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Modeling the process of force load generation at the initial periodic change in pressure (a plane problem)

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Abstract. The article deals with modeling the process of force load generation at an initial periodic change in pressure (a plane problem). The subject of research is a pulsating flow in a flat channel at an initial periodic pressure change. The determination of flow parameters with a periodic change in the inlet pressure; the changes in the structure of the working fluid associated with the release of various particles from the pipe walls, the addition of impurities to prevent leaks, and the high-speed modes, are given in the article considering the law of molecular and molar transfer between layers. Research methods are based on Newton's rheological law, according to which molecular transfer is described by the law of proportionality of stresses to the derivative of the normal velocity; on the method of accounting for molar transfer by proportionality of stresses to the derivative of normal acceleration; on the method of mathematical modeling and the analytical method for their solutions, based on the provisions of operational calculus. An analytical solution to the problem of pulsating fluid motion in a plane-parallel channel is obtained with allowance for single and group transfer of molecules in the flow. The application of the analytical expressions obtained for the velocities is not limited to the critical Reynolds number, i.e. they are applied for any values of this number. Analytical expressions are obtained for the transverse and longitudinal components of the flow velocity. The resulting solution describes two zones of flow: in the first zone, two types of transfer occur, depending on the flow pattern, either molecular or molar transfer of fluid volumes between the layers prevails. In the second zone, only molecular transfer occurs.

1 Introduction

Pulsating flows of fluid ensure the existence of biological and social objects and they are an integral part of the technological production processes. The flow in a sudden

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expansion channel and its flow through an idealized curved coronary artery with a pulsating velocity at the inlet was studied in [1]. A pulsating flow for thermohydraulic analysis of a nuclear reactor in an oceanic environment was investigated in [2]. Modeling a pulsating inlet flow to study the performance of flutterbased energy harvesters [3] and the effect of pseudo-plastic fluid flow in a manifold microchannel heat sink [4] is the evidence of the widespread use of pulsating flow.

Investigations of pulsating fluid flow are conducted by experimental [5, 6] and theoretical methods [3, 4]. Theoretical studies of this process are conducted using the Navier-Stokes equations. The Navier-Stokes equations are derived with Newton's law, according to which the stress is directly proportional to the derivative of the normal velocity, which describes the molecular transfer of momentum between the layers of the flow. This corresponds to a homogeneous layered fluid flow. Under the pulsating motion, conditions for the accumulation of inhomogeneity are formed in the flow, the alignment of which occurs during the groupped motion of molecules.

In hydraulic drives and other structures, wear products are formed in the working fluid, and to improve performance and prevent leaks in the system, various additives are added to the fluid. Moreover, high speeds and pressures arise in various modes. Taking into account all these factors, hydrodynamic processes cannot be described using classical models.

In this study, to account for the transfer of a substance between layers at the molecular level, it is assumed that the stress is directly proportional to the derivative of the normal velocity, and with all the above circumstances, during molar transfer, the stress is proportional to the derivative of the normal acceleration [7, 11]. Taking into account the new factor, the internal structure of the Navier-Stokes equations undergoes substantial changes – the terms in partial derivatives of the third order are formed in the equation [7, 11]. There are many methods and algorithms for the numerical solution of these equations [8, 9]. Applying these tools to solving problems by involving new complicated equations is laborious work. There are various methods for the analytical solution of problems: the method of separation of variables, the method of linear approximation, the Fourier method [10] or the method of involving the provisions of the operational calculus [7, 11]. In the problem considered below, we used the method of involving the provisions of the operational calculus.

2 Methods

Research methods are based on Newton's rheological law, the equation of continuity, which expresses the law of conservation of mass; the method of mathematical modeling and the analytical methods for their solution, based on the provisions of operational calculus.

3 Materials

Let us consider a plane-parallel pulsating flow of fluid, taking into account the molecular and molar transfer in the flow. The system of equations of fluid motion, in this case, has the following form [7, 11]:

$$\left.\begin{array}{l}\rho\dfrac{\partial v_1}{\partial t}=-\dfrac{\partial P}{\partial x}+\mu\dfrac{\partial^2 v}{\partial y^2}+m_l\left(\dfrac{\partial^3 v_1}{\partial t\partial y^2}+v_1\dfrac{\partial^3 v_1}{\partial x\partial y^2}\right),\\ \dfrac{\partial v_1}{\partial x}+\dfrac{\partial v_2}{\partial y}=0\end{array}\right\} \tag{1}$$

Where x,y –are the coordinates; t is time; V_1, V_2 are the velocity components; P is the pressure; μ is the dynamic viscosity; m_e is the molar transfer coefficient.

The pressure gradient is given as:

$$-\frac{\partial P}{\partial x}=a+b\cos\omega t \tag{2}$$

The initial and boundary conditions are as follows:

$$v_1(x,y,0)=\frac{\partial v_1}{\partial y}_{|t=0}=\frac{\partial^2 v_1}{\partial y^2}_{|t=0}=0,$$

$$v_1(0,y,t)=u_0\left(1-e^{-\gamma t}\right);\frac{\partial v_1}{\partial y}_{|x=0}=\frac{\partial^2 v_1}{\partial y^2}_{|x=0}=0, \tag{3}$$

$$\frac{\partial v_1}{\partial y}_{|y=0}=v_2(x,0,t)=0,$$

$$v_1(x,L,t)=v_2(x,L,t)=0,$$

$$\lim_{x\to\infty}v_1(x,y,t)=M=const.$$

here γ is a parameter that takes into account the instantaneous transition of the velocity at $x = 0$ from the state of rest in terms of velocity $u_0=const$.

In order to obtain an analytical solution to this problem, we introduce the following function:

$$u(x,y,t)=v_1(x,y,t)-u_0(1-e^{-\gamma t}) \tag{4}$$

applying the Laplace transform in the variable t , we obtain

$$\rho\left[p\bar{u}+\frac{u_0\gamma}{p+\gamma}\right]=\frac{a}{p}+\frac{bp}{p^2+\omega^2}+\mu\left(\frac{\partial^2\bar{u}}{\partial y^2}\right)+m_l\left(p\frac{\partial^2\bar{u}}{\partial y^2}+u_0\frac{\partial^3\bar{u}}{\partial x\partial y^2}\right), \tag{5}$$

which is subject to the following conditions:

$$u(0,y,p)=\frac{\partial\bar{u}}{\partial y}_{|x=0}=\frac{\partial^2\bar{u}}{\partial y^2}_{|x=0}=0(y>0) \tag{6}$$

$$\frac{\partial\bar{u}}{\partial y}_{|x=0}=0(x>0),$$

$$\bar{u}(x,L,p) = -\frac{u_0\gamma}{p(p+\gamma)}$$

We introduce function $w(x,r,p) = \bar{u}(x,r,p) - A(p)$, and apply the Laplace transform in x to the resulting equation, and a second-order differential equation with respect to the function w is obtained:

$$\frac{\partial^2 w}{\partial y^2} = \frac{\rho p}{\mu + m_l p + m_l u_0 s} w = 0 \tag{7}$$

there is a solution:

$$w = -\left[\frac{a}{\mu s p^2} + \frac{b}{\rho s(p^2+\omega^2)}\right]\frac{ch[\xi(p,s)y]}{ch[\xi(p,s)L]}. \tag{8}$$

Here *P, S* are the parameters of the Laplace transform in *t* and *x*, respectively.

$$\xi^2(p,s) = \frac{\rho p}{\mu + m_l p + m_l u_0 s}. \tag{9}$$

Using the Cauchy theorem [12], we obtain:

$$\frac{ch[\xi(p,s)y]}{ch[\xi(p,s)L]} = \frac{\pi}{L^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2n-1)}{\xi^2(p,s) + \frac{\pi^2}{4L^2}(2n-1)^2}\cos\frac{\pi y(2n-1)}{2L} \tag{10}$$

now (2.8) is written as:

$$w(s,y,p) = -\frac{4}{\pi m_l u_0 s}\left[\frac{a}{\mu p^2} + \frac{b}{\rho\left(p^2+\omega^2\right)}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\left[\mu + m_l p + m_l u_0 s\right]\cos\frac{\pi y(2n-1)}{2L}}{(2n-1)\left[s + \frac{\mu}{m_l u_0} + p\frac{4\rho L^2 + m_l\pi^2(2n-1)^2}{m_l u_0 \pi^2 (2n-1)^2}\right]} \tag{11}$$

Performing the inverse transformation sequentially with respect to parameters *s* and *p* from [12], we obtain:

$$\begin{aligned} w(s,y,t) = &-\frac{4}{\pi\mu}\sum_{n=1}^{\infty}\left\{\psi(t,n)\delta(t) - \exp\left[-\frac{\mu x}{m_l u_0}\right]\psi\left(t - \frac{\mu x}{m_l u_0 \delta_n}, n\right)\delta\left(\frac{m_l u_0 \delta_n t}{\mu} - x\right)\right\}\cos\frac{\pi y(2n-1)}{2L} - \\ &-\frac{4}{\pi\mu}\exp\left[-\frac{\mu x}{m_l u_0}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cos\frac{\pi y(2n-1)}{2n-1}}{(2n-1)}\left\{a\left(t - \frac{\mu x}{m_l u_0 \delta_n}\right) + \frac{b\mu}{\rho\omega}\sin\omega\left[t - \frac{\mu x}{m_l u_0 \delta_n}\right]\right\}\delta\left(\frac{m_l u_0 \delta_n t}{\mu} - x\right), \end{aligned} \tag{12}$$

where $\delta(t) = \begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases}$ is the unit Heaviside function.

From (2.12) it follows that there are two zones in the flow $0 \le x \le u_0 m_l \delta_n t/\mu$ and $x > u_0 m_l \delta_n t/\mu$. For the second zone, the solution takes the following form:

$$w^*(x,y,t) = -\frac{4}{\pi\mu}\sum_{n=1}^{\infty}\left\{\psi(t,n)\cos\frac{\pi y(2n-1)}{2L}\right\}. \tag{13}$$

Moving on to the initial functions, we finally determine:

$$v_1(x,y,t) = w(x,y,t) + \frac{at}{\rho} + \frac{b}{\rho\omega}\sin\omega t,$$

$$v_2(x,y,t) = \frac{8L^2}{\pi^2 m_l u_0}\exp\left[-\frac{\mu x}{m_l u_0}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin\dfrac{\pi y(2n-1)}{2L}}{(2n-1)^2}\times \tag{14}$$

$$\times\left\{a\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)+\frac{b}{\rho}\left[\frac{\mu}{\omega}\sin\omega\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)-m_l\cos\omega\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)\right]\right\}\delta\left(\frac{m_l u_0\delta_n t}{\mu}-x\right)$$

4 Results and discussion

An analytical solution to the problem of plane-parallel pulsating flow of fluid is obtained, taking into account the molar transfer in the flow. From the expressions obtained for the velocity components, it is seen that two flow zones are formed in the flow, and in one of them, the longitudinal velocity does not depend on the *x* coordinate.

The following results (conclusions) were obtained based on the study:

1. The application of the analytical expressions obtained, for the velocities is not limited to the critical Reynolds number, i.e. they are applied for any values of this number, and also describe the annular Richardson effect, which is of great practical importance in reducing hydro-erosion in pipes transporting suspensions, dusty gases and other substances.

2. The solution obtained shows that there are two flow zones: in the first zone there are two types of motion, depending on the nature of the flow, where either molecular or molar transfer of fluid volumes between the layers prevails. In the second zone, only molecular transfer occurs. This means that with time, the flow passes into the limiting mode.

3. The calculations have shown that, with the corresponding molar transfer coefficients for different values of the Reynolds number, analytical expressions for the velocities describe the fluid flow with a periodic law of pressure change in laminar and turbulent flow modes.

References

1. Wu. Wei-Tao, Journal of Computational and Applied Mathematics **376**, 112856 (2020) https://doi.org/10.1016/j.cam.2020.112856
2. B.H. Yan, Annals of Nuclear Energy **38(9)**, 1833-1841 (2011) https://doi.org/10.1016/j.anucene.2011.05.020
3. S. Chawdhury, Computer&Structures **209**, 130-149 (2018) https://doi.org/10.1016/j.compstruc.2018.08.008
4. Hongna Zhang, Applied Thermal Engineering **129**, 1092-1105 (2017) https://doi.org/10.1016/j.applthermaleng.2017.10.124
5. N.I. Mikheev, International Journal of Heat and Mass Transfer **109**, 254-265 (2017) https://doi.org/10.1016/j.ijheatmasstransfer.2017.01.125

6. Yu. Li, Applied Thermal Engineering **171**, 115098 (2020) https://doi.org/10.1016/j.applthermaleng.2020.115098
7. M.K. Khudjaev, Journal of Physics: Conference Series **1614**, 012093 (2020) doi:10.1088/1742-6596/1614/1/012093
8. R.D. Enikeev, G.A. Nozdrim, A.A. Chernousov, Procedia Engineering **176**, 461-470 (2017) https://doi.org/10.1016/j.proeng.2017.02.345
9. V. Gurua, A. Zlotnik, Applied Mathematical Modelling **65**, 34-51 (2019) https://doi.org/10.1016/j.apm.2018.07.051
10. I. Khujaev, S. Akhmadjanov, M. Khujaev, A. Ismailov, International Scientific Journal ISJ Theoretical & Applied Science "Clarivate Analytics" **78(10)**, 32-40 (2019) http//t-science.org/arxivDOI/2019/10-78.html
11. K.A. Karimov, M.K. Khudjaev, E.H. Nematov, T.D. Hojibekov, International Scientific Journal ISJ Theoretical & Applied Science "Clarivate Analytics" **82(2)**, 563-569 (2020) https://t-science.org/arxivDOI/2020/02-82.html
12. M.A. Lavrentiev, B.V. Shabat, *Methods of the theory of functions of a complex variable* (Nauka, M., 1987)