

MATHEMATICAL MODELS OF THE ENERGY CHARACTERISTICS OF A BACKPRESSURE STEAM TURBINE BASED ON A SIMULATION STUDY

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Abstract. The backpressure steam turbine installations for combined production generate thermal energy and electricity. The produced electricity depends on the heat load determined by consumers. Heat energy is released in the district heating system through water, which is heated with the exhausted steam in boiler-condenser (district heater). The operation regime of the installation is influenced by load of the released district heat and the temperature of the district heat water at the input and output of the boiler-condenser. The variables are heat load of the district heating system and temperature of district heat water at power plant output. The temperature of the district heat water at the boiler-condenser input is an uncontrollable variable, whose value depends on heat consumption. It influences the steam pressure in the boiler-condenser and the available enthalpy drop of the cogeneration installation. In this report, a verified simulation model is applied on a backpressure steam turbine installation, type SST-300 CE2L/V36S. A multivariate simulation is carried out, studying the performance of the installation in off-design regimes. The results obtained are used for the creation of regression models of the installation's energy characteristics. They reveal the dependence between the energy characteristics of the cogeneration steam turbine by the released heat load in the district heating system and the temperature of incoming and outgoing district heat water. The accuracy of the developed models is evaluated through multiple correlation coefficients. The mathematical models could be successfully used to optimize the operating regime of the backpressure steam turbine installation.

1 Introduction

The district heating systems provide the heat load to the domestic and industrial consumers connected to it. The heat load depends mainly on climatic factors, of which the air temperature is decisive [1]. Combined heat and power generation is an efficient method of

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using primary energy resources. This method achieves fuel savings compared to the method of separate heat and power generation.

The most widely used cogeneration technology is based on the Rankine cycle, on which steam turbines operate [2].

On the figure 1 present the technological scheme of the backpressure steam turbine installation SST-300 CE2L / V36S, this is the subject of research in this article.

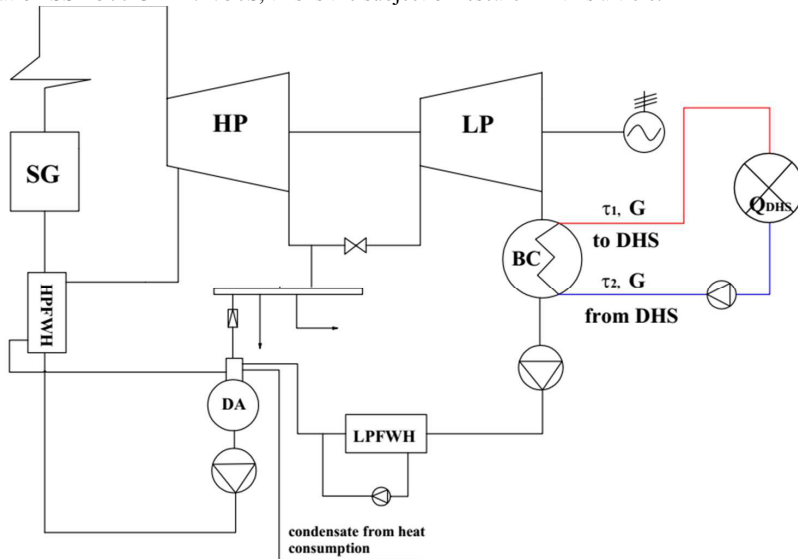


Fig. 1. Technological scheme of backpressure steam turbine type SST-300 CE2L/V36S.

The electricity generated by the backpressure steam turbine installations depends on the size of the released heat load (Q_{DHS}) [3], as the district water is heated with the exhaust steam, which enters after the last turbine stage in the boiler-condenser. The production of electricity in this type of turbine is also influenced by the flow of district water (G) and the temperatures of district water inlet (τ_2) and outlet (τ_1) of the boiler-condenser. The outlet temperature after boiler-condenser depends of temperature of the district water and the magnitude of its heating ($\Delta\tau$).

During operating installation for cogeneration, it is necessary to forecast the production of electrical power and fuel consumption at a certain heat load supplied to consumers in district heating systems. Present study aims to develop mathematical models for forecasting electricity power and the amount of fuel for operating the cogeneration installation.

2 Mathematical modeling

Mathematical modeling requires the compilation of a mathematical model that describes the properties of a real object, called plant in control theory. Usually, the mathematical model does not cover all the components and interdependencies of the plant, but only those that are most important and significant. One of the main advantages of mathematical models is that the plant can be forecasted or operated without laboratory experiments or additional tests.

The main objects of modeling are multifactorial objects, which are characterized by a set of independent or mutually influencing factors (control parameters) [4, 5].

The creation of mathematical models can be realized on the basis of experimental data from the measurement of the inlet and outlet parameters of the plant. To collect this information requires expensive equipment, human resources and technological time to prepare and perform the experiment, as well as time to process the collected data.

Experimental data collection is essentially of two types: active and passive. In the passive experimental data collection, the performer of the experiment does not have an impact on the studied object, but only registers the value of the factors changing in the process of the experiment and the magnitudes characterizing its operation. The active data collection is realized through purposeful actions on specific parameters of the plant in order to study their influence on the parameters characterizing its operation. The creation of mathematical model based on an active data collected from experiment gives a more complete description of the influence of factors on the work of the plant. It is important when performing an active data collection not to consider technological regimes that would lead to an increase in technological losses, accident or destruction of the plant.

The modern approach to the study of objects is the simulation modeling. It develops a computer model of the object by specialized software, which introduces the analytical dependences of the chemical and physical processes which govern the operation of the plant. In order to use the data from the simulation model, it is necessary for it to reflect with sufficient accuracy the considered real object, which presupposes its validation or verification. The main advantage of the simulation modeling is that its application does not violate the mode of operation of the object, and the results obtained after analysis can be used to improve its operation.

Mathematical models are most often described by polynomials, which can be linear or higher degrees. The higher degree of the polynomial makes it possible to achieve a higher accuracy of the study. On the other hand, the higher degree complicates the computational procedures in the development and use of the model. In most cases, the coefficients in the mathematical model after the quadric degrees do not have a significant impact on the final result, therefore the models up to the quadric degree are most often used.

The mathematical model used is a quadric polynomial and has the form [5]:

$$\hat{y}(X_1, X_2, X_3) = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_{11} X_1^2 + a_{22} X_2^2 + a_{33} X_3^2 + a_4 X_1 X_2 X_3 \quad (1)$$

where:

\hat{y} – forecasted variable of the mathematical model;

X_1, X_2 and X_3 - control parameters of the mathematical model;

$a_0, a_1, a_2, a_3, a_{11}, a_{22}, a_{33}, a_4$ - correlation coefficients of the mathematical model.

The accuracy of the mathematical model is determined by the multiple correlation coefficient R [4]. It is defined by the following expression:

$$R = \sqrt{1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (\hat{y}_i - \bar{y}_i)^2 + \sum_{i=1}^N (y_i - \hat{y}_i)^2}} \quad (2)$$

where:

N - number of experiments performed (obtained by measurement or simulation modeling);

y_i – value of the magnitude obtained as a result of the experiment;

\bar{y}_i - arithmetic mean result of the value obtained in the performed experiments.

To use the mathematical model to determine the output variables as a function of the control parameters it is necessary to determine if the coefficient of multiple correlation is significant. This estimation is performed by Fisher's criterion [5,6], as the condition for significance of the multiple correlation coefficient is:

$$F^* > F(\alpha, v_1, v_2)$$

where:

F^* – variance ratio;

F – critical value by Fisher's distribution;

α – probability associated with the Fisher's cumulative distribution;

v_1 – numerator degrees of freedom;

v_2 – denominator degrees of freedom.

3 Mathematical models for determination energy characteristics of cogeneration steam turbines installation

The main energy characteristics of a cogeneration installation are the electrical power and the fuel consumption for its operation. Statistically regressive mathematical models are assumed for their mathematics. For their compilation, data from multivariate calculation by the simulation software GateCycle of the developed and validated simulation model [7] of the steam turbine installation SST-300 CE2L/V36S.

The electric power (P , MW) of the steam turbine is calculated according to the following equation:

$$P = D \cdot H_i \cdot \eta_{TG} \quad (3)$$

where:

D – the steam mass flow rate through steam turbine, kg/s

H_i – enthalpy drop processed by the steam turbine, kJ/kg;

η_{TG} – overall efficiency on the turbo-generator unit, including the mechanical efficiency of the steam turbine and the electrical efficiency of the generator %.

The fuel consumption during the operation of the cogeneration installation is determined by:

$$F = P \frac{3600}{LCV \cdot \eta_{el}^{chp}} + Q_{DHS} \frac{3600}{LCV \cdot \eta_Q^{chp}} \quad (4)$$

where:

LCV – low caloric value, kJ/kg;

η_{el}^{chp} – efficiency of electricity production of the cogeneration unit, %;

η_Q^{chp} – efficiency of heat production of the cogeneration unit, %.

The mathematical models for determining the electrical power of the cogeneration installation (P , MW) and its fuel consumption (F , kg/h) are to be determined in the present research. For the models the following control parameters are identified: the temperature of the district water at the outlet of the boiler-condenser - τ_1 , °C, the temperature of the district water returned from the district heating system for heating in the boiler-condenser τ_2 , °C

and the released heat load in the district heating system Q_{DHS} , MW. These magnitudes correspond to the control parameters of the mathematical model X_1 , X_2 , X_3 . The schematic diagram of the installation for cogeneration as an object for modeling is shown in Fig. 2.

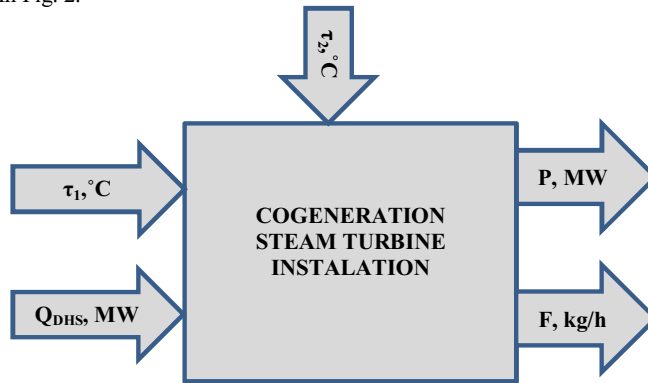


Fig. 2 Visualization of the cogeneration steam turbine installation as an object for mathematical modeling.

The left part of fig. 2 shows the factors that are controllable for the cogeneration unit. These factors are the district water temperature at the outlet of the installation and the output heat load. At the top of the figure is the factor on which the object of study cannot have a regulatory effect. This factor is the inlet district water temperature; its value depends on the operation mode of the district heating systems. The right part of the figure shows the forecasted variables from the mathematical model - the electric power and the fuel consumption for the installation.

The multivariate calculations for obtaining the data necessary for compiling the mathematical model is performed in accordance with the modes and parameters set in the technical documentation of the steam turbine. The main technical limitations under which the calculations were performed are the following:

- Maximum district water flow through the boiler-condenser - 650 kg/s;
- Heating of the district water in the boiler-condenser ($\Delta\tau$) does not exceed 50 °C;
- Steam pressure in the condenser does not exceed 0.25MPa.

The simulation calculation is performed for heat output from the boiler-condenser in the range from 60 to 90 MW, through the adjustable steam extraction no heat is released to external users. The temperature of the district water (τ_2), which flows into the boiler-condenser, varies from 50 to 60 °C. The change of these values is in accordance with the technical documentation of the turbine. The simulation model used a fuel with a lower caloric value - 29330 kJ / kg.

In the simulation study, the values are changed in the following steps: the district water temperature at the outlet of the cogeneration unit (τ_1) by 2 °C, the water temperature at the installation inlet (τ_2) by 5 °C and the output heat by 10 MW.

Under these conditions, 65 simulation studies for its operation were performed with the simulation model of the steam turbine installation.

The data for the simulation study, necessary to create the mathematical model for determining the electric power of the cogeneration steam turbine installation and the fuel consumption, are presented in Table 1.

Table 1. Data necessary to develop mathematical models for determining the electrical power and fuel consumption of cogeneration steam turbine installation

Number of experiments	τ_1	τ_2	Q_{DHS}	P	F
	°C	°C	MW	MW	kg/h
1	110	60	90	32.7	17706
2	108	60	90	33.2	17770
.....					
65	96	50	60	21.8	11775

The correlation coefficients of the mathematical model for determining the electric power of the steam turbine are determined after statistical processing of the data obtained from the multivariate calculations. The values of these coefficients are show in Table 2.

Table 2. Correlation coefficients for determining the electrical power of the cogeneration steam turbine installation

a_0	a_1	a_2	a_3
-18.41	$-40.39 \cdot 10^{-2}$	$55.61 \cdot 10^{-2}$	$89.16 \cdot 10^{-2}$
a_{11}	a_{22}	a_{33}	a_4
$21.68 \cdot 10^{-4}$	$-18.03 \cdot 10^{-4}$	$-86.89 \cdot 10^{-5}$	$-51.36 \cdot 10^{-5}$

The mathematical model for determining the electrical power on the cogeneration steam turbine acquires the following form:

$$P(\tau_1, \tau_2, Q_{DHS}) = -18.41 - 40.39 \cdot 10^{-2} \tau_1 + 55.61 \cdot 10^{-2} \tau_2 + 89.16 \cdot 10^{-2} Q_{DHS} + 21.68 \cdot 10^{-4} \tau_1^2 - 18.03 \cdot 10^{-4} \tau_2^2 - 86.89 \cdot 10^{-5} Q_{DHS}^2 - 51.36 \cdot 10^{-5} \tau_1 \tau_2 Q_{DHS} \quad (5)$$

In a similar way were obtained the correlation coefficients of the mathematical model for determining the fuel consumption of the cogeneration installation. Their values are presented in Table 3.

Table 3. Coefficients of the mathematical model for determining the fuel consumption for the cogeneration unit.

a_0	a_1	a_2	a_3
6846.49	-144.07	32.66	175.60
a_{11}	a_{22}	a_{33}	a_4
$70.00 \cdot 10^{-2}$	$98.91 \cdot 10^{-3}$	$47.89 \cdot 10^{-2}$	$-63.06 \cdot 10^{-4}$

The mathematical model for determining the fuel consumption of the cogeneration steam turbine acquires the following form:

$$F(\tau_1, \tau_2, Q_{DHS}) = 6846.49 - 144.07 \tau_1 + 32.66 \tau_2 + 175.60 Q_{DHS} + 70.00 \cdot 10^{-2} \tau_1^2 + 98.91 \cdot 10^{-3} \tau_2^2 + 47.89 \cdot 10^{-2} Q_{DHS}^2 - 63.06 \cdot 10^{-4} \tau_1 \tau_2 Q_{DHS} \quad (6)$$

The coefficient of multiple correlation for the mathematical model for determining the electrical power of the cogeneration steam turbine is $R = 0.988$, and the value of this coefficient for the mathematical model for obtaining the fuel consumption is $R = 0.993$.

The value of the variance ratio F^* for the two mathematical models is higher than the critical value by Fisher's distribution $F(\alpha, v_1, v_2)$, therefore the obtained coefficients of multiple correlation are significant. The results in solving the models can be used for analysis and control of the cogeneration steam turbine installation.

4 Solving the mathematical models of the steam turbine installation SST-300 CE2L/V36S.

The mathematical models are solved and the results are presented graphically depending on the heat load of the district heating system, the change in the temperature of the return district water and the magnitude of its heating in the boiler-condenser.

On figure 3 are shown the results of solving the mathematical model for determining the electrical power of the cogeneration steam turbine at the maximum heat load of the boiler-condenser of 90 MW.

The obtained results show that with a decrease in the temperature of the district water at the inlet of the installation and a decrease in its heating (respectively a decrease in the temperature of the district water outlet from the installation) the electric power increases. Low values of these parameters increase the available processed enthalpy drop from the steam turbine installation, which increases the electrical output obtained on the basis of heat consumption.

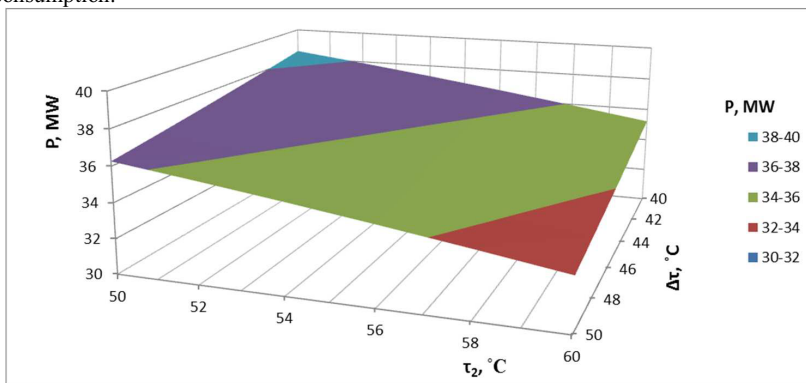


Fig. 3. Change of the electric power of the installation for cogeneration depending on the temperature of the inlet district water and the magnitude of its heating in the boiler-condenser when releasing 90 MW of heat load in the heat supply system.

The mathematical model predictions of the fuel consumption during operation of the cogeneration installation are shown in figure 4.

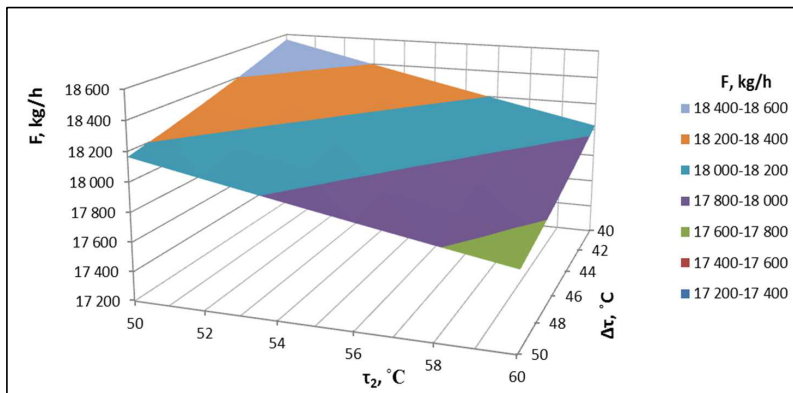


Fig. 4 Change of the fuel consumption of the installation for cogeneration depending on the temperature of the inlet district water and the magnitude of its heating in the boiler-condenser when releasing 90 MW of heat load in the heat supply system

From the graphically presented results it is observed that at the same heat load of the boiler-condenser, the fuel consumption increases with lowering the temperature of the return district water and its heating. The increase in consumption is due to the increase in electricity produced based on heat consumption. These operation modes are associated with an increase of the district water flow through the boiler-condenser, and in them the installation realizes higher energy efficiency [7, 8].

The change in the electrical power and fuel consumption of the installation at other thermal loads of the boiler-condenser is subject to the same trend line as for the operation mode of maximum heat load.

5 Conclusion

The mathematical models for determining the electric power and fuel consumption of a backpressure steam turbine installation are developed in this study. The control parameters in the models are the magnitude of the released heat load and the district water temperatures at the inlet and outlet of the boiler-condenser.

The mathematical models were obtained by processing data from a validated simulation model of a steam turbine plant SST-300 CE2L/V36S.

The significance of mathematical models is assessed by multiple correlation coefficients according to Fisher's criterion. This allows mathematical models to be used to study the installation for cogeneration.

The results of the mathematical study show that as the temperature of the district water at the outlet of the boiler-condenser decreases, the electricity power of the steam turbine increases. This is realized by increasing the flow of district water through the boiler-condenser. The maximum flow rate depends on the throughput of the boiler-condenser.

An increase in the electrical power, while maintaining the heat load and district water temperature at the outlet of the installation, is observed at low temperatures of the district water returned from the districting heating system due to an increase in the processed available enthalpy drop for the steam turbine. The decrease of this temperature depends on the operation mode of the district heating system.

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