E3S Web of Conferences **264**, 05016 (2021) https://doi.org/10.1051/e3sconf/202126405016
CONMECHYDRO - 2021

Dynamic stability of anisotropic fiber-reinforced plate

Bakhtiyor Eshmatov[1,2*], *Rustam* Abdikarimov[3], *Kholida* Komilova[2], and *Nigora* Safarbayeva[2]

[1]Branch of Russian state university of oil and gas (NRU) named after I.M.Gubkin, Tashkent, Uzbekistan
[2]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan
[3]Tashkent Institute of Finance, Tashkent, Uzbekistan

Abstract. The dynamic stability problem of an anisotropic fiber-reinforced plate under increasing compressing load is considered in a geometrically nonlinear formulation using the Kirchhoff-Love's shell theory. The problem is solved using the Bubnov-Galerkin method based on a polynomial approximation of the deflections in combination with a numerical method based on quadrature formulas. For a wide range of variations of physical, mechanical, and geometrical parameters, the dynamic behavior of the plate is studied.

1 Introduction

During the intense development of the modern industry, a reduction in the materials consumption of machine structures is one of the main problems of mechanical and civil engineering. For material saving, the need arises to manufacture thin-walled structures. The thinner the element, and the more flexible it is, the more strongly its susceptibility to buckling and loss of stability is manifested. The latter is accompanied by a catastrophic development of deformations and, as a rule, by a structural failure. From this standpoint, in the production of lightweight, durable and reliable structures, it is reasonable to use the materials that make it possible not only to improve their operating characteristics but also to create the structures unfeasible with traditional materials. Here, the calculation procedure and structural design involving the consideration of their actual properties are rather complicated. Today, efficient solution algorithms for nonlinear problems of dynamic stability of shells, panels, and plates are the most pressing issue. The problems with a similar mathematical formulation were considered in [1-12].

* Corresponding author: ebkh@mail.ru

2 Materials and methods

To construct the mathematical model of the problem of dynamic stability of a plate made of a material having anisotropic properties in a geometrically nonlinear formulation, we use the classical Kirchhoff-Love's shell theory. In this case, the normal and tangential forces N_x, N_y, T, as well as the bending moments and torques M_x, M_y, H, have the form [13-17]:

$$\begin{gathered}
N_x = A_{11}\varepsilon_x + A_{12}\varepsilon_y + A_{16}\gamma_{xy} + B_{11}\chi_x + B_{12}\chi_y + B_{16}\chi_{xy},\\
N_x = A_{12}\varepsilon_x + A_{22}\varepsilon_y + A_{26}\gamma_{xy} + B_{12}\chi_x + B_{22}\chi_y + B_{26}\chi_{xy},\\
T = A_{16}\varepsilon_x + A_{26}\varepsilon_y + A_{66}\gamma_{xy} + B_{16}\chi_x + B_{26}\chi_y + B_{66}\chi_{xy},\\
M_x = B_{11}\varepsilon_x + B_{12}\varepsilon_y + B_{16}\gamma_{xy} + D_{11}\chi_x + D_{12}\chi_y + D_{16}\chi_{xy},\\
M_x = B_{12}\varepsilon_x + B_{22}\varepsilon_y + B_{26}\gamma_{xy} + D_{12}\chi_x + D_{22}\chi_y + D_{26}\chi_{xy},\\
H = B_{16}\varepsilon_x + B_{26}\varepsilon_y + B_{66}\gamma_{xy} + D_{16}\chi_x + D_{26}\chi_y + D_{66}\chi_{xy},
\end{gathered} \tag{1}$$

where the A_{ij}'s are extensional stiffnesses, the B_{ij}'s are bending-extension coupling stiffnesses, and the D_{ij}'s are bending stiffnesses having the following form:

$$A_{ij} = \sum_{k=1}^{K} \bar{Q}_{ij}(z_k - z_{k-1}),\ B_{ij} = \frac{1}{2}\sum_{k=1}^{K} \bar{Q}_{ij}\left(z_k^2 - z_{k-1}^2\right),\ D_{ij} = \frac{1}{3}\sum_{k=1}^{K} \bar{Q}_{ij}\left(z_k^3 - z_{k-1}^3\right)$$

$$\bar{Q}_{11} = Q_{11}cos^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}sin^4\theta,$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(cos^4\theta + sin^4\theta),$$

$$\bar{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta,$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sin\theta cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})cos\theta sin^3\theta,$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cos\theta sin^3\theta + (Q_{12} - Q_{22} + 2Q_{66})sin\theta cos^3\theta,$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta cos^2\theta + Q_{66}(cos^4\theta + sin^4\theta),$$

$$Q_{11} = \frac{E_1}{1 - \mu_{12}\mu_{21}}, Q_{12} = \frac{E_1\mu_{21}}{1 - \mu_{12}\mu_{21}} = \frac{E_2\mu_{12}}{1 - \mu_{12}\mu_{21}},$$

$$Q_{22} = \frac{E_2}{1 - \mu_{12}\mu_{21}}, Q_{66} = G_{12}$$

Here K is the number of plate layers, E_1, E_2 are the elastic modulus, G_{12} is the shear modulus, μ_{12} and μ_{21} are the Poisson ratios, θ is the angle characterizing the direction of the fibers relative to the axis Ox.

The relations between the deformations in the median surface ε_x, ε_y, γ_{xy}, χ_x, χ_y, χ_{xy} and displacements u, v, w in directions x, y, z have the form [18]:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y},$$

$$\chi_x = -\frac{\partial^2 w}{\partial x^2},\ \chi_y = -\frac{\partial^2 w}{\partial y^2},\ \chi_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \tag{2}$$

Substituting (1) and (2) into the equations of motion:

$$\frac{\partial N_x}{\partial x}+\frac{\partial T}{\partial y}+p_x=\rho h\frac{\partial^2 u}{\partial t^2},\qquad \frac{\partial T}{\partial x}+\frac{\partial N_y}{\partial y}+p_y=\rho h\frac{\partial^2 v}{\partial t^2},$$

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2\frac{\partial^2 H}{\partial x\partial y}+\frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x}+T\frac{\partial w}{\partial y}\right)+$$

$$+\frac{\partial}{\partial y}\left(T\frac{\partial w}{\partial x}+N_y\frac{\partial w}{\partial y}\right)+q+P(t)h\frac{\partial^2 w}{\partial x^2}=\rho h\frac{\partial^2 w}{\partial t^2}$$

we obtain a system of nonlinear differential equations in partial derivatives that satisfies the boundary conditions of the problem (the edges are simply supported). The solution of this system is sought in the form:

$$u(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}u_{mn}(t)cos\frac{m\pi x}{a}sin\frac{n\pi y}{b},$$

$$v(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}v_{mn}(t)sin\frac{m\pi x}{a}cos\frac{n\pi y}{b}, \tag{3}$$

$$w(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}w_{mn}(t)sin\frac{m\pi x}{a}sin\frac{n\pi y}{b},$$

where $u_{mn}(t)$, $v_{mn}(t)$, $w_{mn}(t)$, are the unknown functions of time. Substituting the approximating functions (3) into the resulting system of equations and performing the procedure of the Bubnov-Galerkin method, we obtain a system of nonlinear ordinary differential equations that, in turn, is integrated using the numerical method based on the use of quadrature formulas [19-20].

3 **Results and Discussion**

Let's consider the problem of dynamic stability of anisotropic fiber-reinforced rectangular plate of thickness h with the sides a and b, subjected to dynamic compression along one of the sides by force $P(t)=vt$ (v is the loading rate).

In the calculations, the following parameters of the plastic (KAST-V) rectangular plate have been used: $E_1=25.5\,GPa$, $E_2=14.91\,GPa$, $G_{12}=4.41\,GPa$, $\mu_{12}=0.2$, $\rho=1900\,kg/m^3$, $a=b=0.5\,m.$, $h=0.5\,sm.$, $\theta=45^o$, $v=2\,MPa/s$.

As a criterion determining the critical time, we assume that the sag of the deflection should not exceed a value equal to the thickness of the plate. In shell structures, the greater the critical time, the more stable it is to dynamic loads. The following graphs correspond to the results obtained for the midpoint of the hinged plate. On the graphs, m (meter) is the dimension for the deflection, and s (second) is for time.

Figure 1 shows a graph of the changes in the deflections of the midpoints of the plates of various thicknesses. The results show that an increase in plate rigidity due to an increase in plate thickness leads to a proportional increase in the critical time value.

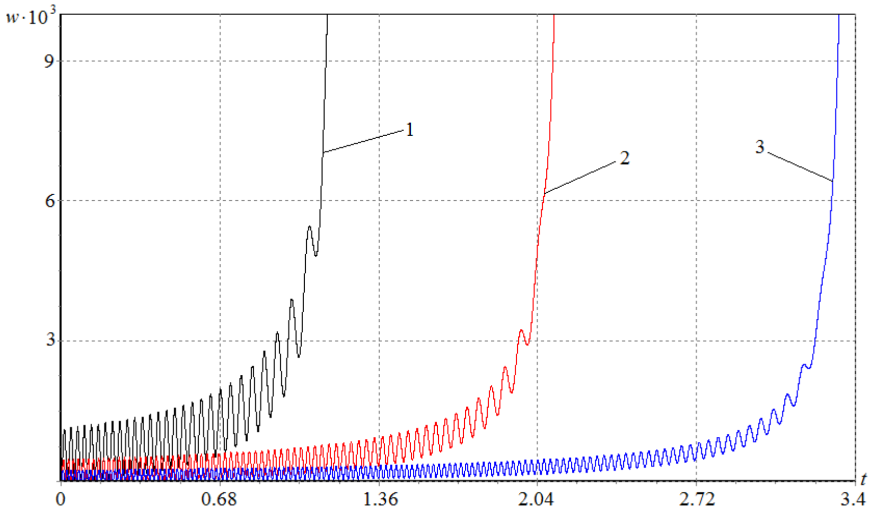

Fig.1. Dependence of the deflection on time for various values of the thicknesses of the plate
1 - $h = 0.3\ sm$; 2 - $h = 0.4\ sm$; 3 - $h = 0.5\ sm$

The various curves in Fig. 2 correspond to cases of changes in the deflections of the midpoint of a reinforced rectangular plate at different loading speeds. It should be noted here that in all cases, at the initial moments of time, the changes in the deflections are oscillations that are harmonic in shape, which begin to increase rapidly at certain points in time.

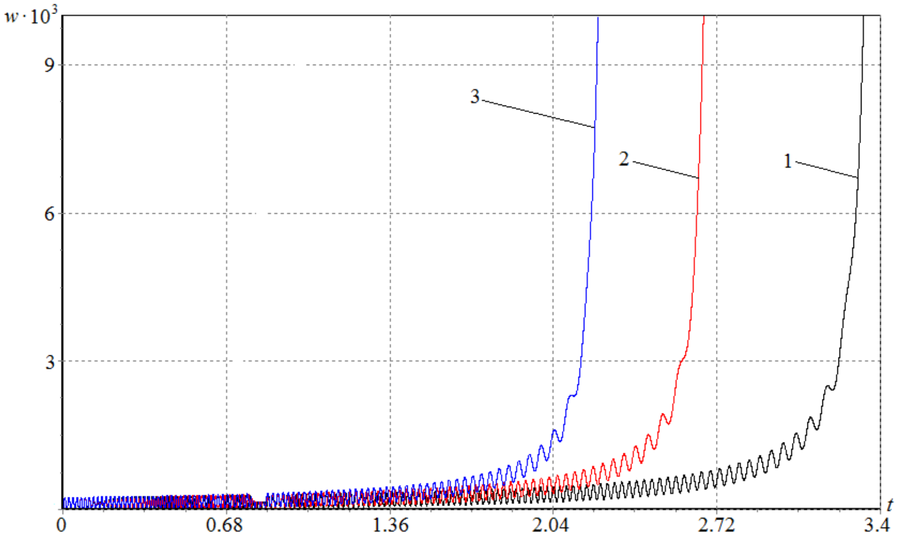

Fig. 2. Dependence of the deflection on time for various values of the velocities of loading

1 - $v = 2\ MPa/s$; 2 - $v = 2.5\ MPa/s$; 3 - $v = 3\ MPa/s$

The influence of changes in the direction of the fibers of the reinforced plate on the dynamic process is shown in (Figure 3). As the angle of direction of the fibers increases from 0 to 45 degrees, an increase in the critical time is observed. The difference between the critical time values for single-layer plates with fiber directions of 0 and 45 degrees is 20.7%.

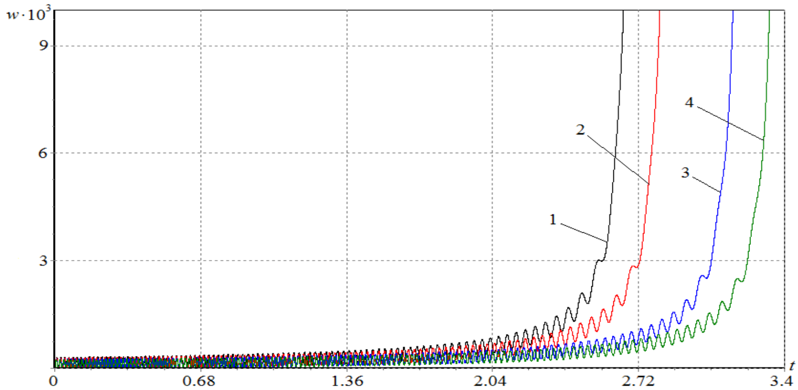

Fig.3. Dependence of the deflection on time for plates with different fiber orientations
1 - $\theta = 0^o$; 2 - $\theta = 15^o$; 3 - $\theta = 30^o$; 4 - $\theta = 45^o$

Modern reinforced composites are a set (composition) of several reinforced layers, each of which has its own mechanical properties. Thus, by changing the composite structure, it is possible to create constructions, the behavior of which can be predicted in advance. Their behavior depends on various factors such as loads, temperatures, and humidity. In this regard, the study of the behavior of laminated reinforced plates with different directions of fibers is of particular interest. Fig.4 shows the changes in the deflections of the midpoints of laminated reinforced plates made of KAST-V. Moreover, although all these plates have different fiber directions, however, their thickness is the same. The results show that for two-layer plates with fibers located at an angle of -45 degrees relative to the OX axis in one layer and 45 degrees in another, the critical time values are higher than the others. The layered fiber plate, which is parallel and perpendicular to the OX axis, has a lower critical time (i.e., it is less stable) than other plates with similar mechanical properties. The difference between the critical time values for the above two-layer plates is 21.8%.

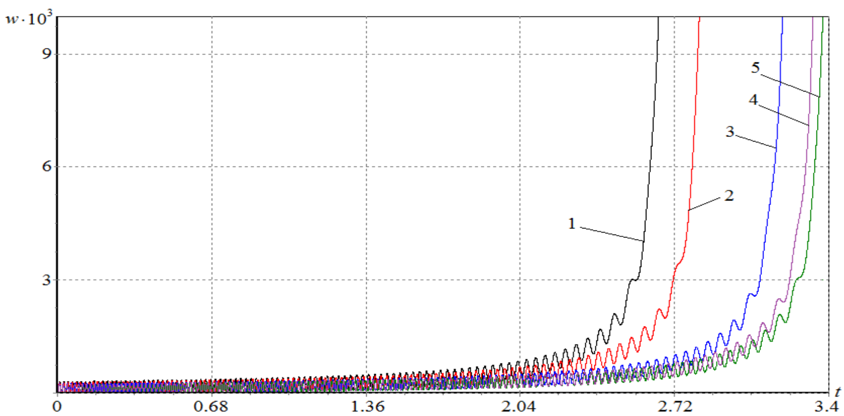

Fig. 4. Dependence of the deflection on time for sandwich plates with different fiber orientated layers
1 - $0^o/90^o$; 2 - $15^o/-15^o$; 3 - $30^o/-30^o$; 4 - $45^o/45^o$; 5 - $45^o/-45^o$

The results of studies of the behavior of reinforced plates for a wide range of changes in their mechanical, physical, and geometric parameters under dynamic compression of one of their sides are shown in Table 1.

Table 1.

№	Geometrical parameters of the plate			Physical parameters		Number of layers	Fiber orientations	The values of critical time
	a, m	b, m	h, sm	q, Pa	$v, MPa/s$			
1	0.5	0.5	0.5	100	2	1	45^0	3.2798
2	0.5	0.5	0.5	100	2	1	45^0	3.2798
3	0.6	0.5	0.5	100	2	1	45^0	3.3358
4	0.7	0.5	0.5	100	2	1	45^0	3.5262
5	0.5	0.5	0.4	100	2	1	45^0	2.0238
6	0.5	0.5	0.3	100	2	1	45^0	0.9192
7	0.5	0.5	0.5	200	2	1	45^0	3.2110
8	0.5	0.5	0.5	300	2	1	45^0	3.1422
9	0.5	0.5	0.5	100	2.5	1	45^0	2.6268
10	0.5	0.5	0.5	100	3	1	45^0	2.1858
11	0.5	0.5	0.5	100	2	1	0^0	2.5984
12	0.5	0.5	0.5	100	2	1	15^0	2.7640
13	0.5	0.5	0.5	100	2	1	30^0	3.1046
14	0.5	0.5	0.5	100	2	2	$0^0/90^0$	2.5984
15	0.5	0.5	0.5	100	2	2	$15^0/-15^0$	2.7860
16	0.5	0.5	0.5	100	2	2	$30^0/-30^0$	3.1396
17	0.5	0.5	0.5	100	2	2	$45^0/-45^0$	3.3242
18	0.5	0.5	0.5	100	2	3	$45^0/-45^0/45^0$	3.2900

4 Conclusion

The study of the problems of the dynamic stability of anisotropic reinforced plates shows that when subjected to dynamic compression along one of their sides, the critical time values mainly depend on the direction of the reinforced fibers in each layer. In single-layer and double-layer plates, the difference in the critical time values depending on the direction of the reinforced fibers in places is 20.7% and 21.8%, respectively. An analysis of the results shows that the most resistant to these types of loads are double-layer plates with fibers located at an angle of - 45 degrees relative to the OX axis in one layer and 45 degrees in another.

References

1. Kozlov M and Sheshenin S, Modeling the progressive failure of laminated composites Mechanics of Composite Materials **51,** https://doi.org/10.1007/s11029-016-9540-0. (2016)
2. Hoksbergen J Ramulu M Reinhall P and Briggs T, A comparison of the vibration characteristics of carbon fiber reinforced plastic plates with those of magnesium plates, Applied Composite Materials 16 263 https://doi.org/10.1007/s10443-009-9093-7.(2009).

3. Allam M Zenkour A and El-Mekawy H, Bending response of inhomogeneous fiber reinforced viscoelastic sandwich plates Acta Mechanica https://doi.org/10.1007/s00707-009-0157-4. (2010)
4. Chen TJ Chen CS and Chen CW, Dynamic response of fiber-reinforced composite plates Mechanics of Composite Materials 47, (2010)
5. Kumar R and Ray M. Active damping of geometrically nonlinear vibrations of sandwich plate with fuzzy fiber reinforced composite facings, International Journal of Dynamics and Control 5 314 https://doi.org/10.1007/s40435-015-0180-3. (2015).
6. Eshmatov B, Dynamic stability of viscoelastic circular cylindrical shells taking into account shear deformation and rotatory inertia Applied Mathematics and Mechanics, 28 1319 https://doi.org/10.1007/s10483-007-1005-y (2005)
7. Eshmatov B. Nonlinear vibrations and dynamic stability of viscoelastic orthotropic rectangular plates, Journal of Sound and Vibration, (2007) **300**, https://doi.org/10.1016/j.jsv.2006.08.024.
8. Eshmatov B, Nonlinear oscillations of a viscoelastic anisotropic reinforced plate Mechanics of Solids 53 568 doi: https://doi.org/10.3103/S0025654418080101. (2019)
9. Khudayarov B and Turaev F, Nonlinear supersonic flutter for the viscoelastic orthotropic cylindrical shells in supersonic flow Aerospace Science and Technology, 84 120 https://doi.org/10.1016/j.ast.2018.08.044. (2019)
10. Khudayarov B Komilova Kh and Turaev F, Numerical simulation of vibration of composite pipelines conveying pulsating fluid International Journal of Applied Mechanics 11 https://doi.org/10.1142/S175882511950090X. (2019)
11. Abdikarimov R and Khudayarov B, Dynamic stability of viscoelastic flexible plates of variable stiffness under axial compression, International Applied Mechanics 50 389 https://doi.org/10.1007/s10778-014-0642-x (2014)
12. Abdikarimov R and Khodzhaev D, Computer modeling of tasks in dynamics of viscoelastic thin-walled elements in structures of variable thickness Magazine of Civil Engineering 5 83https://doi.org/10.5862/MCE.49.9. (2014)
13. Ashton J.E and Whitney J.M, Theory of laminated plates. Technomic Publishing Co. Inc. Stamford (1970).
14. Jones RM, Mechanics of composite materials. McGraw-Hill Book Co. New York. (1970).
15. Qatu M.S, Vibration of laminated shells and plates. Elsevier Ltd. (2004)
16. Reddy J. N, Mechanics of laminated composite plates and shells. Theory and analysis. CRC Press. (2004)
17. Yi-Ming Fu, Nonlinear analyses of laminated plates and shells with damage. WIT Press. (2013)
18. Volmir A.S, The nonlinear dynamics of plates and shells. Nauka Publishers Moscow. (1972)
19. Badalov F Eshmatov Kh and Yusupov M, On certain methods of solving systems of integro-differential equations encountered in viscoelasticity problems Journal of Applied Mathematics and Mechanics 51, **683**, https://doi.org/10.1016/0021-8928(87)90025-6. (1987)
20. Eshmatov B and Khodjaev D 2007 Nonlinear vibration and dynamic stability of a viscoelastic cylindrical panel with concentrated mass Acta Mechanica 190 165 https://doi.org/10.1007/s00707-006-0418-4
21. Abdullayev A.A., Ergashev T.G. Poincare-tricomi problem for the equation of a mixed elliptico-hyperbolic type of second kind. Vestnik Tomskogo Gosudarstvennogo Universiteta, Matematika i Mekhanika, (65), pp. 5–21, DOI 10.17223/19988621/65/1. (2000)

22. Islomov B. I. Abdullayev A.A. On a problem for an elliptic type equation of the second kind with a conormal and integral condition. Nanosystems: Physics, Chemistry, Mathematics, 9 (3), p. 307-318, (2018
23. Vahobov V., Abdullayev A.A., Kholturayev Kh., Hidoyatova M., Raxmatullayev A. On asymptotics of optimal parameters of statistical acceptance control. Journal of Critical Reviews 2020, 7 (11), pp. 330-332, (2020).
24. Yuldashev T.K., Islomov B.I., Abdullaev A.A. On solvability of a Poincare-Tricomi Type Problem for an Elliptic-hyperbolic Equation of the Second Kind. Lobachevskii Journal of Mathematics, 42, (3). pp. 662–674.