Hidden Markov Model of System Elements Technical Maintenance by Age

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Abstract. Technical maintenance is between the methods of operation reliability and effectiveness increasing for systems of different purposes including power systems. In the paper the hidden semi-Markov model of technical maintenance is built basing on the semi-Markov model of two-component system elements technical maintenance by age. The hidden Markov model is used to solve the problems of dynamics analyzing, predicting the states of a system modelled based on the vector of signals obtained during its operation.

Introduction

Technical maintenance (TM) is between the methods of operation reliability and effectiveness increasing for systems of different purposes including power systems [1 – 4]. If it is used in the system, a preemptive recovery of the parameters of the system elements is carried out according to certain rules in order to reduce the probability of system failures and maintain the efficiency of its operation. Semi-Markov processes with a discrete or discrete-continuous set of states are widely used to model systems [5–8]. In the process of functioning of the system for which the semi-Markov model is built, it is necessary to assess how the constructed model agrees with the data obtained during the functioning of the system, to refine it, to analyze the functioning of the system and to predict its state based on the information received. Hidden Markov models and hidden semi-Markov models can be used to solve these problems [9–12]. In this paper, a hidden Markov model of maintenance by age of elements of a two-component system is constructed. Using a hidden model, the tasks of analyzing of the dynamics, predicting of the states of the system, based on the signal vector obtained during its operation, are solved.

1 Description of the system

Let us consider a two-element system. Let us describe the maintenance strategy by the age of its elements [4]. At the initial instant the operation of the system begins and the admissible operating time (age) h_i, i = 1,2 of each i-th element of the system is being assigned, upon reaching of which its scheduled maintenance is to be carried out. The time to failure of the i-th element of the system is a random variable (RV) - α_i with a distribution function (DF) $F_i(t)$, i = 1,2. If by the appointed time h_i the i-th element of the system has not failed, then the scheduled maintenance of the element begins, which completely renews it. The duration of this maintenance is a RV τ_i with DF $R_i(t)$, i = 1,2. If the i-th element of the system fails before the predefined moment h_i, then the failure is detected instantly and its emergency recovery (ER) begins. The duration of this restoration work is RV β_i with DF $G_i(t)$, i = 1.2. The ER also results in that element is being also completely updated and the entire maintenance process is being repeated a new. It is assumed that the second element of the system does not turn off as a result of emergency failure or in connection with maintenance start of any element.

2 Construction of a semi-Markov model of the system

To describe the functioning of the system, let us use the semi-Markov process (SMP) $\xi(t)$. Let us introduce a space of states of the form

$$E=\{1,\ i\bar{d}\bar{x}:\bar{d}=(d_1,d_2),\ \bar{x}=(x_1,x_2),\ x_k>0, k=1,2\}, \quad (1)$$

where i = 1,2 indicates the number of the element in which the change in physical state has occurred. The d_k component of the vector describes the physical state of the element with the number k:

$$d_k=\begin{cases}1, \text{if } k\text{ - th element has resumed after maintenance or ER};\\ 0, \text{if } k\text{ - th element has failed, and its ER has begun;}\\ 2, if\ k\text{ - th element had worked for time } h_k \text{ without}\\ \ \text{failure and its maintenance has begun.}\end{cases}$$

The continuous component x_k of the vector indicates the time elapsed since the last change in the physical state of the element with the number k, k = 1,2; note that x_i = 0.The timing diagram of the described system operation is shown in Fig. 1.

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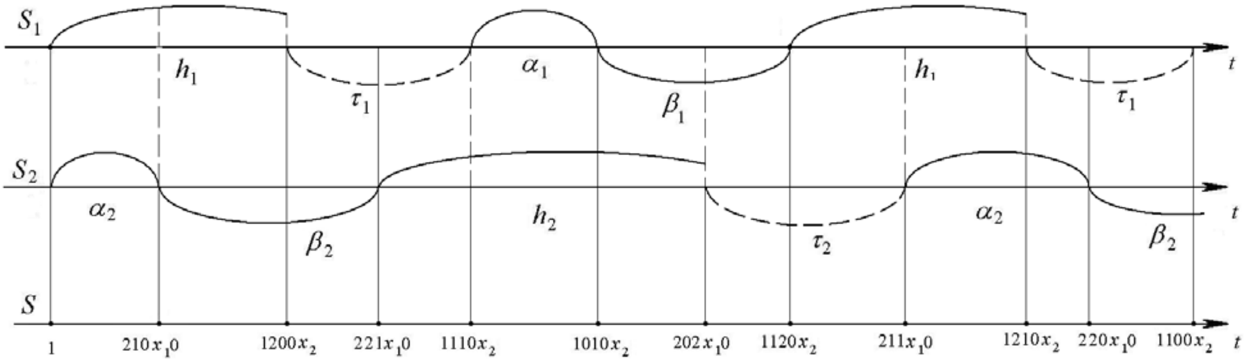

Fig. 1. Timing diagram of the two-component system operation with a maintenance strategy based on the age of elements.

In expanded form, the phase space of system states is:

$E = \{1, 1110x_2, 1100x_2, 1120x_2, 1010x_2, 1000x_2, 1020x_2, 1210x_2, 1200x_2, 1220x_2, 211x_10, 201x_10, 221x_10, 210x_10, 200x_10, 220x_10, 212x_10, 202x_10, 22x_10\}$.

Let us find the sojourn times in the states of the system. To do this, let us introduce RV:

$$\delta_z^{(k)} = \begin{cases} \alpha_k \wedge h_k, \text{ if } z = 1, \\ \beta_k, \text{ if } z = 0, \\ \tau_k, \text{if } z = 2, \ \text{k} = 1{,}2, \end{cases}$$

where $\wedge$ is the minimum sign. RV $\delta_z^{(k)}$ are the sojourn times in the states of the SMP $\xi_k(t)$, which describes the operation of the k-th element of the system, $k = 1{,}2$.

Let us define $V_z^{(k)}(t) = P(\delta_z^{(k)} < t)$ as DF of the RV $\delta_z^{(k)}$, $\overline{V}_z^{(k)}(t) = 1 - V_z^{(k)}(t)$, $v_z^{(k)}(t)$ asdistribution densities (DD) of the RV $\delta_z^{(k)}$, then

$$\overline{V}_1^{(k)}(t) = \overline{F}_k(t)\cdot 1(h_k - t), \overline{V}_0^{(k)}(t) = \overline{G}_k(t), \overline{V}_2^{(k)}(t) = \overline{R}_k(t),$$
$$v_1^{(k)}(t) = f_k(t)\cdot 1(h_k - t),\ v_0^{(k)}(t) = g_k(t),\ v_2^{(k)}(t) = r_k(t),$$

k= 1,2.

Then sojourn time in the state $i\overline{dx}$ is defined by the equality

$$\theta_{i\overline{dx}} = \mathop{\wedge}_{k=1}^{2}\left[\delta_{d_k}^{(k)} - x_k\right]^+, x_i = 0.$$

Let us determine the probabilities the EMC $\{\xi_n;\ n \geq 0\}$ transitions for the states $1000x_2$, $210x_10$, $1020x_2$, for other states they are defined similarly:

$$p_{1000x_2}^{1100y_2} = \frac{g_1(y_2 - x_2)\cdot \overline{G}_2(y_2)}{\overline{G}_2(x_2)}, y_2 > x_2,$$

$$p_{1000x_2}^{201y_10} = \frac{g_2(x_2 + y_1)\cdot \overline{G}_1(y_1)}{\overline{G}_2(x_2)}, y_1 > 0,$$

$$P_{210x_10}^{1200h_1 - x_1} = \frac{\overline{F}_1(h_1)\cdot \overline{G}_2(h_1 - x_1)}{\overline{F}_1(x_1)},$$

$$p_{210x_10}^{1000y_2} = \frac{f_1(x_1 + y_2)\cdot \overline{G}_2(y_2)}{\overline{G}_2(x_2)}, 0 < y_2 < h_1 - x_1,$$

$$p_{210x_10}^{211y_10} = \frac{g_2(y_1 - x_1)\cdot \overline{F}_1(y_1)}{\overline{F}_1(x_1)}, x_1 < y_1 < h,$$

$$p_{1020x_2}^{1120y_2} = \frac{g_1(y_2 - x_2)\cdot \overline{R}_2(y_2)}{\overline{R}_2(x_2)}, y_2 > x_2,$$

$$p_{1020x_2}^{201y_10} = \frac{r_2(x_2 + y_1)\cdot \overline{G}_1(y_1)}{\overline{R}_2(x_2)}, y_1 > 0.$$

Let us find the stationary distribution of the constructed SMP $\xi(t)$ that represents the superposition of the independent SMP $\xi_1(t)$, $\xi_2(t)$ describing the operation of the system's elements.

In accordance with [8], the stationary distribution of n independent SMP superposition EMC is determined by the formula:

$$\rho(dz_1,\ldots,dz_n; x^{(k)}) = c\rho_k(dz)\prod_{i \neq k}\rho_i(dz_i)\cdot \overline{F}_{z_i}(x_i), \quad (2)$$

where $F_{z_i}(x_i)$ is a DF of SMP $\xi^{(i)}(t)$ in the state z_i, $\overline{F}_{z_i}(x_i) = 1 - F_{z_i}(x_i)$.

Let us introduce the notation

$$\rho_z^{(k)} = \begin{cases} \rho, \text{if } z = 1, \\ \rho \cdot F_k(h_k), \text{if } z = 0, \\ \rho \cdot \overline{F}_k(h_k), \text{if } z = 2, \rho = \dfrac{1}{2}, k = 1{,}2, \end{cases}$$

$\left\{\rho_z^{(k)}\right\}$ is an EMC stationary distribution for the SMP $\xi_k(t)$, describing the operation of k-th element of the system. Using (2) let us obtain that SMP $\xi(t)$ EMC stationary distribution takes form:

$$\rho\left(i\overline{dx}\right)=c\cdot\rho_{d_i}^{(i)}\cdot\prod_{\substack{k=1\\k\neq i}}^{2}\rho_{d_k}^{(k)}\cdot\overline{V}_{d_k}^{(k)}(x_k), \tag{3}$$

where the constant c is being found from the normalization requirement.

E.g., using (3) let us obtain

$$\rho(1110x_2)=c\cdot\rho^2\cdot\overline{F}_2(x_2)\cdot 1(h_2-x_2),$$

$$\rho(200x_2 0)=c\cdot\rho^2\cdot F_1(h_1)F_2(h_2)\cdot\overline{G}_1(x_1).$$

3 Example of electric power system modelling

To simplify the model of the system let us merge the constructed semi-Markov model using the stationary phase merging algorithm proposed in [5, 6]. Phase space of states of the initial model is being split into $N = 9$ classes

E_{00}={$1000x_2$,$200x_1 0$}, E_{11}={1,$1110x_2$, $211x_1 0$},

E_{22}={$1220x_2$,$222x_1 0$},E_{21}={$1210x_2$,$221x_1 0$},

E_{12}={$1120x_2$,$212x_1 0$}, E_{10}={$1100x_2$,$210x_1 0$},

E_{01}={$1010x_2$,$221x_1 0$}, E_{02}={$1000x_2$,$200x_1 0$},

E_{20}={$1200x_2$,$220x_1 0$},

each of which is "glued" into one state of the merged model.

Phase space of states of the merged model takes form

E = {00, 11, 22, 21, 12, 10, 01, 02, 20}.

It is not difficult to establish the physical meaning of the merged model states. Thus the state 21 means that the first element is on maintenance, the second is operational.

Let us determine the merged model EMC transitions probabilities that, in accordance with [5], can be found by the formula

$$\hat{p}_k^r=\int_E \rho(de)P(e,E_r)/\rho(E_k), \qquad k,r=\overline{1,N}, \tag{4}$$

where $\rho(de)$ is the EMC stationary distribution, $P(e,E_r)$ are the merging model EMC transition probabilities.

Let us use (4) to find transition probabilities $\hat{p}_k^r$ of the merged model. Thus, for the states 00, 11, 12 they take the next form:

$$\hat{P}_{00}^{01}=\frac{M\beta_1}{M\beta_1+M\beta_2}, \quad \hat{P}_{00}^{10}=\frac{M\beta_2}{M\beta_1+M\beta_2},$$

$$\hat{P}_{11}^{12}=\frac{\overline{F}_2(h_2)\cdot\int_0^{h_1}\overline{F}_1(x)dx}{\int_0^{h_1}\overline{F}_1(t)dt+\int_0^{h_2}\overline{F}_2(t)dt}, \quad \hat{P}_{11}^{21}=\frac{\overline{F}_1(h_1)\cdot\int_0^{h_2}\overline{F}_2(x)dx}{\int_0^{h_1}\overline{F}_1(t)dt+\int_0^{h_2}\overline{F}_2(t)dt}$$

$$\hat{P}_{11}^{01}=\frac{F_1(h_1)\cdot\int_0^{h_2}\overline{F}_2(x)dx}{\int_0^{h_1}\overline{F}_1(t)dt+\int_0^{h_2}\overline{F}_2(t)dt}, \quad \hat{P}_{11}^{10}=\frac{F_2(h_2)\cdot\int_0^{h_1}\overline{F}_1(x)dx}{\int_0^{h_1}\overline{F}_1(t)dt+\int_0^{h_2}\overline{F}_2(t)dt},$$

$$\hat{P}_{22}^{21}=\frac{M\tau_1}{M\tau_1+M\tau_2}, \quad \hat{P}_{22}^{12}=\frac{M\tau_2}{M\tau_1+M\tau_2},$$

$$\hat{P}_{21}^{22}=\frac{\overline{F}_2(h_2)\cdot M\tau_1}{M\tau_1+\int_0^{h_2}\overline{F}_2(t)dt}, \quad \hat{P}_{21}^{20}=\frac{F_2(h_2)\cdot M\tau_1}{M\tau_1+\int_0^{h_2}\overline{F}_2(t)dt},$$

$$\hat{P}_{21}^{11}=\frac{\int_0^{h_2}\overline{F}_2(x)dx}{M\tau_1+\int_0^{h_2}\overline{F}_2(t)dt}, \quad \hat{P}_{12}^{02}=\frac{F_1(h_1)\cdot M\tau_2}{M\tau_2+\int_0^{h_1}\overline{F}_1(t)dt},$$

$$\hat{P}_{12}^{11}=\frac{\int_0^{h_1}\overline{F}_1(x)dx}{M\tau_2+\int_0^{h_1}\overline{F}_1(t)dt}, \quad \hat{P}_{12}^{22}=\frac{\overline{F}_1(h_1)\cdot M\tau_2}{M\tau_2+\int_0^{h_1}\overline{F}_1(t)dt},$$

$$\hat{P}_{10}^{20}=\frac{\overline{F}_1(h_1)\cdot M\beta_2}{M\beta_2+\int_0^{h_1}\overline{F}_1(t)dt}, \quad \hat{P}_{10}^{00}=\frac{F_1(h_1)\cdot M\beta_2}{M\beta_2+\int_0^{h_1}\overline{F}_1(t)dt},$$

$$\hat{P}_{10}^{11}=\frac{\int_0^{h_1}\overline{F}_1(x)dx}{M\beta_2+\int_0^{h_1}\overline{F}_1(t)dt}, \quad \hat{P}_{01}^{02}=\frac{\overline{F}_2(h_2)\cdot M\beta_1}{M\beta_1+\int_0^{h_2}\overline{F}_2(t)dt},$$

$$\hat{P}_{01}^{00}=\frac{F_2(h_2)\cdot M\beta_1}{M\beta_1+\int_0^{h_2}\overline{F}_2(t)dt}, \quad \hat{P}_{01}^{11}=\frac{\int_0^{h_2}\overline{F}_2(x)dx}{M\beta_1+\int_0^{h_2}\overline{F}_2(t)dt},$$

$$\hat{P}_{02}^{01}=\frac{M\beta_1}{M\beta_1+M\tau_2}, \quad \hat{P}_{02}^{12}=\frac{M\tau_2}{M\beta_1+M\tau_2},$$

$$\hat{P}_{20}^{10}=\frac{M\beta_1}{M\beta_2+M\tau_1}, \quad \hat{P}_{20}^{21}=\frac{M\tau_1}{M\beta_2+M\tau_1}. \tag{5}$$

4 Hidden Markov model based on merged semi-Markov model

Let $\{X_n, n=1,2,\ldots\}$ be the merged model EMC, the transition probabilities of which are defined by the formulas (5).

Let us suppose that during the system S operation the merged model EMC states are not observed (hidden states), but only the number of operable components of the system is observed at the instant of the system transition to a new state.

Hence, the set of the signals takes form:

J = {0, 1, 2}.

 https://doi.org/10.1051/e3sconf/202021601030

Let us consider the connection between the merged system EMC states and the signals, i.e. define the function $R(S|\boldsymbol{x})$ [9]:

$$R(s\,|\,\boldsymbol{x}) = P(S_n = s\,|\,X_n = \boldsymbol{x}), \boldsymbol{x} \in \hat{E}, s \in J, \sum_{s\in J} R(s\,|\,\boldsymbol{x}) = 1 \quad (6)$$

where S_n is the n-th signal.

The connection function between the states of the merged model EMC and the signals is represented in the Table 1.

5 Solving of the problems of Hidden Markov model theory

Following [9, 10], let us consider the main problems of the theory of hidden Markov models in relation to the constructed hidden Markov model.

Let $\bar{S}^n = (S_1, S_2, \ldots, S_n)$ be a random vector of the first n signals. For the vector of the signals given $\bar{s}_n = (s_1, s_2, \ldots, s_n)$ let it be $\bar{s}_n = (s_1, s_2, \ldots, s_n)$ $k \le n$.

It is required to evaluate the EMC characteristics of of the merged (hidden) model based on the vector of signals $\bar{s}_n$. It is supposed that at the initial instant the system was in the state 11.

Let us introduce the functions $F_k(i)$ [9]:

$$F_k(i) = P(S^k = \bar{s}_k, X_k = i), k = 1, 2, \ldots, n, \quad (7)$$

called forward variables. For these functions the next recurrent formula is true

$$F_k(i) = R(s_k\,|\,i)\sum_j F_{k-1}(j)P_j^i,$$

$$F_1(i) = R(s_1\,|\,i)p_i, \quad (8)$$

where P^i_j are the merged model EMC transition probabilities defined by the formulas (5), (p_i) is the EMC initial state distribution.

Using the formula (8) let us find three first functions $F_k(i)$:

$$F_1(i) = \begin{cases} 0, i \ne 11, \\ R(s_1\,|\,11), i = 11, \end{cases}$$

$$F_2(i) = \begin{cases} R(s_2\,|\,21)R(s_1\,|\,11)P_{11}^{21}, i = 21, \\ R(s_2\,|\,12)R(s_1\,|\,11)P_{11}^{12}, i = 12, \\ R(s_2\,|\,10)R(s_1\,|\,11)P_{11}^{10}, i = 10, \\ R(s_2\,|\,01)R(s_1\,|\,11)P_{11}^{01}, i = 01, \\ 0, \text{for the rest of the states}, \end{cases}$$

$$F_3(i) = \begin{cases} R(s_3\,|\,00)R(s_1\,|\,11)\left[R(s_2\,|\,10)P_{11}^{10}P_{10}^{00} + R(s_2\,|\,01)P_{11}^{01}P_{01}^{00}\right], i = 00, \\ R(s_3\,|\,11)R(s_1\,|\,11)\left[R(s_2\,|\,21)P_{11}^{21}P_{21}^{11} + R(s_2\,|\,12)P_{11}^{12}P_{12}^{11} + \right. \\ \left. + R(s_2\,|\,10)P_{11}^{10}P_{10}^{11} + R(s_2\,|\,01)P_{11}^{01}P_{01}^{11}\right], i = 11, \\ R(s_3\,|\,22)R(s_1\,|\,11)\left[R(s_2\,|\,21)P_{11}^{21}P_{21}^{22} + R(s_2\,|\,12)P_{11}^{12}P_{12}^{22}\right], i = 22, \\ R(s_3\,|\,02)R(s_1\,|\,11)\left[R(s_2\,|\,10)P_{11}^{01}P_{01}^{02} + R(s_2\,|\,12)P_{11}^{12}P_{12}^{02}\right], i = 02, \\ R(s_3\,|\,20)R(s_1\,|\,11)\left[R(s_2\,|\,21)P_{11}^{21}P_{21}^{20} + R(s_2\,|\,10)P_{11}^{10}P_{10}^{20}\right], i = 20, \\ 0, \text{for the rest of the states.} \end{cases}$$

The functions $F_4(i)$, $F_5(i)$ are being found similarly using the recurrent formula (8).

Other functions applied for hidden model characteristics estimate are the functions $B_k(i)$ called backward variables [9,10]

$$B_k(i) = P(S_{k=1} = s_{k=1}, \ldots, S_n = s_n\,|\,X_k = i), k = \overline{1, n-1},$$

for which the next recurrent formula takes place

$$B_k(i) = \sum_j R(s_{k+1}\,|\,j)B_{k+1}(j)P_i^j,$$

$$B_{n-1}(i) = \sum_j P_i^j R(s_n\,|\,j). \quad (9)$$

For the probability $P(\bar{S}^n = \bar{s}_n)$ the next formulas are true

$$P(\bar{S}^n = \bar{s}_n) = \sum_i F_n(i) = \sum_i R(s_1\,|\,i)B_1(i)p_i, \quad (10)$$

and also [9]

$$P(\bar{S}^n = \bar{s}_n) = \sum_i F_k(i)B_k(i), \quad (11)$$

for any fixed k.

As an example of hidden merged model characteristics estimates, let us consider a system S, in which the uptime of the first and second components have a second-order Erlang distribution with average values $E\alpha_1$ = 20.0 h, $E\alpha_2$ = 25.0 h. Average values of scheduled maintenance $E\tau_1$ and $E\tau_2$ are 2.5 and 3.0 hours, the average ER values $E\beta_1$ and $E\beta_2$ are 4.5 and 6.7 hours, respectively. Nonrandom values of the permissible operating time levels are 6.0 and 10.0 hours, respectively.

Let the next vector of signals be defined (2, 1, 0, 1 , 2, 1, 2, 1, 2, 1, 0, 1, 2, 1, 2, 1, 2, 1, 2, 1,2, 1, 2, 1, 2, 1, 0, 1, 0, 1) (n= 30). Let us consider the next problems on hidden model characteristics evaluation.

Table 1. The connection function $R(S|j)$ between the merged model EMC states and the signals.

State, $\boldsymbol{x}$ / Signal, s	00	11	22	21	12	10	01	02	20
s=0	1	0	1	0	0	0	0	1	1
s=1	0	0	0	1	1	1	1	0	0
s=2	0	1	0	0	0	0	0	0	0

1. Let us determine the probabilities of the states of the hidden model at the moment of emission of the 30th signal. Let us use the formula [9]:

$$P\left(X_n = i \mid \bar{S}^n = \bar{s}_n\right) = \frac{F_n(i)}{\sum_j F_n(j)}. \qquad (12)$$

Then, at the 30th step, the model was in state 21 with a probability of 0.3277, in state 12 with a probability of 0.3489, in state 10 with a probability of 0.2332, and in state 01 with a probability of 0.0901. For other states, this probability is zero.

2. Let us find the probabilities with which the hidden model will perform the transition to the states at the next step. For this let us use the formula [9]:

$$P(X_{n+1} = j \mid \bar{s}_n) = \sum_i P(X_n = i \mid \bar{s}_n) P_i^j, \qquad (13)$$

We get the following probabilities of the hidden model transition at the 31st step: to state 00 with a probability of 0.0210; in state 11 - 0.6554; in state 22 - 0.1615; to state 21 - 0; to state 12 - 0; to state 10 - 0; to state 01 - 0; in state 02 - 0.0384; to state 20 - 0.1237.

3. Let us determine the probability of the appearance of signals at the next step, with the help of the formula [9]:

$$P\left(S_{n+1} = s_{n+1} \mid \bar{s}_n\right) = \sum_i P\left(X_{n+1} = i \mid \bar{s}_n\right) R\left(s_{n+1} \mid i\right), \qquad (14)$$

in this case the formula (13) is used.

We get that the probability of signal 2 appearing at the 31st step is 0.6554; signal 1 - 0; signal 0 - 0.3446.

4. Let us find the probability of appearance (emission) of a given vector of signals.

To do this, you can use formulas (10) - (11).

The occurrence probability of a given vector of signals (2, 1, 0, 1, 2, 1, 2, 1, 2, 1, 0, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 0, 1, 0, 1) with the initial parameters of the system S is equal to 0.00023.

Table 2. The probability of the appearance of a given vector of signals at various permissible operating time levels.

1	2	3
h_1	h_2	$P(\bar{S}^n = \bar{s}_n)$
0	0	0
0	14.0	0
2.0	8.0	0.0001944
2.0	12.0	0.0002098
4.0	4.0	0.0001335
6.0	4.0	0.0001800
6.0	10.0	0.0002341
8.0	8.0	0.0002318
10.0	10.0	0.0002303
10.0	8.0	0.0002323
12.0	12.0	0.0002222
50.0	0	0
100	100	0.0001731

Table 2 shows the probability of occurrence of a given vector of signals depending on the value of the operating time.

Table 2 shows that with an increase in the permissible operating time levels, the probability of the appearance of a given signal vector first increases, then decreases, which makes it possible to pose the problem of finding the point $\{h_1, h_2\}$ = argmax $P\left(\bar{S}^n = \bar{s}_n\right)$. The values of one or two levels equal to zero represent the appearance of the signal chain under consideration as an impossible event, since in the limit at $h_i \to 0$ one or two elements are constantly on scheduled maintenance, making, in particular, the appearance of signal 2 impossible, while such a signal is present in the chain. In the limit at $h_i \to \infty$, i = 1.2, the planned maintenance is not carried out and the hidden model turns into the hidden Markov model considered in [12] based on the superposition of two independent alternating renewal processes [5].

5. Predicting the states of the hidden model for a given vector of signals.

Table 3. The most probable states of the hidden model on its transitions.

Transition #	1	2	3	4	5	6	7
The most probable state	11	21	22	12	11	21	11
The probability of the state	1	0.5427	0.6532	0.4024	1	0.5427	1

The Viterbi algorithm [9, 10] was used to find the most probable chain of states (Table 3).

Using the Baum-Welsh algorithm [10], you can select the parameters of the hidden model (conduct training) so that it most closely matches the given signal vector.

Trellis diagram of merged model operation is depicted in the Fig. 2 using for simplicity the next notation: $00 \leftrightarrow 1$, $11 \leftrightarrow 2$, $22 \leftrightarrow 3$, $21 \leftrightarrow 4$, $12 \leftrightarrow 5$, $10 \leftrightarrow 6$, $01 \leftrightarrow 7$, $02 \leftrightarrow 8$, $20 \leftrightarrow 9$. The thick line shows the most likely transitions.

Conclusion

In this paper, a hidden Markov model of maintenance by age of elements of a two-component system is constructed. The constructed hidden model is used to solve the problems of analyzing of the dynamics, predicting of the states of the modeled system based on the vector of signals obtained during its operation.

In the future, it is planned to build hidden Markov models of multicomponent systems, taking maintenance into account.

The results of the work can be used to analyze the functioning and predict the states of systems in which maintenance is performed.

The work was supported by the Russian Foundation for Basic Research No. 18-01-00392a.

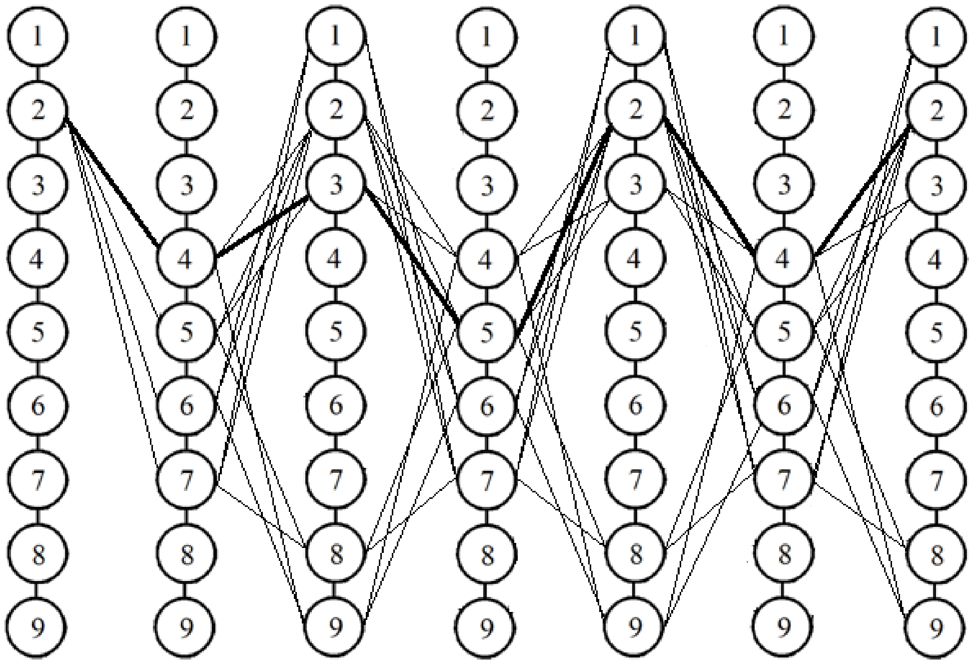

Fig. 2. The trellis of maximized transition probabilities for considered chain of signals.

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