 https://doi.org/10.1051/e3sconf/202122901019

Unknown Input Observer Design for a Class of Linear Descriptor Systems

Karim Bouassem[1,2,*], *Abdellatif* El Assoudi[1,2,**], *Jalal* Soulami[1,2,***], and *El Hassane* El Yaagoubi[1,2,****]

[1]Laboratory of High Energy Physics and Condensed Matter, Faculty of Science, Hassan II University of Casablanca. B.P 5366, Maarif, Casablanca, Morocco.
[2]ECPI, Department of Electrical Engineering, ENSEM, Hassan II University of Casablanca. B.P 8118, Oasis, Casablanca, Morocco.

Abstract. This paper addresses the problem of unknown inputs observer (UIO) design for a class of linear descriptor systems. The unknown inputs affect both state and output of the system. The basic idea of the proposed approach is based on the separation between dynamic and static relations in the descriptor model. Firstly, the method used to separate the differential part from the algebraic part is developed. Secondly, an observer design permitting the simultaneous estimation of the system state and the unknown inputs is proposed. The developed approach for the observer design is based on the synthesis of an augmented model which regroups the differential variables and unknown inputs. The exponential stability of the estimation error is studied using the Lyapunov theory and the stability condition is given in term of linear matrix inequality (LMI). Finally, to illustrate the efficiency of the proposed methodology, a heat exchanger pilot model is considered.

1 Introduction

Descriptor systems variously called implicit systems or singular systems or differential-algebraic equations (DAEs) have been widely used in the modeling of dynamic processes to describe the behavior of many chemical and physical processes. Known as a generalization of standard models, such descriptor systems constitute a powerful modeling tool allowing to describe the dynamic behaviour of processes. They represent physical phenomenas that can not be described by standard systems. We may cite [1–3] for some real applications of implicit models. The numerical simulation of such models usually combines an ODE numerical method together with an optimization algorithm.

The aim of the paper is the development of an UIO for a class of continuous-time linear descriptor systems. Notice that, the UIO design problem widely used in the area of fault detection and design of fault tolerant control strategy has received considerable attention and is still an active area of research. Indeed, many works on observer design and its application to fault detection for linear descriptor systems exist in the literature see e.g. [4–17].

The main contribution of this paper is to give an UIO design for a class of linear descriptor models allowing the simultaneous estimation of the unknown states and unknown inputs. A new design methodology through judicious use of the separation between the dynamic and static relations in the descriptor model is proposed. Based on the Lyapunov theory, the exponential stability condition of the UIO is given in term of LMI. Besides, for reasons of ease of the implementation, the main result of this paper consists in showing that the UIO problem for the considered class of linear descriptor systems can be achieved by using an observer having only an ODE structure.

This paper is organized as follows: The structure of the considered class of linear descriptor systems subject to unknown input is presented in section 2. The main result concerning the design of the proposed observer permitting to estimate simultaneously the unknown states and the unknown inputs is established in section 3. In section 4, we illustrate the performance of the proposed UIO in simulation through an electrical circuit descriptor model. Finally, some conclusions are drawn in section 5.

Throughout this paper we take up the following notations:

- X^T is the transpose of matrix X;
- $X > 0$ means that matrix X is symmetric and positive definite;
- The symbol I (or 0) represents the identity matrix (or zero matrix) with the appropriate dimension.

*e-mail: karim.bouassem@gmail.com
**e-mail: a.elassoudi@ensem.ac.ma
***e-mail: jalal.soulami@gmail.com
****e-mail: h.elyaagoubi@ensem.ac.ma

2 Linear descriptor systems with unknown input

In this paper, the following class of continuous-time linear descriptor systems with unknown inputs is considered:

$$\begin{cases} \Gamma\dot{\xi} &= A\xi + Bu + Ed \\ y &= C\xi + Du + Fd \end{cases} \quad (1)$$

where $\xi = [\xi^{1^T}\ \xi^{2^T}]^T \in \mathbf{R}^n$ is the state vector with $\xi^1 \in \mathbf{R}^{n_1}$ is the vector of differential variables, $\xi^2 \in \mathbf{R}^{n_2}$ is the vector of algebraic variables with $n_1 + n_2 = n$, $u \in \mathbf{R}^m$ is the control input, $d \in \mathbf{R}^q$ is the unknown control input, $y \in \mathbf{R}^p$ is the measured output.
$\Gamma \in \mathbf{R}^{n\times n}$ with $rank(\Gamma) = n_1$, $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{p\times n}$, $D \in \mathbf{R}^{p\times m}$, $E \in \mathbf{R}^{n\times q}$, $F \in \mathbf{R}^{p\times q}$ are all constant matrices with:

$$\begin{cases} A = \begin{pmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{pmatrix};\ B = \begin{pmatrix} B^1 \\ B^2 \end{pmatrix} \\ C = \begin{pmatrix} C^1 & C^2 \end{pmatrix};\ D = \begin{pmatrix} D^1 \\ D^2 \end{pmatrix};\ \Gamma = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \end{cases} \quad (2)$$

where constant matrix A^{22} is supposed invertible.

Assumption 1 *Suppose that [1]:*

- $(\Gamma,\ A)$ *is regular, i.e.* $det(s\Gamma - A) \neq 0\ \forall s \in \mathbf{C}$
- *The model (1) is impulse observable and detectable.*

In order to investigate the UIO design for linear descriptor system (1), the approach is based on the separation between difference and algebraic equations in the model (1). So, using (2), system (1) can be rewritten as follows:

$$\begin{cases} \dot{\xi}^1 &= A^{11}\xi^1 + A^{12}\xi^2 + B^1u + E^1d \\ 0 &= A^{21}\xi^1 + A^{22}\xi^2 + B^2u + E^2d \\ y &= C^1\xi^1 + C^2\xi^2 + Du + Fd \end{cases} \quad (3)$$

The form (3) for system (1) is also known as the second equivalent form [1].
From (3) and using the fact that $(A^{22})^{-1}$ exists, the algebraic equations can be solved directly for algebraic variables, to obtain:

$$\xi^2 = -(A^{22})^{-1}(A^{21}\xi^1 + B^2u + E^2d) \quad (4)$$

Substitution of the resulting expression of ξ^2 (equation (4)) in equation (3) yields the following model:

$$\begin{cases} \dot{\xi}^1 &= M\xi^1 + Nu + Pd \\ \xi^2 &= J\xi^1 + Ku + Ld \\ y &= R\xi^1 + Su + Td \end{cases} \quad (5)$$

where

$$\begin{cases} M &= A^{11} - A^{12}(A^{22})^{-1}A^{21} \\ N &= B^1 - A^{12}(A^{22})^{-1}B^2 \\ P &= E^1 - A^{12}(A^{22})^{-1}E^2 \\ J &= -(A^{22})^{-1}A^{21} \\ K &= -(A^{22})^{-1}B^2 \\ L &= -(A^{22})^{-1}E^2 \\ R &= C^1 - C^2(A^{22})^{-1}A^{21} \\ S &= D - C^2(A^{22})^{-1}B^2 \\ T &= F - C^2(A^{22})^{-1}E^2 \end{cases} \quad (6)$$

Assumption 2 *Suppose that d is considered as a constant unknown control input per time interval i.e.:*

$$\dot{d}(t) = 0 \quad t \in [T_1\ \ T_2] \quad \forall T_1,\ T_2 \in \mathbf{R}^+ \quad (7)$$

Let us define the augmented state vector:

$$\begin{cases} \eta^1 &= \begin{pmatrix} \xi^1 \\ d \end{pmatrix} \\ \eta^2 &= \xi^2 \end{cases} \quad (8)$$

Thus, the system (5) can be represented as:

$$\begin{cases} \dot{\eta}^1 &= \tilde{M}\eta^1 + \tilde{N}u \\ \eta^2 &= \tilde{J}\eta^1 + Ku \\ y &= \tilde{R}\eta^1 + Su \end{cases} \quad (9)$$

where

$$\begin{cases} \tilde{M} = \begin{pmatrix} M & P \\ 0 & 0 \end{pmatrix};\quad \tilde{N} = \begin{pmatrix} N \\ 0 \end{pmatrix} \\ \tilde{J} = \begin{pmatrix} J & L \end{pmatrix};\quad \tilde{R} = \begin{pmatrix} R & T \end{pmatrix} \end{cases} \quad (10)$$

3 Main result

Based on the transformation of the linear descriptor system (1) into the equivalent form (8), the proposed UIO permitting to estimate simultaneously the unmeasurable states and unknown inputs takes the following form:

$$\begin{cases} \dot{\hat{\eta}}^1 &= \tilde{M}\hat{\eta}^1 + \tilde{N}u - G(\hat{y} - y) \\ \hat{\eta}^2 &= \tilde{J}\hat{\eta}^1 + Ku \\ \hat{y} &= \tilde{R}\hat{\eta}^1 + Su \end{cases} \quad (11)$$

where $(\hat{\eta}^1,\ \hat{\eta}^2)$ and $\hat{y}$ denote the estimated augmented state vector and the output vector respectively. G is the gain of UIO which is determined such that $(\hat{\eta}^1,\ \hat{\eta}^2)$ asymptotically converges to $(\eta^1,\ \eta^2)$.
In order to establish the conditions for the asymptotic convergence of the observer (11), we define the state estimation error:

$$\varepsilon = \begin{pmatrix} \varepsilon^1 \\ \varepsilon^2 \end{pmatrix} = \begin{pmatrix} \hat{\eta}^1 - \eta^1 \\ \hat{\eta}^2 - \eta^2 \end{pmatrix} \quad (12)$$

It follows from (9) and (11) that the observer error dynamic is given by the differential-algebraic equation:

$$\begin{cases} \dot{\varepsilon}^1 &= \Omega\varepsilon^1 \\ \varepsilon^2 &= Q\varepsilon^1 \end{cases} \quad (13)$$

where

$$\Omega = \tilde{M} - G\tilde{R} \quad (14)$$

To prove the convergence of the estimation error ε toward zero, it suffices to prove from (13) that ε_1 converges toward zero. Then, the following result can be stated.

Theorem 1 *: There exists an UIO (11) for linear descriptor system (1) if given* $\alpha > 0$ *there exist matrices* $Q > 0$ *and W, verifying the following LMI:*

$$\tilde{M}^TQ + Q\tilde{M} - \tilde{R}^TW^T - W\tilde{R} + 2\alpha Q < 0 \quad (15)$$

The observer gain G is given by:

$$G = Q^{-1}W \quad (16)$$

Proof of Theorem 1 : Let us consider the following quadratic Lyapunov function as follows:

$$V(\varepsilon^1) = (\varepsilon^1)^T Q \varepsilon^1 , \quad Q > 0 \tag{17}$$

Estimation error convergence is exponentially ensured if the following condition is guaranteed:

$$\dot{V}(\varepsilon^1) = (\dot{\varepsilon}^1)^T Q \varepsilon^1 + \varepsilon^1 Q \dot{\varepsilon}^1 < -2\alpha V(\varepsilon^1) \qquad \alpha > 0 \tag{18}$$

By using (13), the condition (18) can be written as:

$$\dot{V}(\varepsilon^1) = (\varepsilon^1)^T (\Omega^T Q + Q\Omega)\varepsilon_1 < -2\alpha V(\varepsilon^1) \tag{19}$$

which is equivalent to the following stability condition:

$$\Omega^T Q + Q\Omega + 2\alpha Q < 0 \tag{20}$$

Letting $W = QG$, from (14) it follows that (20) is equivalent to (15). From the Lypunov stability theory, if the LMI condition (15) is satisfied, the error dynamic equation (13) is exponentially asymptotically stable.

4 Application to a heat exchanger plant

In order to demonstrate the effectiveness and applicability of the proposed UIO design approach (10), let us consider the heat exchanger process described by the following Figure 1 (see [18] for more detail):

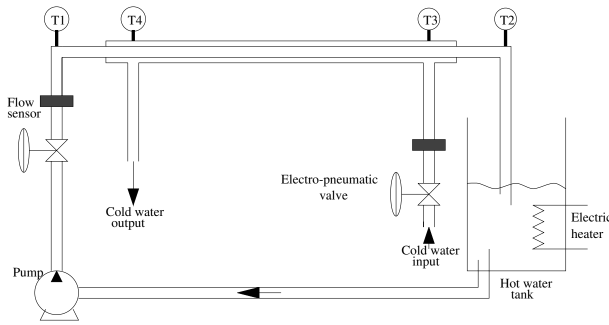

Figure 1. Heat exchanger process.

In this illustrative application the considered process is supposed described by a linear descriptor model obtained by tangent linearization around an operating point of the given nonlinear model in [18]. The considered linear descriptor model which is supposed to be affected by unknown input takes the following form:

$$\begin{cases} \Gamma\dot{\xi} &= A\xi + Bu + Ed \\ y &= Cx + Du + Fd \end{cases} \tag{21}$$

where $\xi = [\xi^{1^T} \ \xi^{2^T}]^T \in \mathbf{R}^8$ is the state vector with $\xi^1 = [\xi_1 \ \ldots \ \xi_6]^T \in \mathbf{R}^6$ is the vector of differential variables, $\xi^2 = [\xi_7 \ \ \xi_8]^T \in \mathbf{R}^2$ is the vector of algebraic variables. $u \in \mathbf{R}^2$ is the control vector, $y \in \mathbf{R}^2$ is the vector of output measurements. $d \in \mathbf{R}$ is the unknown input. As mentioned in (2), the sub-matrices of A, B, C, E, Γ and D, F have the following expressions:

$$\Gamma = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \text{ with } rank(\Gamma) = 6$$

$$A^{11} = \begin{pmatrix} -1.1423 & 5525.9 & 0 & 0.2856 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0952 & 0 & 0 & -0.2455 & -1749.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A^{12} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}; \ B^1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \ E^1 = \begin{pmatrix} 0 \\ 0.2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$A^{21} = \begin{pmatrix} 0 & -39.48 & -8.80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -39.48 & -8.80 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 36.7149 & 0 \\ 0 & 36.7149 \end{pmatrix}; \ A^{22} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$C^1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}; \ C^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E^2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \ D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}; \ F = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Since A^{22} is invertible, the linear descriptor system (21) can be rewritten in the following equivalent system:

$$\begin{cases} \dot{\xi}^1 &= M\xi^1 + Nu + Pd \\ \xi^2 &= J\xi^1 + Ku + Ld \\ y &= C^1\xi^1 \end{cases} \tag{22}$$

where M, N, P, J, K, L can be calculated by using (6).
Let define the augmented state vector $\eta^1 = [\xi^{1^T} \ \ d]^T$ and $\eta^2 = \xi^2$. Thus, under Assumption 2 (see Figure 6), the system (22) can be represented as:

$$\begin{cases} \dot{\eta}^1 &= \tilde{M}\eta^1 + \tilde{N}u \\ \eta^2 &= \tilde{J}\eta^1 + Ku \\ y &= \tilde{R}\eta^1 \end{cases} \tag{23}$$

where $\tilde{M}$, $\tilde{N}$, $\tilde{J}$, $\tilde{R}$ can be calculated by using (10).
By Theorem 1, considering $\alpha = 5$ the following observer gain G is obtained:

$$G = \begin{pmatrix} 417.6165 & -33.2526 \\ 8.2508 & -3.0020 \\ 0.5058 & -28.6113 \\ -0.4063 & 26.6194 \\ 1.5304 & -36.5944 \\ 5.4385 & 0.4945 \\ 26.0116 & -99.4944 \end{pmatrix}$$

Simulation results with initial conditions:
$\eta^1(0) = [10 \ \ 0.0021 \ \ 0 \ \ -9 \ \ -0.0046 \ \ 0 \ \ 0]^T$
$\eta^2(0) = [0.3671 \ \ 0.3671]^T$
$\hat{\eta}^1(0) = [10 \ \ -0.0011 \ \ 0.001 \ \ -9 \ \ -0.0036 \ \ 0.001 \ \ 0.06]^T$
$\hat{\eta}^2(0) = [0.3789 \ \ 0.3789]^T$
are given in Figure 2 to Figure 6. These simulation results show the performances of the proposed UIO (11) with the gains G, where the dashed lines denote the state variables and unknown input estimated by the observer. They show that the observer gives a good estimation of unknown states and unknown input of the considered heat exchanger pilot.

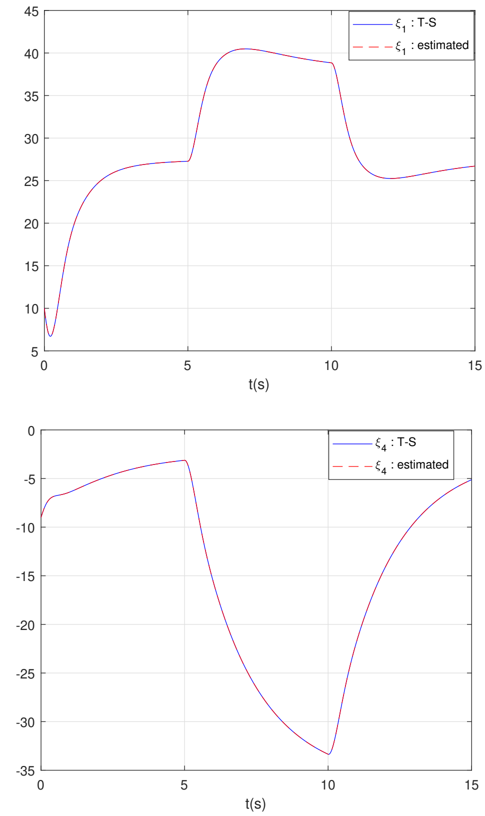

Figure 2. ξ_1 and ξ_4 with their estimates

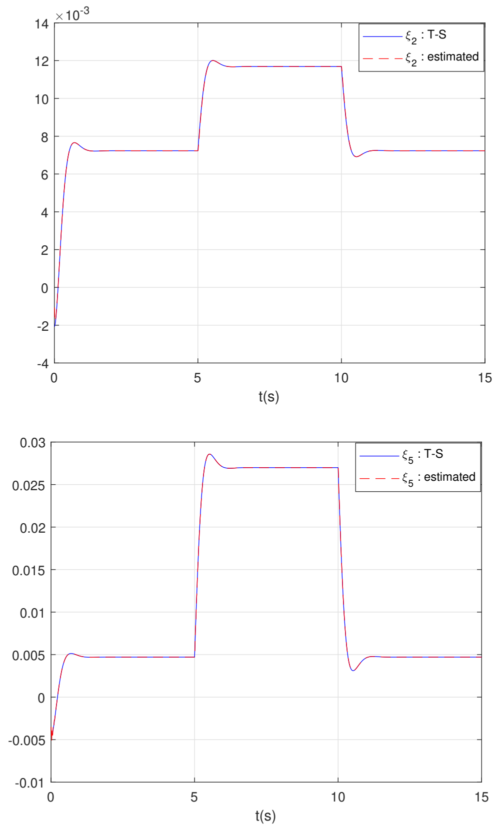

Figure 3. ξ_2 and ξ_5 with their estimates

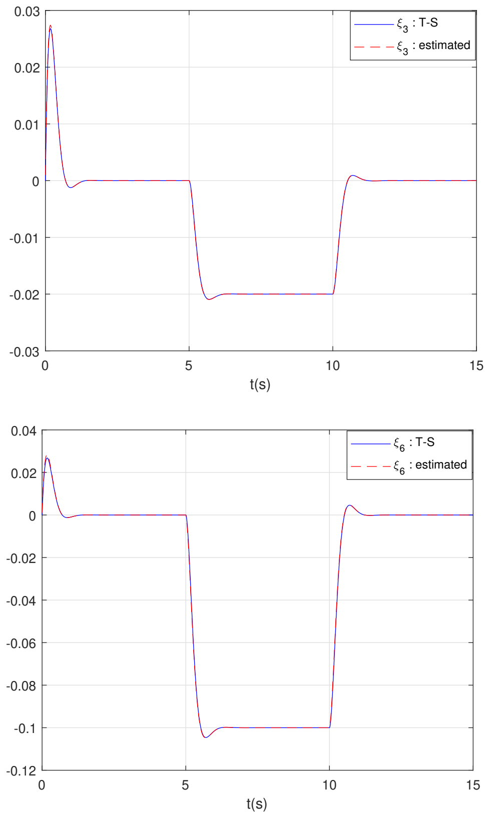

Figure 4. ξ_3 and ξ_6 with their estimates

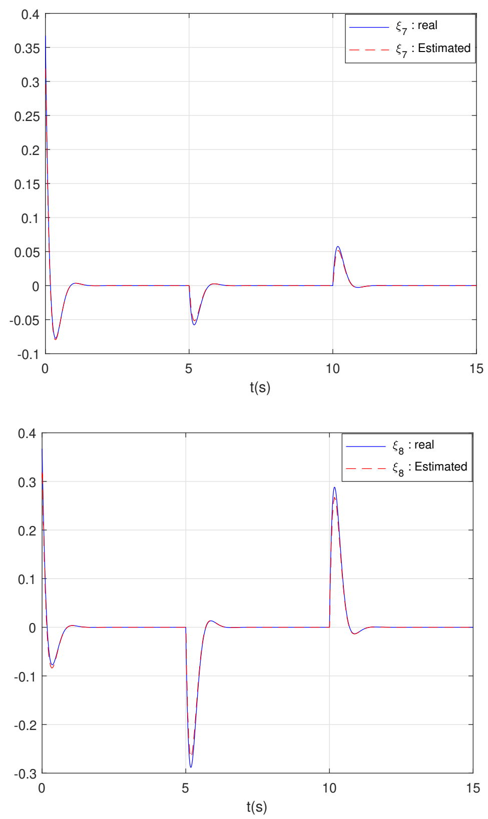

Figure 5. ξ_8 and ξ_9 with their estimates

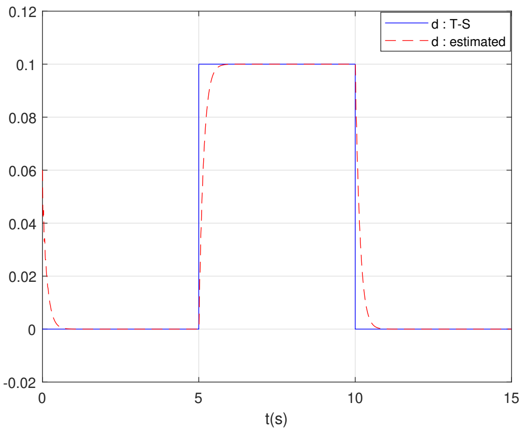

Figure 6. Unknown input $d = \xi_7$ with its estimate

5 Conclusion

In this paper, a new observer design approach permitting to estimate simultaneously the unknown states and the unknown inputs for a class of linear descriptor systems with unknown input is proposed. The main idea of the present work is based on the separation between dynamic and static relations in the linear descriptor model. The exponential convergence of the state estimation error is studied by using the Lyapunov theory and the stability condition is given in term of LMI. Simulation results are given and demonstrate the good performance of the proposed UIO design.

References

[1] L. Dai. Singular Control Systems. Lecture Notes in Control and Information Sciences. Springer-Verlag. Berlin, 1989.

[2] A. Kumar and P. Daoutidis, Control of nonlinear differential algebraic equation systems. Chapman & Hall CRC, 1999.

[3] G. R. Duan. Analysis and Design of Descriptor Linear Systems. Springer 2010.

[4] P. Kudva, N. Viswandam, and A. Ramakrishna. Observers for linear systems with unknown inputs. IEEE Trans. Automat. Contr., vol. AC25, pp. 113-115, 1980.

[5] F. Yang and R. W. Wild. Observers for linear systems with unknown inputs. IEEE Trans. Automat. Contr., vol. AC-33, pp. 677-681, 1988.

[6] Cheng-Wu Yang and Hua-Lin Tan. Observer design for singular systems with unknown inputs, International Journal of Control, 49:6, 1937-1946, 1989.

[7] Y. Guan and M. Saif. A novel approach to the design of unknown input observers. IEEE Transactions on Automatic Control, vol. 36, no. 5, pp. 871–875, 1992.

[8] M. Darouach and M. Boutayeb. Design of observers for descriptor systems. IEEE Transactions on Automatic Control, vol. 40, no. 7, pp. 1323–1327, July 1995.

[9] B. Gaddouna, S. Giuliani and J. Ragot. Design of observers for descriptor systems affected by unknown inputs, International Journal of Systems Science, 27:5, 465-471, 1996.

[10] H.S. Kim, T.K. Yeu,S. Kawaji. Fault detection in linear descriptor systems via unknown input PI observer. Trans. Control Autom. Syst. Eng. 3(2), 77–82, 2001.

[11] H. L. Trentelman, A. A. Stoorvogel, and M. L. J. Hautus. Control theory for linear systems. Communications and control engineering. New York, London: Springer, 2001.

[12] B. Sfaihi, O. Boubaker. Full order observer design for linear systems with unknown inputs. IEEE International Conference on Industrial Technology, 8 – 10, December, Hammamet, Tunisia, 2004.

[13] D. Koenig. Unknown input proportional multiple-integral observer design for linear descriptor systems: application to state and fault estimation. IEEE Transactions on Automatic Control, vol. 50, no. 2, pp. 212–217, 2005.

[14] O. Boubaker, B. Sfaihi. Robust observers for linear systems with unknown inputs: A comparative study. IEEE SSD'2005, Sousse, Tunisia, 2005.

[15] F. J. Bejarano, T. Floquet, W. Perruquetti, G. Zheng. Observability and Detectability Analysis of Singular Linear Systems with Unknown Inputs. 50th IEEE Conference on Decision and Control and European Control Conference, Dec 2011, Orlando, Florida, United States.

[16] M. K. Gupta, N. K. Tomar and S. Bhaumik. Detectability and Observer Design for Linear Descriptor Systems. 22nd Mediterranean Conference on Control and Automation (MED). University of Palermo. June 16-19, 2014. Palermo, Italy.

[17] M. K. Gupta, N. K. Tomar and S. Bhaumik. Full- and reduced-order observer design for rectangular descriptor systems with unknown inputs. Journal of the Franklin Institute Volume 352, Issue 3, March 2015, Pages 1250-1264.

[18] M. Essabre, J. Soulami, A. El Assoudi, E. Elyaagoubi and E. El Bouatmani, Fuzzy Observer Design for a Class of Takagi-Sugeno Descriptor Systems, Contemporary Engineering Sciences, vol. 7, no. 4, 149-164, 2014.