the

determination of the factors λ_j by a simple selection. We accept this result as a reference. In this case, the optimal result was obtained in 1385 iterations.

Table 4. Reference optimum result.

$t, h.$	P_L, MW	Optimal loads of power plant, MW				
		TPP-1	TPP-2	HPP-1	HPP-2	HPP-3
1	2850	1140.39	581,86	294,72	417,97	415,06
2	2750	1064.54	581,62	270,97	417,83	415,04
3	2700	1026.62	581,51	259,09	417,76	415,03
4	2750	1064.54	581,62	270,97	417,83	415,04
5	2850	1140.39	581,86	294,72	417,97	415,06
6	2900	1178.31	581,97	305,60	418,04	415,08
7	3000	1239.43	582,40	344,74	418,31	415,13
8	3100	1296.23	582,92	387,04	418,63	415,18
9	3150	1324.66	583,12	408,21	418,79	415,21
10	3200	1350.08	583,32	432,38	418,97	415,25
11	3250	1350.80	583,72	480,83	419,34	415,31
12	3200	1350.08	583,32	432,38	418,97	415,25
13	3000	1239.43	582,40	344,74	418,31	415,13
14	2950	1211.02	582,14	323,59	418,15	415,10
15	3050	1267.83	582,66	365,89	418,47	415,16
16	3100	1296.23	582,92	387,04	418,63	415,18
17	3250	1350.80	583,72	480,83	419,34	415,31
18	3400	1395.56	586,05	581,18	421,51	415,71
19	3500	1495.33	586,11	581,27	421,56	415,72
20	3600	1500.00	634,59	608,38	438,28	418,75
21	3550	1500.00	603,22	597,68	431,57	417,53
22	3400	1395.56	586,05	581,18	421,51	415,71
23	3250	1350.80	583,72	480,83	419,34	415,31
24	3000	1239.43	582,40	344,74	418,31	415,12
Lagrange multipliers and average daily water consumption in HPP						
HPP		λ_j			$Q_{c.av.day}, m^3/s.$	
HPP-1		0.2604			570.03	
HPP-2		0.3924			199.80	
HPP-3		0.3603			159.84	
Total consumption of equivalent fuel in thermal power plants per day: $B=13799.04$ t.f.e.						

When obtaining the reference result, the accuracy of the calculation for the discrepancies of the water flow rates in the HPP is taken as 0.1% of the given average daily water flow rate, i.e. $\varepsilon_{\Delta Q} = 0.1\%$.

The experimental calculations have shown that at large values of the step $\Delta\lambda_j$, in some cases, the result can be obtained in a smaller number of iterations, and in the general case, the process may not converge or looping. In particular, when calculating with $\varepsilon_{\Delta Q} = 0.1\%$ and a step of $\Delta\lambda_i = 0.00005$, the calculation process did not

converge to a solution - it looped, and at $\Delta\lambda_i = 0.00002$ it was ensured with the accepted accuracy.

Table 5 shows the λ_j factors, the discrepancy between the average daily water consumption at the HPP ΔQ_j and the total consumption of the equivalent fuel in the TPP per day B , obtained as a result of the optimal coverage of daily load graph of the power system by the methods described in [17, 18] with the determination of values of λ_j multipliers proposed here algorithm, where the optimal results are obtained in 3-4 iterations.

Table 5. The results of optimal coverage of daily load schedule of power systems with the choice of λ_j by the combined algorithm.

$\varepsilon_{\Delta Q}, \%$ from Q_j	Number of iterations	HPP-1		HPP-2		ГЭС-3		B t.f.e.
		λ_1	$\Delta Q_1, m^3/s.$	λ_2	$\Delta Q_2, m^3/s.$	λ_3	$\Delta Q_3, m^3/s$	
1	3	0.26048	-2.55	0.39390	-0.89	0.35775	0.12	13809.69
0.5	3	0.26048	-2.38	0.39390	-0.89	0.35798	0.09	13808.86
0.1	4	0.26045	-0.46	0.39239	-0.18	0.35793	0.09	13799.76

To analyze the convergence of the iterative optimization process, Table 6 shows the results obtained in iterations

when choosing the multipliers by the proposed algorithm.

Table 6. Changes in λ_j multipliers and discrepancies in water flow rates during iterations.

Number of iteration	HPP-1		HPP-2		HPP-3	
	λ_1	$\Delta Q_1, m^3/s.$	λ_2	$\Delta Q_2, m^3/s.$	λ_3	$\Delta Q_3, m^3/s$
0	0.2602	18.4	0.3939	-0.9295	0.3880	-2.787
1	0.2702	-232.202	0.3839	8.883	0.3780	-1.0877
2	0.26093	-29.922	0.39295	-0.268	0.37160	-1.267
3	0.26048	-2.579	0.39269	-0.307	0.35794	0.0967
4	0.26045	-0.63	0.39239	-0.1759	0.35794	0.0941

The results obtained allow us to conclude that the proposed algorithm for choosing the values of the factors λ_j provides fast and reliable convergence of the iterative computational process.

It should be noted that in the problem under consideration, it is also important to correctly choose the permissible discrepancy of the water flow rate in the HPP (the accuracy of the calculation based on the daily water consumption in the HPP) $\epsilon_{\Delta Q}$.

To study this issue, the optimization of the daily mode of the hydrothermal power system (in the example under consideration) was carried out at various values of $\epsilon_{\Delta Q}$. Based on the results obtained, it has been established that for large values of $\epsilon_{\Delta Q}$, the result can be obtained with lower total daily consumption of equivalent fuel in TPPs and large excessive consumption of water in HPPs. Such modes cannot be considered optimal, since the deviation of water flow rates in hydroelectric power plants from its preset values by large values violates the system-wide optimality due to the violation of the optimal regimes of reservoirs determined in advance in long-term planning. In this regard, for the permissible maximum values of $\epsilon_{\Delta Q}$, we can take those values at which the deviation of the total daily consumption of equivalent fuel in TPP from their values in the reference result does not exceed 0.1%.

4 Conclusion

1. An algorithm for the optimal planning of short-term modes of hydrothermal power systems which characterized by effective calculation of indefinite Lagrange multipliers, which are costly or fuel equivalent of water consumption in hydroelectric power plants is proposed.
2. Calculation of indefinite Lagrange multipliers for the functions of imbalances of water flow rates on the basis of the proposed combined method allows to obtain an optimal solution of the problem with sufficient accuracy rather reliably and in a small number of iterations.
3. The proposed algorithm can be used by dispatching services for optimal planning of short-term modes of hydrothermal power systems.

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