Catastrophes, fractals and chaos in geoengineering and water treatment systems

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Abstract. The present paper provides the assessing scale of the actual state of the geoengineering complexes and the model for predicting the behavior of supporting structures. To predict accidents in geoengineering and water treatment facilities it is proposed to apply the results of the theory of catastrophe theory, fuzzy sets, chaos theory and the theory of possibilities for the selection of optimal models of system behavior for a particular situation. It is shown that determination the limit boundaries in which the operation of the system can exist is impossible without the consideration of geoengineering treatment facilities and their components as fractal structures, functioning under conditions of "chaos". To minimize risks of damages of sewage geoengineering systems the monitoring with measurement of the toxic gases concentration and comparing it with the obtained one should be provided. The use of models of internal and external corrosion which includes the elements of the theory of fuzzy sets helps to evaluate completely the state of water supply and treatment facilities network.

1 Introduction

In the processes of water treatment and water purification, there are very often the problems that are clearly nonlinear and, at the same time, can not be analysed with the methods used in "classical" nonlinear systems, or give the results that are far from the optimal ones (and even plausible). These problems include quasiperiodic changes in the states of shallow water basins (such as, for example, water intakes and artificial seas), which may receive stormwater or wastewater; behavior of biological treatment systems; general behavior of the entire complex of water treatment systems in different spatial and temporal boundaries and scales, etc.; and finally, the behavior of individual components of treatment facilities, pipelines and collectors under the conditions of a man-made disaster emerging in their depths. All these problems can be successfully solved only when we turn to the use of a new mathematical apparatus based on the results of the theories of fuzzy sets, catastrophes, fractals and chaos [1-5].

The aim of the work is to determine the conditions that ensure the normal functioning of a complex system under conditions of uncertainty, as well as an effective assessment of its parameters.

As the World Health Organization underlines, corrosion monitoring is very important for the water supplies functioning [6]. Concrete corrosion is one of the most significant failure mechanisms of sewer pipes, and can reduce the sewer service life significantly [7]. Result

of presented research indicate that water is corrosive at 10.6%-89.4% of drinking water supply reservoirs [6].

The wastewater system as a system functioning under conditions of fuzzy data needs the regular monitoring and control of its parameters. The proposed performance-rating system evaluates each parameter and combines them mathematically through a weighted summation and a fuzzy inference system that reflects the importance of the various factors [8].

An analysis of the results of observations and analogies suggests that the structures and functioning of natural and geoengineering water treatment systems demonstrate in a broad sense the self-similarity of water purification processes from pollution both in natural conditions and in engineering treatment facilities – the same principles are applied, the only difference is in productivity and quality of treatment. In other words, both those and other systems more or less equally fit into a wide range of spatial, temporal and quantitative scales, which indicates the presence of a certain symmetry of the scales. In this case, for the study of such systems, one can fully use the methods of fractal analysis and elements of the chaos theory [9-10]. But such an approach is suitable not only for analysing the structures of water treatment and the processes occurring in them, but also for optimizing the geometry of tree-like configurations of water distribution systems, for displaying contamination isolines caused by man-made accidents and natural disasters, and also for using similar displays obtained at

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different stages of monitoring and to analyse the dynamics of pollution migration.

It is also perspective to use the methods mentioned above in the analysis of a number of biological catastrophes associated with changes in water quality under the influence of external factors. Finally, it is interesting to apply the proposed approach together with the results of the theory of fuzzy sets and the theory of possibilities for the selection of optimal models of system behavior for a particular situation. The proposed approach also assumes the use of catastrophe theory, fuzzy set theory and chaos theory to predict accidents in geoengineering and treatment facilities.

2 Reliable assessment of the state of geoengineering structures

Let's consider some special cases of the proposed approach implementation. The behavior (dynamics) of the bottom layer and bottom sediments in shallow lake

ecosystems, as well as in most artificial seas formed during the construction of hydroelectric power plants, is an example of a classic catastrophe with several attractors [11]. Usually two different states of such shallow water bodies are identified. In the state of the bottom layer under conditions of high transparency, there is a vegetation ecosystem at the bottom. With an increase in the intake of biogenic components of wastewater (municipal wastewater or storm water), turbidity in the water increases and the ecosystem crosses the catastrophic threshold and transforms into a hypertrophic turbid phytoplankton ecosystem in bottom sediments. In reality, there are lakes that regularly pass from one state to another (Lake Ontario in North America, for example, regularly experiences similar cataclysms). In some water bodies, the beginning of spring (loading of biogenic components) determines the state of the ecosystem in summer. This is an example of a bifurcation in the behavior of an ecosystem when it gets to the point of return of the seasonal "eight" (see Fig. 1).
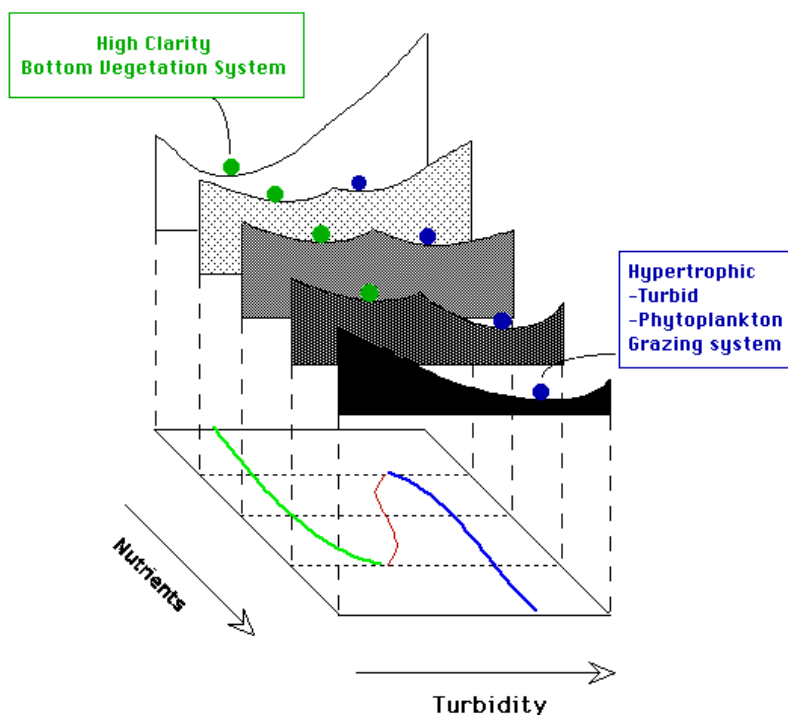

Fig. 1. Ecological disaster such as a catastrophic leap, when small changes in the value of the manager (nutrients supply) major changes in the state value (turbidity).

Discontinuous transitions from one mode to another regimes occur when the parameter in the system changes smoothly:

$$F(x; a, b) = (1/4) \cdot x^4 + (1/2) \cdot a \cdot x^2 + b \cdot x \qquad (1)$$

with a phase state variable (turbidity a). On the plane of values of the parameter b (nutrients) and the phase variable, stationary modes form a smooth curve, which can be described by the equations:

$$(-a/3)^{1/2} = (b/2)^{1/3}, \qquad (2)$$

$$(-a/3)^3 = (b/2)^2, \qquad (3)$$

$$(a/3)^3 + (b/2)^{2 = 0}. \qquad (4)$$

Its projection onto the parameter value axis has some peculiarities. When the parameter changes, one of the stable equilibrium positions disappears, merging with the unstable one, and the system is forced to jump to a new regime with sharply different characteristics. Understanding the essence of a disaster allows to anticipate a leap and develop heuristics that either prevent bifurcation or change the water treatment technology (by

using other reagents, their concentration or the sequence of using different treatment technologies).

Predicting the behavior of supporting structures (columns, supports, brackets, etc.) of a treatment plant complex is an extremely important task, and the amplitude of the initial bending of the supporting structure can serve as a hazard indicator. In other words, the safe load is determined by the maximum safe bend (deforiation) [2]:

$$\Delta V(a_1 = s; F_s, \varepsilon) = \varepsilon + \tfrac{1}{2}(\pi/l)^2(F_1 - F_s)s + (3/2^4)F_s l(\pi/l)^4 s^3, \quad (5)$$

where

$$F_s = F_c(s) - k(s)\,\varepsilon, \quad (6)$$

$$F_c(s) = \{F_1/(l - 3/8[as/l]^2)\}, \quad (7)$$

$$k(s) = \{(2/l - s/l)(l/\pi)^2\}/\{l - 3/8(\pi s/l)^2\}. \quad (8)$$

Here $F_c(s)$ – the safe load in the absence of defects, F_1 – the maximum bearing capacity at which the bending amplitude reaches s, l – the coordinate of another end of the supporting structure, ε – the coefficient of dynamic sensitivity to defects, k – a positive constant.

Thus, the problem of assessing the actual state of the geoengineering complexes that are part of water treatment systems can be considered as a procedure for finding the maximum safe bend (stress), for which it is possible to use the data of a set of measurements (observations) of features studied in order to classify the state of structures as or another cluster. The first step in the assessing procedure is to define a scale of states (for example, a five-point one), where "0" corresponds to the absence of any problems (according to all estimates, the structure meets the requirements of the standard), and "4" – to an emergency state requiring destruction and reconstruction of the structure. All intermediate classes of states provide appropriate recommendations for repair and restoration work. From the point of view of the theory of fuzzy sets, these classes (d) can be characterized using Table 1.

The amount of physical wear is associated with the cost indicators of the structure. So, with an increase in physical wear, the initial cost of the structure decreases by the same percentage. Considering the fact that overhaul eliminates physical wear to a certain extent (if we do not take into account the general physical aging of structures, which is a consequence of the manifestation of material fatigue), there is an economic feasibility of overhaul if the cost of this repair does not exceed 40 % of the primary cost of the structure.

The set of signs suitable for classifying or assessing the state on the basis of visual inspection may include the identification of deformations and cracks in columns, beams, joints, etc. In addition, during the assessment of the state of structures there should be considered building materials, the height of structures, the actual duration of operation after next repair, as well as external conditions, conditions and total operating time, characteristics of the soil and foundation.

The formulation of the state evaluation problem is related to the method of decision-making under difficult conditions, when the information received from experts is characterized by uncertainty. It mostly determines the final decision, which has the nature of an inaccurate or approximate conclusion pursuing the achievement of the most reliable answer to the problem. If to take the numerical form of expressing the intermediate diagnosed state in the form, for example, of the function of membership $\mu(d)$ to the corresponding problem, requiring major repairs (see Table 2), and to interpret the values of the estimation truth in the following form: 0 – not true; 0.1-0.3 – weak level of truth; 0.4-0.5 – significant level of truth; 0.6-0.7 – high level of truth; 0.8-0.9 – the truth is almost revealed; 1.0 – the revealed truth, then it is possible to use the following scoring rule:

IF (the number of cracks *CN* is estimated at the level 0.3) *OR* (the presence of large cracks *LC* – at 0.2) *OR* (excessive deformation of structural elements – at 0.1), *THEN* (the overhaul of the *CN* is not needed);
or
IF (*CN* is estimated at 0.6) *OR* (*LC* is at 0.5) *OR* (deformation of structural elements – at the level of 0.5), *THEN* (overhaul of the *CN* is necessary).

Table 1. Scale for assessing the deterioration of structural elements.

Classes, *d*	Physical wear, %	Assessment of technical conditions	General characteristics of technical condition
1-2	0-20	Good	No damages or deformations. Some faults that do not affect operation and are eliminated during routine repair
3-4	21-40	Satisfying	The elements of the structure are generally serviceable, but in need of repair
5-6	41-60	Not satisfying	Operation of the elements of the structure is possible only under the condition of urgent repair
7-8	61-80	Decrepit	The state of the load-bearing structural elements is emergency, and of the not load-bearing ones is very decrepit. The limited performance by the elements of the structure of their functions is possible only under conditions of restrictive and protective measures, or the complete replacement of these elements
9-10	81-100	Kaput	Elements of the structure are in an unusable state. The structure needs to be demolished or radically rebuilt

Table 2. Evaluation of the overhaul problems.

$\mu(d)$	0,2	0,5	0,7	1,0	0,8	0,2
d	2	3	4	5	6	7

Belonging to the state corresponding to the need for the overhaul can be determined from the expression

$$\mu = max\{\mu_1, \mu_{2, ...} \mu_n\}, \quad (9)$$

where the need for overhaul for data group *n*:

$$\bigcup_{uu\,=\,1}^{b} B_i \,, \quad (10)$$

where B_i – the state requiring the overhaul, obtained in accordance with the *i-th* data group (*i*=1 – information on the registered cracks, *i*=2 – data of physical measurements of stresses in structures, etc.).

B_i can be considered as the algebraic sum of damages to each of the components of the structure *j(j≤n)*:

$$B_i = \sum_{o\,=\,1}^{m} D_{ij} \,, \quad (11)$$

where *Dij* – the state of serious damage to the *j-th* component, and

$$\mu(B_i) = 1 - \prod_{j\,=\,1}^{n} (1 - \mu_{Dij}), \quad (12)$$

For example, if there are three main components with the registered cracks, for which it is known that μ_{D11}=0,2, μ_{D12}=0,8, μ_{D13}=0,6, then

$$\mu_{B1} = 1 - \{(1-0,2)(1-0,8)(1-0,6)\} = 0,984. \quad (13)$$

Another approach is also possible. Let $X = \{x_1, x_2,... x_k\}$ – a set of signs, for example, x_1 – a big amount of cracks, x_2 – large cracks, x_3 – large deformations. Let $Y = \{y1, y2,... y_m\}$ – a set of types of potential damages, for example, y_1 – damage from fatigue or breaking, y_2 – plastic deformation, y_3 – instability, y_4 – progressive damage. Let Z – a state of severe damage. If we can find fuzzy relations R (from X to Y) and S (from Y to Z), then the signs of X can be associated with the state of severe damage to the structure Z using the composition $R \cdot S$. If to set the relationship R and S by data from the Tables 3 and 4, then the result can be presented in the form of Table 5.

This result indicates that the presence of the characteristics x_2 (large cracks) and x_3 (large deformations leads to a high assessment of the degree of belonging to a set of structures in a state of severe damage.

Table 3. Fuzzy relation *R*.

		y_1	y_2	y_3	y_4
	x_1	0,9	0,2	0,4	0,4
R =	x_2	0,8	0,3	0,7	0,8
	x_3	0,3	0,8	0,9	0,7

Table 4. Fuzzy relation *S*.

		Z
	y_1	0,4
S =	y_2	0,3
	y_3	0,8
	y_4	1,0

Table 5. Relationship *R* and *S*.

		Z
	x_1	0,4
$R \cdot S$ =	x_2	0,8
	x_3	0,8

In other words, if large cracks and large deformations are observed, then the structure is classified as badly damaged and requiring replacement or overhaul.

The considered approach allows for a more qualified and more reliable assessment of the state of treatment structures and at the same time taking into account the influence of individual components on the state of structures as a whole, which helps to minimize accidents and the corresponding environmental consequences.

3 "Chaos" and unpredictable development of events problem elimination

In the concrete vaults of the gravity collectors of the city sewage system, there is a specific problem: microbiological corrosion of concrete due to biogenic acid aggression (under the influence of hydrogen sulphide escaping from the collector vaults) reduces the resource of these objects in 3-3.5 times (i.e., from 50 up to 10-15 years). Up to 74% of accidents on reinforced concrete pipelines of drainage systems are caused by just such corrosion. Concrete, of course, is protected by waterproofing anti-corrosion coatings, but the last ones do not fulfill their role fully, mainly due to the presence of microcracks. The concentration of hydrogen sulfide C_{H2S} (mg/m^3) in a membrane of condensate moisture on the surface of a concrete structure, exposed to biogenic hydrogen sulfide aggression, at which the coating still retains its properties, can be determined from the following empirical formula:

$$C_{H_2S} = \left\{\frac{414,6*t_{real}}{t_{demand}}\right\}^{0,45}, \quad (14)$$

where t_{real} – the real durability of the coating (in days), t_{demand} – the required durability of the coating (in days).

It should be noted that the standard deviation of the concentration C_{H2S} from the mathematical expectation of this concentration in the atmosphere of real sewerage facilities is $\sigma = \pm 50\%$, which, if the distribution of concentration values is close to the normal law, corresponds to a data uncertainty bandwidth of about 33%. Thus, periodical measurement of the C_{H2S} concentration and comparing it with the obtained one in accordance with the above formula, allows to determine the limit to which operation is permissible without changing the coating, i.e. with minimal risk.

It is also should be noted that the activity of microorganisms that ultimately cause the corrosion is carried out cyclically, and since the initial conditions for each cycle, as a rule, are not the same, there are opportunities for the emergence of "chaos", that is, unpredictable development of events [1], which is very typical for biological treatment systems. In other words,

in such systems it is only possible to predict the boundaries within which the processes can occur. These boundaries are set by "strange attractors". So, for example, the development of a population is under conditions of different initial data characteristic for the operation of treatment facilities, the expected population density of biologically active bacteria N at the time $t+1$, $N_t + 1$ can be represented as a nonlinear autoregressive model

$$N_{t+1} = N_t[1 + \check{r}(1 - N_t/K)], \qquad (15)$$

where N_t – population density at time t, $\check{r}$ – the internal growth rate of the population (Malthusian parameter), and K – the maximum volume of a given population, possible under these conditions.

After certain transformations, this model can be represented as

$$x_{t+1} = rx_t(1 - x_t), \qquad (16)$$

and

$$x_t = N_{t\check{r}} / [(1 + \check{r}K)], \qquad (17)$$

$$r = 1 + \check{r}, \qquad (18)$$

so that x_t turns out to be a dimensionless quantity characterizing the population. If the given model is used to carry out a number of iterations, choosing the values of r in the range 3-4 and at the same time changing the initial conditions, it is possible to obtain solutions within a fairly wide range, which, on the one hand, do not allow to unambiguously predict the result ("chaotic" solution), on the other hand, they determine the boundaries within which this result can be expected.

At the minimal value of r, the first type of dynamic behavior is observed (stable stationary state), in the interval middle, different limit cycles are realized. With a further increase in r, chaotic regimes arise (the values of x_n randomly fill a certain interval), alternating with the so-called r-windows in which limit cycles are observed. Each r-window is characterized by its main limit cycle with the smallest period. This limit cycle allows to determine the boundaries in the middle of which the system behaves to a certain extent predictably, which allows to assess the reliability of the data of specific measurements.

In water distribution systems, disasters such as pipe breaks often occur. Elimination of the consequences is associated with significant resource costs and interruptions in providing consumers. However, it is not giving much attention to how the process of gradual transition from the normal operation mode to the accident development mode is carried out in time, as well as under what qualitative and quantitative variations of variables and factors. Moreover, it is generally accepted that an accident occurs "suddenly", "inevitably", "accidentally".

If to ignore such factors as terrorism, sabotage or natural disasters (earthquake, flood, tornado, etc.), any technogenic accident develops in the bowels of a normally operating system as a result of negligence and lack of professionalism, because factors such as aging, wear, drift of parameters, etc. should be taken into account at all

stages of operation of any systems. In other words, along with the performance of their direct functions, automated systems and personnel should ensure constant monitoring of the state of both each system as a whole and all its main components. Here, monitoring should be understood not only as a periodical fixation of the current state of each of the components of the system, but also the use of models of the behavior of these components in time and as a function of internal and external factors to predict future changes in the state of certain components and the system as a whole. Thus, for the state of the water supply network, it is necessary to use models of internal and external corrosion, which have the following general form:

$$\Delta h = 0,5 \ \{ d_o k_o C_o \ exp\left(\frac{\alpha_o t}{\tau T} E\right) - d_i k_i C_i \cdot$$

$$\cdot exp\left(1 - \frac{\alpha_i t}{\tau_i T} E\right) \}, \qquad (19)$$

where Δh – total decrease in pipe thickness due to internal and external corrosion, k_i and k_o – rate constants of oxidation reactions inside and outside the pipe, C_i i C_o – concentrations of oxidants accordingly, α_i and α_o – transfer coefficients (constants) that determine the influence of other factors on the exponential dependence of the reaction rate E accordingly, τ – time constant of the system in which this other reaction occurs, T – absolute temperature outside the pipe and the temperature of the carrier (water), t – operating time of this line link, d_i and d_o – internal and the outer diameters of the pipe (Fig. 2).

Taking into account the indicators of the soil (or insulation), in which the pipe is laid, and the water flowing in the pipe, as well as the corresponding temperatures, the date of commencement of operation of a particular section of the network, or it's the weakest link, as well as the maximum allowable total reduction in the thickness of the pipe Δh_{max} and the corresponding deterioration of the structure of the pipe material, it is possible to envisage the time when the strength characteristics associated with the thinning of the pipe walls due to corrosion and deterioration of the structural characteristics of the pipe material reach the limit Δh_{max}, beyond which (in the presence of certain transient processes in system) the pipe can break.

This model should also take into account pressure fluctuations (including dynamic shocks) and pumping rates. The heuristics for determining the need to update the pipeline are as follows:

IF [(h$_1$ > h$_{1\ min}$) AND (VaR($\Delta p/h_1$) < VaR ($\Delta p/h_1)_{max}$)]
OR [($\sum \Delta p < \sum \Delta p\ _{max}$)

AND

(Turb LF < Turb Lf$_{max}$)], THEN («norm») ELSE
(«renewal») $\qquad (20)$

Here h_1 – actual or predicted pipe thickness; $h_{1\ min}$ – minimum permissible pipe thickness under operating

conditions; $\Delta p/h_1$ – expected operational pressure surges in the pipeline related to the real or predicted pipe thickness; $(\Delta p/h1)_{max}$ – maximum allowable value of $\Delta p/h_1$; $\sum\Delta p$ – integral assessment of stresses that take place during the observation interval; $\sum\Delta p_{max}$ – maximum allowable integral stress for the entire life cycle of the pipeline; *Turb LF* – current (predicted) turbulence in the pipeline; *Turb Lf_{max}* – maximum permissible turbulence.

But turbulence is a consequence of the fractal structure (tree structure) of the water distribution network, imperfect conditions for the flow of water, accompanied by pressure and velocity fluctuations. Therefore, *Turb Lf*

and *Turb Lf_{max}* should be determined on the base on the apparatus of the theory of fractals and chaos.

By the way, the tree-like structure of the network, one of the conditions for preventing (or weakening) of the turbulence, presupposes the fulfillment of the condition

$$\Sigma s_n \geq S_O, \tag{21}$$

where Σs_n – sum of the cross-sections of *n* branch pipelines coming out of this (root) pipeline with a cross-section S_O.
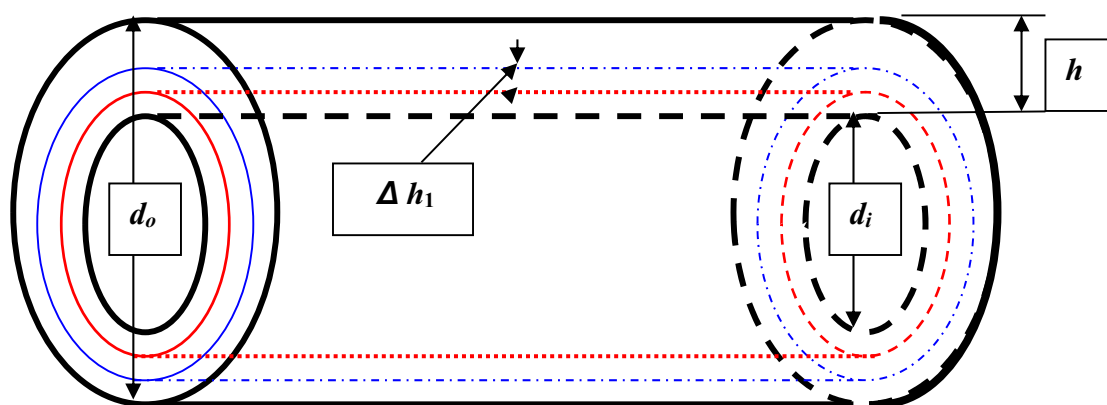

Fig. 2. Change in Δh under the influence of internal and external corrosion.

Conclusions

The consideration of geoengineering treatment facilities and their components as fractal structures, functioning under conditions of "chaos", allows, on the one hand, understanding the processes and connections, and on the other hand, to identify the limit boundaries in which these processes and connections can exist.

The constant or periodical monitoring with measurement of the toxic gases concentration and comparing it with the obtained one allows to determine the limit to which operation is permissible without changing the coating, i.e. with minimal risk.

For the complete evaluation of the state of water supply and treatment facilities network, it is necessary to use models of internal and external corrosion, which includes the elements of the theory of fuzzy sets.

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