A probabilistic and statistical model of rock deformation

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Abstract. A new approach to the study of strength characteristics of the rocks on the basis of probabilistic and statistical models of the deformation processes (models of dilatancy initiation processes) under conditions of uncertainty is presented. The main purpose of the study is to create geological and mathematical tools, which could be used to study the development of volumetric deformation (dilatancy, disintegration) of reservoirs at a constant tension, acting on the array. The information and analytical basis of the performed study consists of methods of mathematical statistics and probability theory and the sedimentary rocks research methods. The probabilistic-statistical approach is formed in the study of deformation processes of the productive stratum structure, taking into account the degree of heterogeneity of the reservoirs. The obtained analytical expressions allow us to determine the moment of the beginning of the volumetric deformation process (dilatancy, disintegration) in the rock, similar to the identification of the bifurcation point in the development of geological and dynamic systems

1 Introduction

In the West Siberian oil-and-gas bearing basin (WSOGB), a large segment of hydrocarbon reservoirs are sedimentary rocks, the primary conditions of accumulation and further conversion of which are represented by arkosic (with fragments of granite), tuff (with sequential build-up structure of the sandstones and fragments of the other rocks), graywackle (fragments of rock-forming minerals), and other structures.

The further transformation degree of sedimentary materials features (secondary transformations) is represented by fibrous, crystalline, and other changes in the structure of the minerals [1].

The structure of a hydrocarbon reservoir is understood as a system of geometric, energetic and morphometric characteristics determined by qualitative and quantitative interactions among the rock components within a single spatial organization of a geological body [2, 3].

In the modern development of the oil-and-gas industry, the deterioration of the reservoirs' structure and, accordingly, the reserves dedicated to them, is characterized by the increased hard-to-recover reserves of oil and gas, as a result of both bringing into development of the deposits in the late stage and insufficient knowledge about the structural quality characteristics of the rocks for the new productive strata at the initial stage of development.

Similar characteristics, worsening the oil production system, are heterogeneous concentrations of abrasive particles in different areas of the deposit, geological and geophysical features of the formations, physical and chemical properties of oil (gas), and others [4].

Therefore, the development of an approach that takes into account the qualitative structural features of the reservoirs will allow us to predict their strength characteristics, thereby minimizing the probability of rock destruction and inefficiency of the planned geological and engineering operations.

2 Materials and methods

The information and analytical basis of the performed study consists of methods of mathematical statistics and probability theory and the sedimentary rocks research methods. The probabilistic-statistical approach is formed in the study of deformation processes of the productive stratum structure, taking into account the degree of heterogeneity of the reservoirs. The paper is based on the materials presented in the works of Katanov Yu. E., Nesterov I. I., Yakovlev Yu. V., Yagafarov A. K. and other domestic and foreign scientists.

3 Results

When moving to the final stage of the development of hydrocarbon deposits, it is necessary to fulfill the conditions for changing the operation modes in accordance with the current nature of structural complications, the factors of which can function both separately and simultaneously, strengthening each other (bifurcation zone of the research of different technologies). The solution to this process is the formation of a complex dynamic system of oil and gas production, which is based on special technologies and methods with different periods of implementation [5, 6].

On the other hand, there is a problem of obtaining reliable qualitative information, both about the structural state of the reservoirs and about other parameters that need to be taken into account when assessing the economic and technological efficiency of the planned technologies.

Constant transformations of the reservoir structure occurring under external influence lead to the notion of their heterogeneity.

The heterogeneity of the productive stratum implies changes in the physical and mechanical features of the reservoir formation, due to constant transformations of lithological and structural-facial characteristics, which affect the filtration of hydrocarbons in the studied wells [7]. Such transformations are probabilistic.

Therefore, mathematical modeling of such random phenomena (statement of a probabilistic research problem) will require correctness and adequacy in the analysis of the simulated process, namely, what probabilistic model should be considered in a particular problem and what its characteristics are fundamental [8, 9].

The process of rock deformation is an inevitable phenomenon. However, it is necessary to understand when the deformation of the reservoir structure is not critical for the hydrocarbon filtration process as a whole and when the volumetric deformation (dilatancy) and disintegration can appear during the operation process [10]).

Mathematically, the process of the reservoir volumetric deformation during the productive well operation can be described as follows, making several assumptions [11]:

- At the initial stage, the reservoirs have a stable structure.
- In the process of well operation, when reservoir fluid influence acts on the reservoir structure (the effect of adsorption reduction of the strength - the Rehbinder effect), point and linear defects of an “avalanche” nature accumulate and further develop (which is the movement of the defects of natural and wave origin in the existing rock volume), which leads to an increase in the probability of volumetric deformation, which can be presented as the beginning of dilatancy, shift, or compaction of rock.

The probability of the Rehbinder effect can be characterized by a complex of the following main reasons: filtration characteristics of the reservoir and the degree of its structure consolidation; the porosity and density of the reservoir; the chemical composition of the cementing material (clay or carbonate) in the structural nodes of the reservoir and filtration activity of the formation fluid in relation to it.

The statistical nature of the reservoir structural features lies in the random distribution of vacancies in the rock mass; therefore, the formation of local zones in the rock structure, weakening due to adsorption processes, is also a random event. Insufficient study of the statistical nature of the reservoir structure and failure to into account their highly heterogeneous features will inevitably lead to inadequate research results, especially in the complex of the sciences such as solid-state physics, materials science, continuum mechanics, and others [12, 13].

In addition, the variability of the strength parameters of the lithological types reservoirs is also not taken into account when building a structural-geological model of a rock mass, which leads to significant errors in obtaining of the modeling results.

For clay reservoirs, we consider the development of the dilatancy process and form a statistical model of changes in the rock structure during the transition from a stable initial state to destruction on the basis of the methods of probability theory [14]

Dispersion, as a characteristic of the normal distribution law of random variables, can be used as a qualitative measure of the reservoir heterogeneity. At the same time, the greater the degree of heterogeneity of the reservoirs, the higher the dispersion value will be.

In this case, the geological characteristics of the normal distribution (humidity, density, permeability, porosity, etc.) will have individual intervals of their values for different areas of the studied rocks, so the degree of their heterogeneity will be tied up to the specifics of the possible characteristics.

The implementation principle of the variance analysis is based on the assessment and allocation of individual factors within a single system of events that cause variability (heterogeneity) of the output characteristics and their “decomposition” into separate composite subsets, which is due to the predominance of independent input factors and their interaction with previously unaccounted random causes.

Consequently, the study of the heterogeneity of a discrete network of observations of a lower-order (studies at the micro-level) can manifest itself as non-random or random patterns; with an increase in the distance between individual observations due to a corresponding increase in the proportion of random variability, the proportion of observed non-random variability decreases, and vice versa.

Application of this principle to geological systems means that in the process of studying the reservoir structure it is necessary to determine the degree of their heterogeneity in the aggregate strength characteristics for each lithological type separately, which corresponds to the search for the weakest link in the structure of an individual reservoir [15].

The concept of finding the weakest link can be interpreted as a mental division of the rock volume into many local areas with different levels of structural defects, the limit of which will be the stage of destruction of the rock starting with the weakest structural link, the occurrence of which is the result of a combination of random factors.

If the random factors are independent from each other, and, denoting $P_i P_i$, as the probability of occurrence of the weakest link in the ith factor, then for the system of normal distribution of the reservoir strength parameters, we can write the following [16]:

$$P_i = \frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot \int_0^{x_i} e^{\left(-\frac{(x-\mu)^2}{2\sigma_i^2}\right)} dx \qquad (1)$$

where $\mu, \sigma \mu, \sigma$ are probabilistic characteristics of the normal distribution, which are represented by specific values when choosing a scale of measurement, wherein $\sigma_i > 0$ $\sigma_i > 0$ represents the mean square deviation or displacement of the studied parameter, which is the systemic scale factor; $\mu \mu$ is the characteristic of the shift of the studied parameter, e.g., the average value, distribution mode, expectation or median; $x_i x_i$ is the current value of the studied parameter for each ith step.

Then the full probability of the beginning of dilatancy (dilatational strain) will be formed by a set of partial probabilities of the weakest link for each selected geological fragment of the rock mass, except for the cases when the possible predominance of such link is excluded (when the structure of the reservoir is relatively stable or in the absence of a clay component in the reservoir, because the Rehbinder effect can be observed only in clay reservoirs):

$$P_i = \sum_{i=1}^{n} \left\{ 1 - \prod_{i=1}^{n} \left[1 - \frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot \int_0^{x_i} e^{\left(-\frac{(x-\mu)^2}{2\sigma_i^2}\right)} dx \right] \right\} \qquad (2)$$

For practical assessments of dilatancy processes, formula (2) a priori provides for obtaining of the frequency distributions of the experimental parameters under study and their statistical characteristics of displacement and dispersion [17].

It should be noted that the manifestations of the heterogeneity of reservoirs can be studied by the parameters of permeability, density, humidity, porosity, physical and chemical composition, microstructure and structure, as well as relative to those other characteristics. At the same time, the analysis of changes in reservoir heterogeneity, taking into account the determination of the dispersion values of their strength indicators, can be considered as the basis for predicting of structural parameters during dilatancy (dilatational strain).

The heterogeneity of the reservoirs can be represented at two levels:

- Quantitative level, when analytically (using special formulas and expressions) it is possible to determine the signs of differences between reservoirs from each other;
- Qualitative level, when the reservoirs are characterized by the signs of differences in their composition, structure, and so on;
- We will highlight the factors that affect the weakening of the reservoir structure:
- A set of indicators that determine the external impact on the reservoir under hydrodynamic loads, physical and technological impact, etc;
- Strength characteristics of the reservoirs, for example, the compressive strength (tensile, shear), parameters of spatial structural bonds, porosity, and others.

As an example of the proposed approach to the study of the process of dilatational deformation of the reservoir (the beginning of dilatancy), let us consider physical and technological factors, such as ultimate compression strain $\delta_{COMPRESSION} \delta_{COMPRESSION}$, which varies according to a normal distribution law from zero to some ultimate value (Fig. 1); in this case, the weighted average value of the reservoir compression strength is $\delta_{COMPRESSION-S} \delta_{COMPRESSION-S}$ («s» - semi-consolidated), at which the structure starts to collapse (irreversible deformation and, as a result, disintegration).

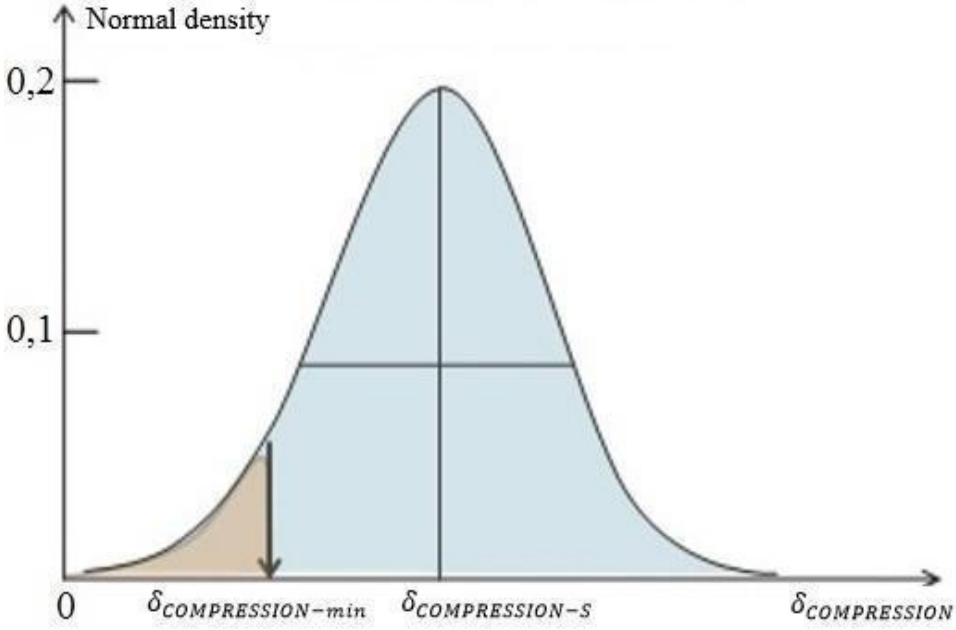

Fig. 1. The sector of local (semi-consolidated) zones of the reservoir (dark triangle), presented on the normal distribution curve with changing compressive strength.

According to Fig. 1, there is a certain minimum value of the strength $\delta_{COMPRESSION-min}\delta_{COMPRESSION-min}$, of a reservoir, the structure of which experiences a complex of loads, below the level of which the reversible (elastic) deformation prevails. The range of values from zero to $\delta_{COMPRESSION-min}\delta_{COMPRESSION-min}$ is represented by a sector of local (semi-consolidated) reservoir areas, relative to which $\delta_{COMPRESSION-min}\delta_{COMPRESSION-min}$ is the cut-off point for the reservoir compressive strength: this is the level of impact (complex stress at the selected reservoir section), at which volumetric deformation of the reservoir structure begins (beginning of dilatancy).The closer the value of $\delta_{COMPRESSION-min}\delta_{COMPRESSION-min}$ to the distribution center of $\delta_{COMPRESSION-S}$ $\delta_{COMPRESSION-S}$ the larger is the area of the sector of semi-consolidated reservoirs and the higher is the probability of dilatancy (volumetric deformation).

Similarly, it is possible to determine the parameters of the strength characteristics of the displacement of the tension structure of the reservoir (shear). In this case, the value of the dispersion of the normal distribution of strength characteristics of the reservoir will be of fundamental importance in the study of the sizes of zones of volumetric deformations and zones of structural stability the reservoir.

Fig. 2 shows wide- and narrow-dispersed distributions of the ultimate strength of the reservoir for the compression process $\delta_{COMPRESSION}$ $\delta_{COMPRESSION}$ under the same mathematical expectations, and Fig. 3 illustrates the sectors of local (semi-consolidated) reservoir areas that are subject to structural destruction (disintegration) during the compression $\delta_{COMPRESSION}\delta_{COMPRESSION}$.

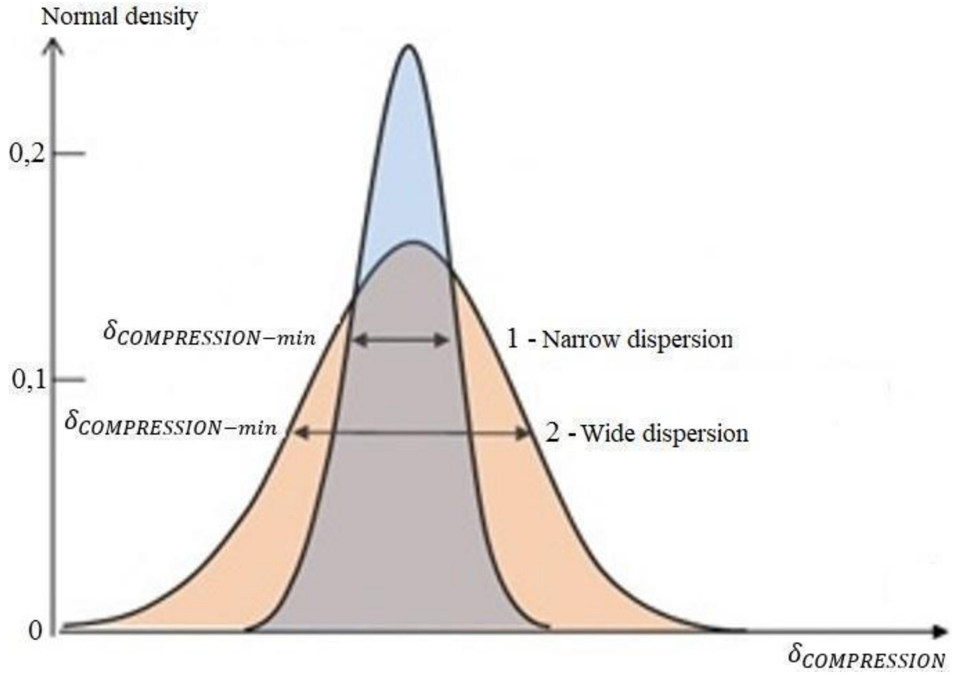

Fig. 2. Reservoir compression strength distribution of narrow and wide dispersion.

At a high level of dispersion (Curve 2, Fig. 2), the size of the volumetric deformation zone will be significantly higher than at a low dispersion level (Curve 1, Fig. 2).

To estimate the reservoir volumetric deformation zones (dilatancy zones) analytically, we will introduce a dimensionless parameter $\alpha\alpha$, which is the relative probability of the reservoir dilatancy (volumetric deformation) when the individual (for each lithological type) compressive ultimate strength of the reservoir is reached

$$\alpha = \frac{p\left(\frac{\mu}{\beta}\right),}{p(\mu)} \tag{3}$$

where $p(\mu)p(\mu)$ is the probability of volumetric deformation at point $x = \mu x = \mu$; $p\left(\frac{\mu}{\beta}\right)p\left(\frac{\mu}{\beta}\right)$ is the probability of volumetric deformation at point $x = \frac{\mu}{\beta}x = \frac{\mu}{\beta}$; $\beta\beta$ is the displacement parameter of the ultimate strength of the reservoir, which characterizes its location in the total volume of the rock mass when lithological types change.

Let's take into account formula (1) for the normal distribution; then formula (3) will take the following form (reducing by a common indicator $\frac{1}{\sigma_i \cdot \sqrt{2\pi}}\frac{1}{\sigma_i \cdot \sqrt{2\pi}}$) in the numerator and denominator):

$$\alpha = \frac{\frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{2\sigma_i^2}\right]}}{\frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot e^{\left[-\frac{(\mu-\mu)^2}{2\sigma_i^2}\right]}} = e^{-\frac{\mu^2}{2\sigma_i^2} \cdot \left(\frac{1-\beta}{\beta}\right)^2} \tag{4}$$

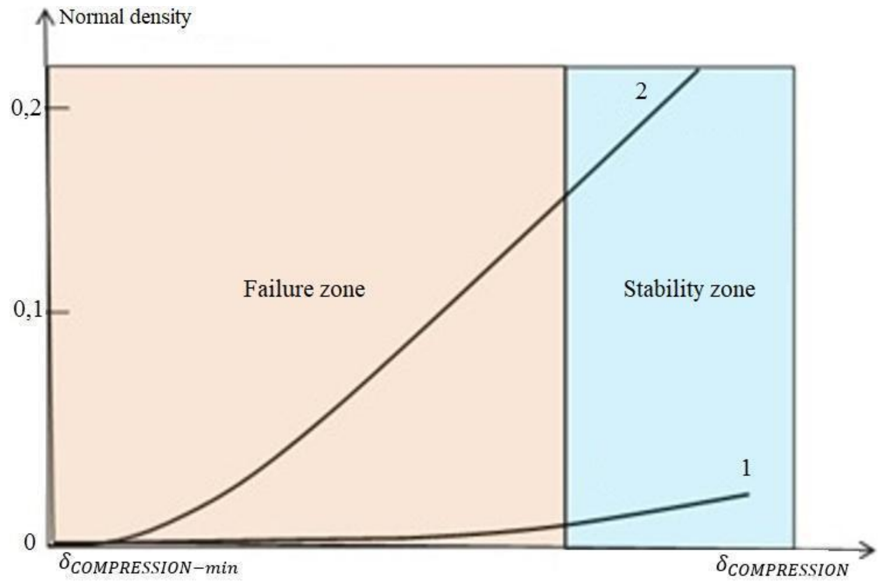

Fig. 3. Structural presentation of the reservoir stability and failure zones at complex impact on the rock mass.

Since it is proposed to use the parameter of the normal dispersion distribution of the reservoir strength characteristics as a measure of their heterogeneity (the higher the width of the normal distribution, the higher the degree of heterogeneity is), this position can be disclosed by the example of the reservoir compressive ultimate strength $\delta_{COMPRESSION}$ $\delta_{COMPRESSION}$ (Fig. 1).

The greater the proportion of the semi-consolidated reservoir in the total mass of the studied rock, the higher the probability of the beginning of dilatancy (volumetric deformation of the rock) is. Since the total mass of the rock is represented by a set of separate geological areas, each of which may have an individual character of changes in their structure under the complex influence, then taking into account Formula (4), we will conditionally combine all local sections of the rock into a single research system, the total number of which will depend on the predominance of a set of different lithological types in a single rock mass:

$$p_{\delta_{COMPRESSION-REDUCED}} = \frac{\frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot \int_0^{\delta_{COMPRESSION-min}} e^{\left[-\frac{(\delta_{COMPRESSION}-\mu)^2}{2\sigma_i^2}\right]} d\delta_{COMPRESSION}}{\frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot \int_0^{\infty} e^{\left[-\frac{(\delta_{COMPRESSION}-\mu)^2}{2\sigma_i^2}\right]} d\delta_{COMPRESSION}} \tag{5}$$

where $p_{\delta_{COMPRESSION-REDUCED}}$ $p_{\delta_{COMPRESSION-REDUCED}}$ is the reduced probability of the beginning of dilatancy (volumetric deformation of the reservoir), determined by the value of the compressive ultimate strength of the reservoir; $\delta_{COMPRESSION-min}$ $\delta_{COMPRESSION-min}$ is the minimum value of the compressive ultimate strength of the reservoir .

By itself, Formula (5) is the proportion of the areas of the selected figures (Fig. 1) - “the area of the dark curved trapezoid” - represented by the elastic deformation sector which corresponds to the local (semi-consolidated) areas of the reservoir, and "the area under the distribution curve of the parameter $\delta_{COMPRESSION}\delta_{COMPRESSION}$", which corresponds to the zone of dilatancy (volumetric deformation).

Since the volumetric deformation of the reservoir (under constantly increasing stress) is an unavoidable event (the probability of this event is 100%), then assuming the denominator in Formula (5) equal to one, we get the following:

$$p_{\delta_{COMPRESSION\ -REDUCED\ .}} = \frac{1}{\sigma_i \cdot \sqrt{2\pi}} \cdot \int_0^{\delta_{COMPRESSION\ -min}} e^{\left[-\frac{(\delta_{COMPRESSION}\ -\mu)^2}{2\sigma_i^2}\right]} d\delta_{COMPRESSION} \tag{6}$$

To obtain a linear record of the formula (6), we use the following scheme for the case of a narrow-dispersed distribution of the parameters of the reservoir compressive strength (Fig. 2, Curve 1 and Fig. 3, Curve 1) when the corresponding ultimate strength is reached (Fig. 4).

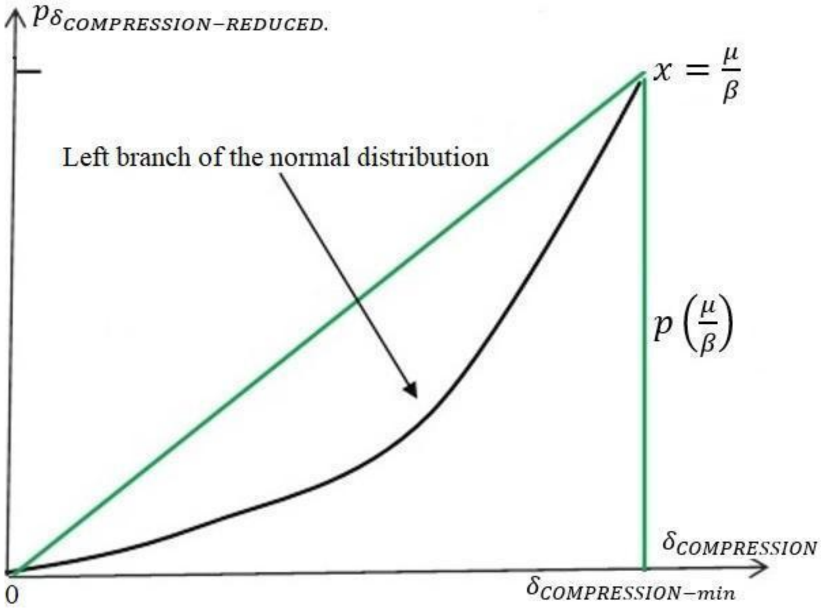

Fig. 4. Graphics visualization of the probability of dilatancy beginning (reservoir volumetric deformation $p_{\delta_{COMPRESSION-REDUCED.}}$ in the case of narrow dispersion distribution of the ultimate strength of its compression.

The area of the "green triangle" (Fig. 4) will correspond to the range of values of the probability of the beginning of dilatancy (volumetric deformation of the reservoir). In accordance with the theorem on finding the area of triangles, the area of the investigated triangle is equal to a half of the product of its base ($\delta_{COMPRESSION-min}$ $\delta_{COMPRESSION-min}$, Fig. 4) by the height ($p\left(\frac{\mu}{\beta}\right)p\left(\frac{\mu}{\beta}\right)$, Fig. 4), and we get the following formula:

$$p_{\delta_{COMPR\ .-RED\ .}} = \frac{1}{2} \cdot \delta_{COMPRESSION\ -min} \cdot p\left(\frac{\mu}{\beta}\right) = \frac{1}{2\sigma_i \cdot \sqrt{2\pi}} \cdot \delta_{COMPRESSION\ -min} \cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{2\sigma_i^2}\right]}$$

(7)

Formula (7) represents “upper estimations” of the probability of the beginning dilatancy (volumetric deformation of the reservoir), since the area of the “green triangle” (Fig. 4) is bigger than the area of the “same (dark) triangle” (Fig. 4), where the so-called hypotenuse is represented by the left branch of the normal distribution of the compression strength parameter (Fig. 4).

Let’s investigate how the dependence of the reduced probability of the beginning of dilatancy changes in the case of a wide-dispersed distribution of the ultimate compressive strength of the reservoir (Fig. 2, Curve 2 and Fig. 3, Curve 2) when the dispersion zone is doubled (with an increase in the rock heterogeneity, as if one lithological type of the reservoir is replaced by another one with a high level of clay content, Fig. 5). Then, taking into account Formula (7), we get the following:

$$\frac{p_{\delta_{COMPR.-RED.}}(\sigma_i)}{p_{\delta_{COMPR.-RED.}}(2\sigma_i)} = \frac{\frac{1}{2\sigma_i\cdot\sqrt{2\pi}}\cdot\delta_{COMPRESSION-min}\cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{2\sigma_i^2}\right]}}{\frac{1}{4\sigma_i\cdot\sqrt{2\pi}}\cdot\delta_{COMPRESSION-min}\cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{8\sigma_i^2}\right]}} = \frac{2\cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{2\sigma_i^2}\right]}}{e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{8\sigma_i^2}\right]}} = 2\cdot e^{\left[-\frac{\left(\frac{\mu}{\beta}-\mu\right)^2}{2\sigma_i^2}\right]^{\frac{3}{4}}} \tag{8}$$

Where $p_{\delta_{COMPR.-RED.}}(\sigma_i)p_{\delta_{COMPR.-RED.}}(\sigma_i)$ is the reduced probability of the beginning of dilatancy when the dispersion $\sigma_i\sigma_i$; $p_{\delta_{COMPR.-RED.}}(2\sigma_i)p_{\delta_{COMPR.-RED.}}(2\sigma_i)$ is the reduced probability of the beginning of dilatancy (volumetric deformation of the reservoir) when the dispersion amounts to $2\sigma_i2\sigma_i$.

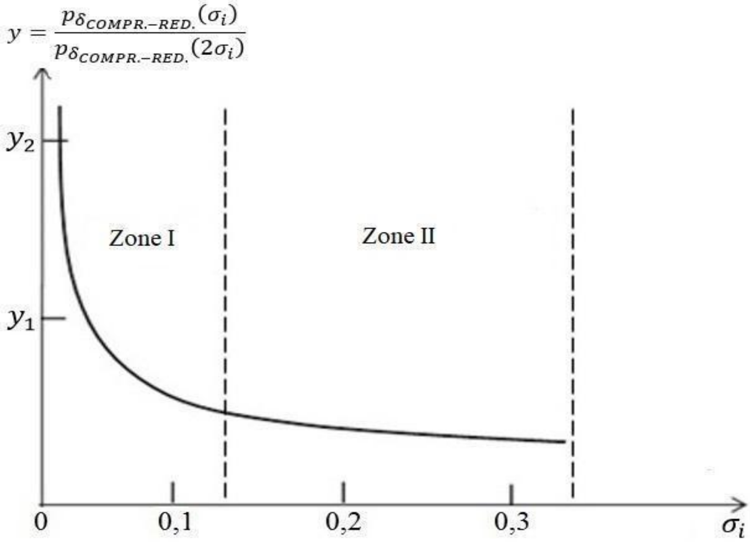

Fig. 5. The general trend of the change in the probability of volumetric deformation (dilatancy beginning) of the reservoir with an increase in the dispersion by a factor of two in the strength characteristics (in the case of a wide-dispersion normal distribution).

The dependence constructed according to Formula (8) is non-monotone; the most interesting part of the curve (Fig. 5) is the part when there is a sharp drop in the ratio $y = \frac{p\delta_{COMPR.-RED.}(\sigma_i)}{p\delta_{COMPR.-RED.}(2\sigma_i)}$ value corresponding to Zone I and, with an increase in dispersion, determining the change in the reduced probabilities of volumetric deformations of the reservoir.

On the right side of the y curve corresponding to Zone II, a slow asymptotic approximation of the probability curve to some constant value prevails.

Consequently, with an increase in the spread of the strength parameters the reservoir, the probability of the prevalence of volumetric deformation (the beginning of dilatancy) rapidly increases.

The obtained formulas (7 and 8) are of practical interest for mathematical modeling of the heterogeneity of reservoirs when changing their strength parameters.

In the same way, we can calculate the probability of the beginning of dilatancy (volumetric deformations) of the reservoirs in the case of a complex impact on the rock mass that leads to stretching (compression) of its structural features.

If the geological and mathematical model of the reservoir deformation process uses their averaged strength characteristics, then during the calculations, the systemic error will accumulate; this is caused by the fact that any rock is a highly heterogeneous geological environment, and for correcting the averaging of the parameters it is necessary to make some adjustments [18].

For example, let's consider the approaches to the parameters averaging, presented in [19, 20]. In this work, the strength of piece-wise heterogeneous rocks and individual rock masses was studied. The following operator was used for weighted averaging of any indicator of the mineral properties (grains) over the entire area $a_i a_i$ with the weight $\Psi_i \Psi_i$,

$$\overline{a} = \sum_{i=1}^{n} a_i \cdot \psi_i \quad . \qquad (9)$$

However, the method of weighted averaging in the case of normal distribution of the quantity leads to estimates that correspond to the mathematical expectation, the center of the distribution of the studied parameters. But this approach is not always justified, since one lithological type of rocks is replaced by the subsequent ones, and it is generally incorrect correct to form a global trend of changing the lithological strength characteristics of a rock mass as a whole.

4 Discussion

The analytic expressions $UV_1^1 UV_1^1$ are applied in the study of real productive strata of the fields in West Siberia, in particular, the Las-Eganskoye reservoir, and the results are presented at scientific conferences, meetings of the Expert Council of the West-Siberian Research Institute of Geology and Geophysics and meetings of the Development and Exploitation of Oil and Gas Field Department, the Oil and Gas Fields Geology Department, and the Department of Applied Geophysics of Industrial University of Tyumen.

5 Conclusions

The management of the West Siberian Research Institute of Geology and Geophysics, Tyumen, Russia, formed a number of recommendations on the use of the presented statistical approach and the results obtained during its implementation for studying the structural features of oil-bearing deposits of the Tyumen formation, Russia.

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