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Analysis of Thermodynamic Modelling for Gamma Type Double Piston Cylinder Engine

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Abstract. The exhaust of an automotive engine is one of the main causes of air pollution. These days, many researchers are investigating the waste heat recovery of automotive engines. A two-cylinder gamma-type Stirling engine is chosen for this purpose. The exhaust of a diesel engine is chosen as a heat input source for this purpose. This work explains the isothermal, ideal adiabatic, and non-ideal simple analysis of the Stirling engine. A set of differential equations are solved using Runge-Kutta 4th order method using MATLAB software. These equations describe the pressure, pressure variation, mass, mass flow, and energy flow in the Stirling engine which estimate the power and efficiency. Using non-ideal simple analysis, pressure drop analysis, piston finite speed, heat transfer losses of Stirling engine are calculated. The power estimated by isothermal, adiabatic, simple, and experimental analysis is 133.82 W, 143.75 W, 93.2 W, 111.43 W, and thermal efficiency is 30.70 %, 30.90%, 21.20%, 24.70% respectively. The results of these models are in close agreement with the experimental results.

Nomenclature

A_{wgh}	heater internal wetted area (m2)	A_{wgk1}	connecting pipe internal wetted area (m^2)
Cp	gas heat capacity at constant pressure (J/kg K)		
Cv	gas heat capacity at constant volume (J/kg K)		
CVC	compression clearance volumes (m3)	CVE	expansion clearance volumes (m3)
dh	hydraulic diameter of heater (m)	f	Reynolds friction factor
h	transfer coefficient (W/m2 K)	lr	regenerator effective length (m)
M	total mass of gas (kg)	m	mass of gas (kg)
$\dot{m}$	mass flow rate (kg/s)	N	engine speed (rpm)
NTU	number of transfer units	P	instantaneous gas pressure (Pa)
p_{ch}	charge pressure (Pa)	$Q_{act,h}$	actual heat transferred to heater (J)
Q_h	heat transferred to heater (J)	Q_k	heat transferred to cooler (J)
Q_r	heat transferred to regenerator (J)	$Q_{r,loss}$	regenerator heat loss (J)

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$Q_{r,max}$ maximum regenerator heat loss (J) $Q_{r,min}$ minimum regenerator heat loss (J)
R gas constant (J/kg K) Re Reynolds number
T_h ideal heater gas temperature (K) T_k ideal cooler gas temperature (K)
T_r ideal regenerator gas temperature (K) T_e expansion space temperature (K)
T_c compression space temperature (K)
T_{he} conditional temperature expansion cell/heater (K)
T_{ck1} conditional temperature compression cell/pipe (K)
Twh heater wall temperature (K)
Twk cooler wall temperature (K) Tgh heater gas temperature (K)
Tgk cooler gas temperature (K) u gas velocity (m/s)
Vc compression volume variation (m^3) Ve expansion volume (m^3)
V_{k1} connecting pipe void volume (m^3) V_{k2} cooler volume (m^3)
V_r regenerator volume (m^3) V_{SC} compression swept volumes (m^3)
V_{SE} expansion swept volumes (m^3)

Greek letters

γ specific heat ratios θ crank angle (rad)
μ absolute gas viscosity (Pa s) λ_e crank radius to expansion connecting rod ratio
λ_c crank radius to compression connecting rod ratio ρ gas density (kg/m^3)

1 Introduction

Environmental change is observed as perhaps the biggest danger confronting the planet. Indeed, the utilization of fossil fuel products adds to the primary impact and could prompt calamitous changes in the world's current circumstances [1]. To manage such trouble, a few countries are producing different sources of energy such as sustainable power sources (for example sun-powered energy, biomass energy, waste heat recovery, geothermal energy) which are viewed as a brilliant substitute for petroleum derivatives. Stirling engine is viewed as a significant component to change over sustainable power sources into mechanical work or power [2]. In the last few decades, an incredible attempt was made to Stirling engine because of its multi-fuel capacity, (for example, solar energy, biogas energy, geothermal energy, waste heat recovery, etc.), Low sound, easy to maintain needs, and reduced carbon, if driven by same fuels compared to those other engines [3].

In 1816, Robert Stirling was the first person who concocted and licensed the Stirling engine. The patent number of the engine is 4081, which is a multi-fuel, closed-cycle thermodynamic heat engine. Ideally, the efficiency of the Carnot engine and Stirling engine are equal. The working of the Stirling engine depends on the expansion and compression of gas i.e. helium, nitrogen, hydrogen at a different temperature [4]. The operation in compression and expansion chamber is done moving the gas to and fro motion in heater and cooler chamber through a regenerator which is a key segment in Stirling engine as it stores heat energy. The regenerator retains heat energy from the gas and delivers to cold working as during expansion and compression process respectively [5]. The regenerator plays a significant part to increase the engine performance. Concerning, the heater part of the Stirling engine needed heat energy five times more to produce the same amount of power as with regenerator [6].

There are three types of Stirling engine i.e., alpha, beta, and gamma. Each type has a different mechanical configuration but the same thermodynamic cycle [2]. These engines have the same parts as a heater, cooler, regenerator, displacer, and power piston. But beta type is a more compact design than another engine because the displacer and power piston are located same cylinder [7]. A variety of researchers have tried to evaluate the output of the Stirling engine based on numerical studies and parametric values. A new dimensionless parameter called Beals number was submitted by Beals [23]. West evaluated an empirical factor and fitted into Beals number in order determine power more accurately [23]. Gustav Schmidt conducted engine cycle analysis in 1817 [8]. He predicted the power of the engine cycle based on changes in sinusoidal volume and isothermal changes. An adiabatic second-order nodal analysis of the Stirling engine was presented by Finkelstein [9], where the heat exchanger was supposed to be isothermally and working space was adiabatic. An advance of the study of Finkelstein et al was developed by Berchowitz and Urieli in 1984 [9]. They proposed a non-ideal heat exchanger and split the Stirling engine into five areas namely heater, cooler, regenerator, expansion space, and compression space.

Stirling engine can convey high power and efficiency. Some scientists trying to develop more Stirling engines particularly by optimizing their thermal and mechanical systems. In 2013, Foster et al. [10] patented a novel configuration of the Stirling engine called the Rotatory displacer Stirling to the engine. This configuration comprises a rotatory displacer, heater, cooler a cylinder, a power piston, and a crankshaft. The rotatory displacer has many benefits and tends to be precise more powerful than the traditional Stirling engine's displacer. Because rotatory displacer engine has only three elements namely as rotatory displacer, power piston, and connecting rods. To examine the efficiency of this engine, Bagheri et al. [11] studied a prototype rotatory displacer Stirling engine. In his investigation, he analyzed heat input, phase angle, heat output, load pressure of helium and air. The results stated that the 125° phase angle could deliver more production power than the 90° phase angle. It is also shown that helium gas is more effective to deliver output power than air.

The production of the Stirling engine component is the primary goal of researchers to obtain higher engine efficiency. Importantly, Nielsen et al. [12] have used a one-dimensional heat transfer approach to determine the regenerator effectiveness and efficiency of the Stirling engine. He used a theoretical and experimental technique and divided the regenerator into further sub-regenerator. The findings of their parametric study showed that the regenerator thermal mass ratio and the sub-regenerators are the two factors that influence the performance of the regenerator. He determined that the improvement in these factors increases the performance of the overall Stirling regenerator and therefore boosts the performance of the Stirling engine. For example, by using 19 sub regenerators, he achieved 95% regenerator effectiveness. Genetic algorithms were used by many researchers to enhance Stirling engine efficiency. Ferreira et al. [13] developed a solar Stirling cogeneration process using a multi-objective optimization method for a residential building. The strategy was centered on optimizing system performance and reducing aggregate investment costs. Their process was able to provide 361 KW electrical power output and 965 KW thermal power at an average purchasing cost of €18,059 at the optimum decision variable prices. More recently, Zare et al. [14] have suggested a novel methodology to test the efficiency of piston-free Stirling engines based on the objective function and genetic algorithm. This innovative technique has been used to calculate the phase angle, the displacement, stroke of power piston, and the frequency of piston-free Stirling engine. Cinar et al. [15] designed a compact alpha-type Stirling engine with 98 cc of swept volume and an electrical heater was used as a source of heat. The engine was able to deliver 30.7 W output power using 3.5 bar pressure and 1000 °C hot source temperature. Almajri et al. [4] suggested a new method incorporating a new thermodynamic model and analysis was done also with a CFD simulation for the Alpha Stirling engine. A parametric analysis was performed to analyses various design parameters i.e. pressure, dead volume, porosity, and others. The results show that CFD is an outstanding tool for improving the efficiency of Stirling engines. As far as power production is concerned, the Alpha type can achieve 240 W at a pressure of 3.5 bar, a speed of 555 rpm, − 20 ℃ cold temperature, 1100 ℃ heater temperature. Many experts have focused study on the beta Stirling engine. Aksoy et al. [16] evaluated the efficiency of a Beta Stirling. He used two mechanisms to drive the engine i.e., rhombic drive and crank drive mechanism. For this objective, nodal analysis was used. It is a method in which the heat exchanger and regenerator are subdivided into cells and volume calculation through adiabatic analysis. Equal physical and geometrical conditions have been developed for both mechanisms. Their finding suggests that the efficiency of the rhombic drive mechanism is greater than the crank drive mechanism at 4 bar pressure and 500 W/m2K heat transfer. This difference was clarified that the cooler volume was larger than the heater volume of the crank engine. The working of Stirling engines at low-temperature difference was investigated by Sripakagorn et al [17]. He designed a prototype beta Stirling engine working at different temperature ranges using concentrated solar power technology. The engine was very effectively working at 7 bar pressure and 500 ℃, achieving a combined output of 95.4 W with a thermal efficiency of 9.35 percent. The results of this prototype were similar to that high-temperature difference which shows that it is easy to operate the Stirling engine at low temperature and also cost development. Yang et al. [18] have established an updated non-ideal adiabatic model for a Beta Stirling engine employing rhombic drive configuration. He developed an empirical for mechanical losses and implemented it in a non-ideal adiabatic model and analyzed in detail the pressure drops in the heat exchanger and regenerator. He used an infrared heater as a source of heat. As a consequence, the engine can deliver 556W shaft power at 1100K heater temperature and 1665 rpm. Gamma Stirling engine has been also examined by many scholars. Damirchi et al. [19] built and fabricated a gamma Stirling engine based upon biomass meant for the micro-CHP. He designed a 220cc gamma Stirling engine using the Schmidt principle. During his experiment, he obtained brake power of 96.7 W at 550 ℃ hot temperature and thermal efficiency of 16 percent.Li et al. [20] used solar energy to fuel the Gamma type SE. They concentrated on researching the effects of many losses on the engine performance using Finite Speed Thermodynamics approaches. He set up an overview of exergy and energy analysis for the calculation of energy dissipations. The findings reveal that mass leakage through from displacer working space into buffer spacer and inaccurate design of regenerator is the most significant phenomena which decrease the performance of engine efficiency. Due to such an enhancement, he recommends a decline in clearance leakage, improves the geometry of the regenerator, and the use of insulation for the wall of the cylinder to minimize the displacement losses.

Thermodynamic of gamma-type Stirling engine has been stated in literature commonly. In this work, gamma type double piston cylinder engine is chosen. In all previous literature, solar energy, electrical energy, biomass energy is used as a heat source for heater part. The waste heat from exhaust of diesel engine is used as heat source. In addition, the cooler connecting pipe is ignored. The connecting pipe (dead volume) has a great effect on the performance of the Stirling engine. The gas mass oscillates between compression to cooler space is not exactly calculated. The heat exchange in the other part of the Stirling engine is based on Reynold numbers. This causes a pressure variation from the real estimated pressure average pressure. In the context of improving the performance of Stirling engines, the objective is to design a program to support the evaluation process for a double piston gamma-type cylinder engine. Moreover, the engine is divided into six parts, namely, compression space, cooler tube, cooler fin, regenerator, heater tubes, and expansion space based on the enhanced adiabatic model presented by Alfarawi et al. [6]. The heater, cooler, mean pressure, and frequency of the engine are 424 K, 294 K, 3.5 bar, and 14.67 Hz with helium as a working fluid inside the Stirling engine. The equation for real piston motion is used in this model. The heat transfer losses, load losses, non-ideal regenerator behavior, and piston finite speed losses are included in the model.

2 Engine Details

2.1 Engine Data

The engine analyzed here is a double piston gamma-type Stirling engine. Table 1 describes the geometrical data of the Stirling engine. Figure 1 shows a double piston Stirling engine.

Table 1. Engine parameters of gamma type double piston Stirling engine

Engine data	Values	Engine data	values
Expansion swept volume (m^3)	221E-6	Number of heater tubes	42
Compression swept volume (m^3)	194E-6	Length of heater tubes (m)	0.108
Expansion clearance volume (m^3)	35.3E-6	Diameter of heater tubes	0.007
Compression clearance volume (m^3)	24.3E-6	Cooler connecting tube length (m)	0.155
Phase angle (°)	88	Cooler void volume	223E-6
Regenerator material (wire matrix)	Copper	Cooler free flow area	29.72E-4
Porosity	96.4%	Cooler wet area	228.55E-4
Rotational Speed (rpm)	882	Cooler temperature (K)	294
Pressure (bar)	3.58	Heater temperature (K)	424

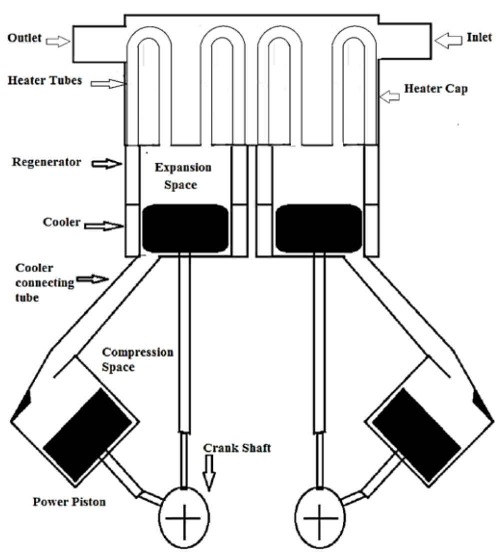

Fig.1. ITS-V2-75 double piston Stirling engine.

2.2 Expansion and Compression Space Volume

Power piston and displacer motion in the Stirling engine leads to volume variation. Therefore, a mathematical model is required as input for our mathematical models. The objective here is to define volume variation as a function of the crank angle. The volume difference of compression and expansion are examined here in terms of compression and expansion swept volume respectively and crank radius to connecting rod ratios λc and λe respectively.

$$Ve = CVE + \frac{V_{SE}}{2}\left[1-\cos\theta+\frac{1}{\lambda_e}\left(1-\sqrt{\left(1-\lambda_e\sin^2\theta\right)}\right)\right] \tag{1}$$

$$Vc = CVC + \frac{V_{SE}}{2}\left[1+\cos\theta-\frac{1}{\lambda_e}\left(1-\sqrt{\left(1-\lambda_e\sin^2\theta\right)}\right)\right]+ \frac{V_{SC}}{2}\left[1-\cos\left(\theta-\frac{22\pi}{45}\right)+\frac{1}{\lambda_c}\left(1-\sqrt{\left(1-\lambda_e\sin^2\left(\theta-\frac{22\pi}{45}\right)\right)}\right)\right] \tag{2}$$

In the above equations, CVE is expansion clearance volume, CVC is compression clearance volume, Vse is expansion swept volume, Vsc is compression swept volume. Equations 1 & 2 are the expansion and compression space volume vary and describe in terms of theta. Figure 2 describes volume variation of expansion, compression, and total volume in the Stirling engine as a function of crank angle. According to volume variation, minimum and maximum volume variation is calculated as 1.8267×10^{-3} m^3 and 2.2155×10^{-3} $m^{3,}$ respectively.

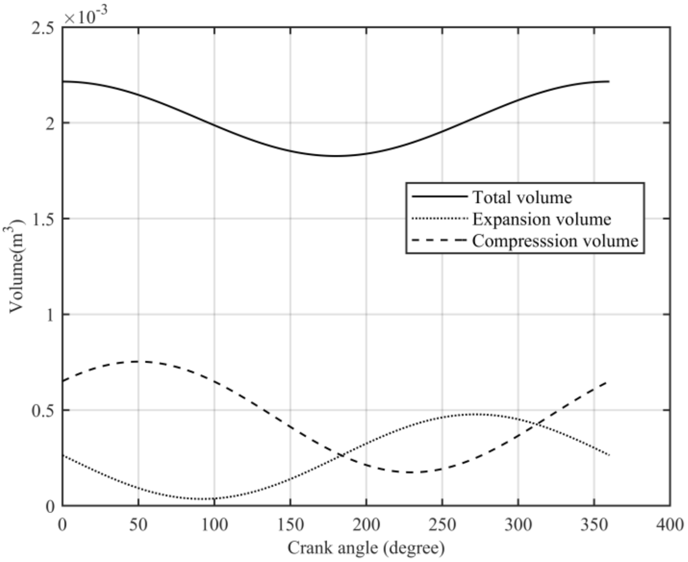

Fig.2. Expansion, compression, and total volume of Stirling engine.

3 Thermal Models

3.1 Ideal isothermal analysis

The working of isothermal analysis is made upon the following statement [21].

- Gas inside the Stirling engine is ideal, so the equation of state can be used.
- There are three dead volumes in the Stirling engine.
  1. Compression space
  2. Expansion space
  3. Regenerator

- The gas in each of the spaces is ideal homogeneous mixed throughout the cycle.
- Compression and expansion processes are isothermal. This hypothesis suggests that heat exchangers are perfectly effective.
- The speed of the machine is constant, and the motion is sinusoidal.
- The temperature of the regenerator is linear and constant. There are no losses and therefore working gas leaves the regenerator with the temperature of space it enters
- There are no fluid losses and dissipation effects in the cycle which leads to one constant pressure over the whole machine.

Following are the equation for an ideal isothermal cycle. The total mass in the engine is a sum of masses in each component of the engine consider constant over time with crank angle

$$M = m_c + m_{k1} + m_{k2} + m_r + m_h + m_e \tag{3}$$

Considering the ideal gas constant, the total mass in the engine is defined as.

$$M = \frac{p_{ch}}{R}\left(\frac{V_c}{T_k} + \frac{V_{k1}}{T_k} + \frac{V_{k2}}{T_k} + \frac{V_r \log\left(\frac{T_h}{T_k}\right)}{T_h - T_k} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right) \tag{4}$$

The pressure as a function of volume is defined as

$$P = \frac{M.R}{\frac{V_c}{T_c} + \frac{V_{k1}}{T_k} + \frac{V_{k2}}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}} \tag{5}$$

The total work delivered by the engine is integral to work done by expansion and compression.

$$W = W_e + W_c = \oint p\left(\frac{dV_e}{d\theta} + \frac{dV_c}{d\theta}\right) d\theta \tag{6}$$

3.2 Ideal adiabatic analysis

The Stirling Engine Analysis (SEA) method included in this research work is based on the program and methods were implemented by Berchowitz and Urieli (1984) [9]. The simulation was proposed by Urieli and Berchowitz (1984) [9] for an alpha-type Stirling engine that included heat exchanger geometries and operating conditions. But Alfarawi et al. [6] suggested an enhanced thermal model for gamma Stirling engine. He divided the Stirling engine into six compartments i.e. compression chamber, cooler tube, cooler fin, regenerator, heater tubes, expansion chamber. This research work is based on that enhanced thermal model. The two-cylinder type gamma Stirling engine used in this study is, however, which is pretty different from that presented by Berchowitz and Urieli (1984) [9]. Berchowitz and Urieli (1984) [9] described three approaches, Schmidt analysis, adiabatic analysis, and simple analysis. The isothermal analysis is the most simplified form of analysis the basis for the other two methods. The most complex procedure for simplification is an adiabatic analysis. The adiabatic analysis does not include any losses

terms like heat transfer losses etc. The simple analysis is an enhancement of adiabatic analysis. The solution procedure for simple analysis is same as for adiabatic analysis and spaces of expansion and compression are considered as adiabatic. In a simple analysis, losses (heat transfer losses, load losses, imperfect regeneration losses) terms are included along with the adiabatic model. By adding these losses, indicated cycle work is reduced from ideal cycle work. In this way, non-ideal analysis is being developed in simple analysis and that predicts the real performance of the Stirling engine. These three methods are developed in MATLAB and the RK-4 method is employed.
Mass of gas within the Stirling engine is a significant parameter for adiabatic analysis. Therefore, Schmidt's analysis is used to calculate the mass of gas inside the Stirling Engine. It required mean pressure inside the Stirling engine as input MATLAB program. The Schmidt analysis required seven, hot and cold gas wall temperature, the phase angle of displacement piston overpower piston, clearance volume, expansion, and compression swept volume, and mean pressure [6].

To get a set of the equation for this engine, the engine is divided into six compartments, expansion chamber, heater tubes, regenerator, cooler, cooler connecting pipes, compression chamber. Heat transfer in heat exchanger is unsteady and temperature in cooler and heater varies concerning crank angle. For formulating a set differential equation for governing equations, the derivation of each parameter with respect to crank angle (θ) is denoted by d and defined as d=d/dθ hereunder. By applying the law of conservation of energy and equation of state on each cell.

$$dQ + c_p\left(T_i m_i - T_0 m_0\right) = dW + dE_{loss} + c_v d\left(mT\right) \tag{7}$$

Whereas, dE_{loss} is a loss term that included all losses occurring in the Stirling engine. The losses occurring in different volume of cell in Stirling engine is given below. In adiabatic analysis, the total mass of gas in the engine is constant.

$$M = m_c + m_{k1} + m_{k2} + m_r + m_h + m_e \tag{8}$$

The derivative of the mass equation is given by

$$dm_c + dm_{k1} + dm_{k2} + dm_r + dm_h + dm_e = 0 \tag{9}$$

Using the equation of state, the mass in each cell of the equation is given by

$$m_i = \frac{P_i V_i}{R T_i} \tag{10}$$

where i= k1,k2, r, h (cooler tube, cooler fin, regenerator, and heater). The gas pressure equation is given by

$$P = \frac{mR}{\dfrac{V_c}{T_c} + \dfrac{V_{k1}}{T_k} + \dfrac{V_{k2}}{T_k} + \dfrac{V_r}{T_r} + \dfrac{V_h}{T_h} + \dfrac{V_e}{T_e}} \tag{11}$$

Temperature and volume of a heater, cooler tube, cooler fin, the regenerator is constant, so differential form of the equation of state is

$$\frac{dm}{m} = \frac{dP}{P} \tag{12}$$

$$dm = \frac{dP}{R}\frac{V_i}{T_i} \tag{13}$$

where i= k, r, h (cooler fin, cooler tube, regenerator, and heater)

The mass accumulation in the compression and expansion chamber is

$$dm_c = \frac{PdV_c + \frac{V_c dP}{\gamma}}{RT_{ck1}} \tag{14}$$

$$dm_e = \frac{PdV_e + \frac{V_e dP}{\gamma}}{RT_{he}} \tag{15}$$

The mass flow rate between two continuous cells is

$$m_{ck1} = -dm_c \tag{16}$$

$$m_{k1k2} = m_{ck1} - dm_{k1} \tag{17}$$

$$m_{k2r} = m_{k1k2} - dm_{k2} \tag{18}$$

$$m_{rh} = m_{k2r} - dm_r \tag{19}$$

$$m_{he} = m_{rh} - dm_h \tag{20}$$

The pressure variation in the equation is given by

$$dP = \frac{\gamma P\left(\frac{dV_c}{T_{ck1}} + \frac{dV_e}{T_{he}}\right)}{\frac{V_c}{T_{ck1}} + \gamma\left(\frac{V_{k1}}{T_k} + \frac{V_{k2}}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) + \frac{V_e}{T_{he}}} \tag{21}$$

The temperature Tck and The are conditional temperatures in direction of flow given
If mass m_{he}>0 then T_{he}=T_h else if T_{he}= T_e
If mass m_{ck}>0 then T_{ck}=T_c else if T_{ck}= T_k
The temperature in compression and expansion space in differential form is

$$dT_e = T_e\left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e}\right) \tag{22}$$

$$dT_c = T_c\left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}\right) \tag{23}$$

The temperature of the regenerator is determining by

$$T_r = \frac{T_h - T_k}{\ln\left(\frac{T_h}{T_k}\right)} \tag{24}$$

The heat transfers in a heater, regenerator, and cooler (combine as the cooler and cooler fin) are by

$$dQ_h = \frac{V_h dPc_v}{R} - c_p\left(T_{rh}m_{rh} - T_{he}m_{he}\right) \tag{25}$$

$$dQ_r = \frac{V_r dPc_v}{R} - c_p\left(T_{kr}m_{kr} - T_{rh}m_{rh}\right) \tag{26}$$

$$dQ_k = \frac{\left(V_{k1} + V_{k2}\right) dPc_v}{R} - c_p\left(T_{ck1}m_{ck1} - T_k m_{k2r}\right) \tag{27}$$

The work done in compression and expansion space is

$$dW_e = PdV_e \tag{28}$$

$$dW_c = PdV_c \tag{29}$$

The total work done is presented

$$dW = dW_e + dW_c \tag{30}$$

The efficiency is outlined as

$$\eta = \frac{W}{Q_h} \tag{31}$$

3.3 Non-ideal analysis

The regenerator stores energy from the gas oscillating in the Stirling engine. Different types of materials are used in the regenerator to store energy. Due to this advantage, it greatly improves the performance of the Stirling engine. The thermal efficiency for the non-ideal model is

$$\eta_i = \frac{W'}{Q_K} = \frac{Q'_H + Q'_K}{Q'_H} \tag{32}$$

The heat exchange in the heater and cooler for the non-ideal case is given by

$$Q_H = Q'_H + Q'_R{}^{(1-\varepsilon)} \tag{33}$$

$$Q_K = Q'_K - Q'_R{}^{(1-\varepsilon)} \tag{34}$$

Substituting the value of Q_h, Q_k, and ideal efficiency

$$\eta = \frac{\eta_i}{\left[1 + \left(\frac{Q'_R}{Q'_H}\right)^{(1-\varepsilon)}\right]} \tag{35}$$

In a simple analysis, the number of heat transfer units are given as

$$NTU = N_{ST}\frac{A_{WG}}{2A} \tag{36}$$

The Stanton number is obtained by

$$N_{ST} = h/\rho u c_p \tag{37}$$

The regenerator effectiveness is obtained as

$$\varepsilon_{reg} = \frac{NTU}{\left(1 + NTU\right)} \tag{38}$$

NTU is also used to estimate the effectiveness of heater and cooler

$$\varepsilon_{he} = 1 - e^{NTU} \tag{39}$$

It is also important to mention here that the wall temperature of the cooler and heater are lower than the respective gas temperature inside the Stirling engine, which reduces the performance of the engine. Using the convective heat transfer equation, it is easy to determine inside the gas temperature of the heater and cooler. For this purpose, the value of Q_k and Q_h are found from the adiabatic cycle. The basic heat transfer equation in adiabatic is

$$Q = hA_{wg}\left(T_w - T\right) \tag{40}$$

By using the above equation, the individual equation for cooler and heater temperature can be written as

$$T_k = T_{wk1} - fQ_k \Big/ \left(h_k A_{wgk1}\right) \tag{41}$$

$$T_h = T_{wh} - fQ_h \Big/ \left(h_h A_{wgh}\right) \tag{42}$$

In a practical Stirling engine, fluid friction is also known as load losses, an important factor in calculating the actual performance of the Stirling engine. Due to fluid motion pressure drops around the heat exchanger resulting in reduces the output power and efficiency of the Stirling engine. It is also important to note here that due to the oscillating flow of nature in the Stirling engine. The pressure drop across three heat exchangers is calculated by the following equation.

$$W = \oint p\left(dV_E + dV_C\right) - \oint \sum \Delta p dV_E = W_i + \Delta W \tag{43}$$

$$\Delta W = \int_0^{2\pi} \left(\sum_{i=1}^{4} \Delta p_i \frac{dV_E}{d\theta} \right) d\theta \tag{44}$$

The pressure drops, friction factor, and Reynold number is given by

$$\Delta p = \frac{-2C_f \operatorname{Re} \mu V}{d_h^2 A} \tag{45}$$

$$C_f = \frac{\tau}{0.5\rho\mu^2} \tag{46}$$

$$\operatorname{Re} = \frac{\rho u d_h}{\mu} \tag{47}$$

It was first time describe by Petrescu et al. [20] that pressure on the surface of a piston will differ from pressure inside a closed system of a Stirling engine. The pressure during the compression process on the piston surface will greater than the pressure inside the engine and similarly, during the expansion, it will be lower. This study leads to fact that an increase of expansion and decrease of compression work reduce the output power of the Stirling engine. So the pressure correction must be concluded. The pressure loss due to finite speed thermodynamics and mechanical friction is calculated as follows.

$$P_{cylinder\_corrected} = P_{cylinder}\left[1 \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}}\right] \tag{48}$$

$$P_{finite\_speed} = \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \tag{49}$$

$$a = \sqrt{3\gamma} \tag{50}$$

$$c = \sqrt{3RT} \tag{51}$$

$$\Delta P_f = 0.97 + 0.15\frac{N}{1000} \tag{52}$$

$$W_{loss\_finite\_speed} = \int \left(\pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \right) dV \tag{53}$$

In the above equations, delta P_f is piston-cylinder friction loss. (+) a sign denotes the compression space, (-) denotes expansion space and N is the speed of the engine. These are set of differential equations with 16 containing derivative and 24 variables in which 7 derivatives are solved by using the Rk-4 order method

4 Results and discussions

4.1 Model Validation

Figure 3 depicts indicated PV diagram related to experimental, ideal, adiabatic, and simple analysis. It can be seen that the simulated PV diagram closely fits with the experimental PV diagram in the figure. It is important to mention here that the simple analysis of the Stirling engine is obtained by excluding losses of Stirling (i.e., regenerator wall heat leakage losses, heat transfer losses, pressure drop losses, and piston finite speed losses) engine which is also known here as non-ideal simulated curve. The evaluated mean pressure of the non-ideal simulated curve is 3.59 bar pressure while the experimental mean pressure is 3.58 bar pressure. However, maximum pressure is the difference for the experimental and non-ideal curves. This is due to consideration of non-ideal heat exchanger causes large change non-ideal curve with an experimental curve. Reader et al. (1983) [22] mentioned that the phenomenon is caused by the fact that cylindrical walls of heat exchangers do not provide sufficiently high conduction heat transfer medium to provide a constant gas temperature within the cylinders, generating substantial curve deviations. This phenomenon is clear from figure 4 which shows Texp and Tcom temperature fluctuation in expansion and compression chamber. Figure 4 also shows the temperature gradient of wall and gas temperature within the heater and cooler heat exchangers. The T_h and T_k are the gas temperatures within heater and cooler respectively which is much higher than the user-defined wall temperature of the heater and cooler, respectively. The wall temperature of the heater and cooler are defined as 424 K and 294 K respectively while the gas temperature is found as 389.6 K and 308.6 K, respectively. There is a big difference between the wall and gas temperature which means heat exchanger walls are not providing enough conduction for constant gas temperature. This proves the fact mentioned by Reader et al. (1983) [22] in his research work. The indicated PV diagram of the experimental curve is obtained from actual pressure drops in all spaces of the Stirling engine. The speed and pressure of the engine are 882 rpm and 3.58 bar, respectively. The indicated work of isothermal, adiabatic, simple, and experimental analysis is 133.82 W, 143.75 W, 93.2W, 111.43W, respectively. The thermal efficiency of Schmidt, adiabatic, and simple analysis is calculating as 30.7 %, 30.9%, 21.20% respectively.

Figure 5 and figure 6 depict the mass and energy flow in each compartment of the Stirling engine. In figure 5, the values of heat energy added to a heater, rejected from a cooler, and stored in a regenerator are shown below. It is important to note that there is a difference in magnitude of energy flow and total energy flow for regenerator is a sum to zero. The heat transfer to a heater and cooler is 30.33 J and -24.08 J respectively. The total work done is calculated as 6.35 J. Figure 6 shows instantaneous mass flow in each compartment of the engine. At the crank angle of 145-degree, the large amount of mass flow takes from

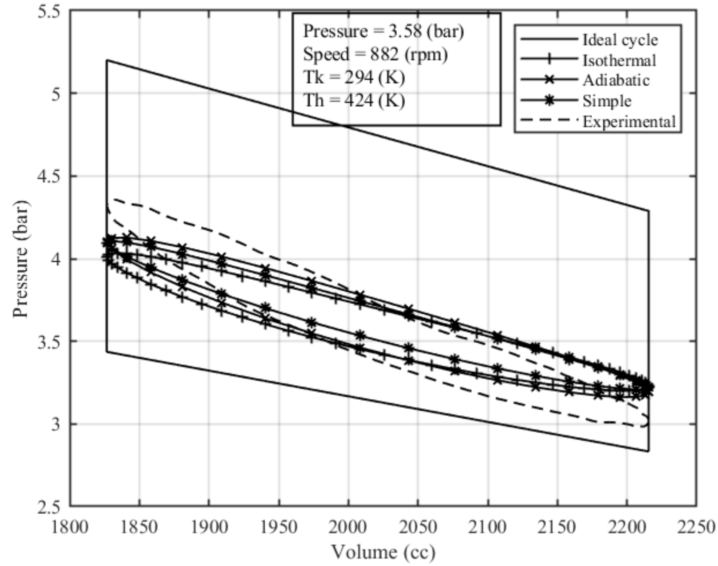

Fig.3. Pressure volume diagram of experimental, adiabatic, isothermal, and simple analysis

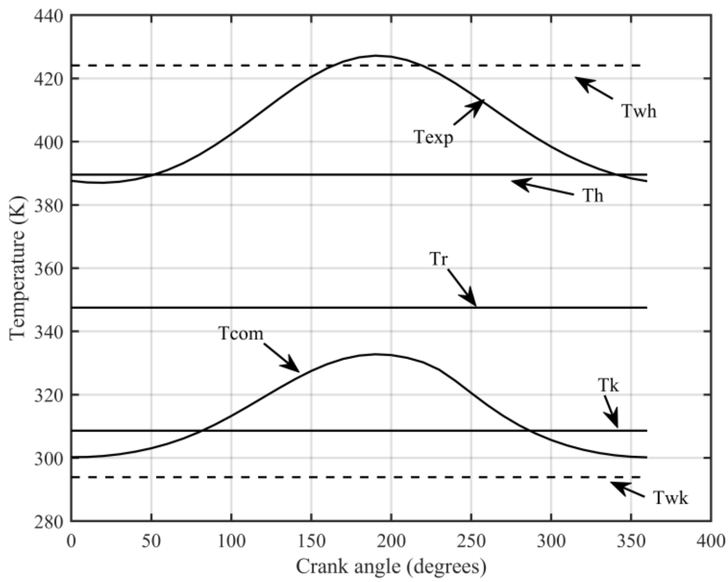

Fig.4. Temperature fluctuation for simple analysis.

compression to cooler (CK1) and less amount of mass flow from heater to expansion space. In a reverse manner, mass flow takes place after crank angle of 150 degree.

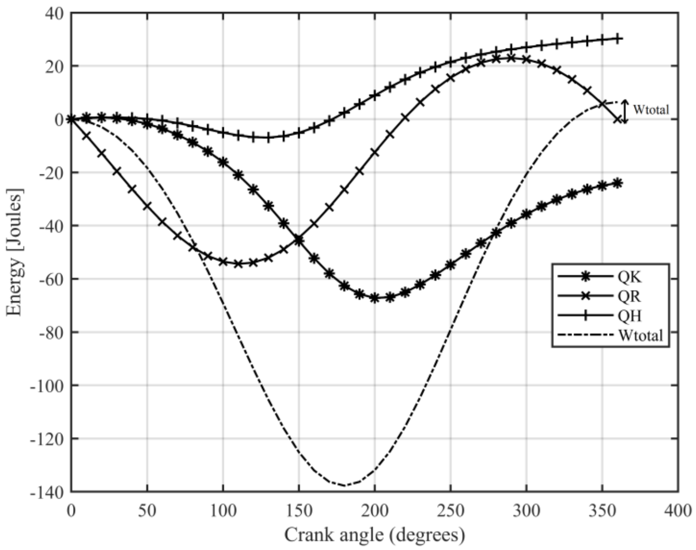

Fig. 5. Energy flow in a heater, cooler, and regenerator

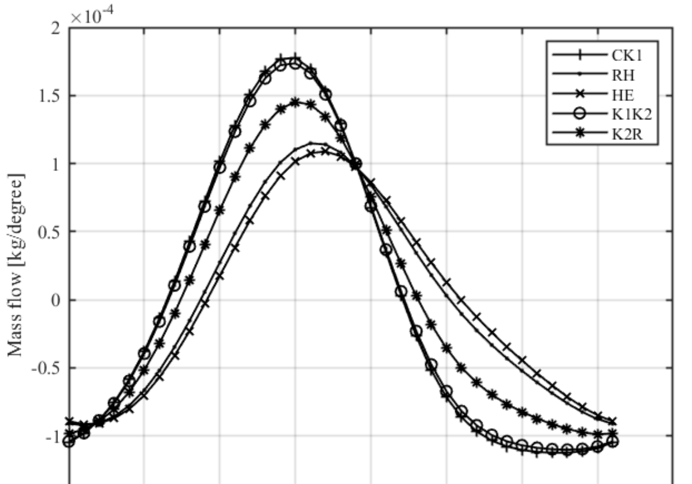

Fig. 6. Mass flow in each compartment of Stirling engine.

Figure 7 represents the work loss due to piston finite speed. Throughout the cycle, work loss fluctuates. At the angles 100- and 260-degree, maximum finite speed loss occurred 1.79 W. At an angle of 10, 170, and 360 degrees, the finite speed loss is calculated as -1.9 W. The sum of loss to the whole of a cycle is calculated as -4.3 W.

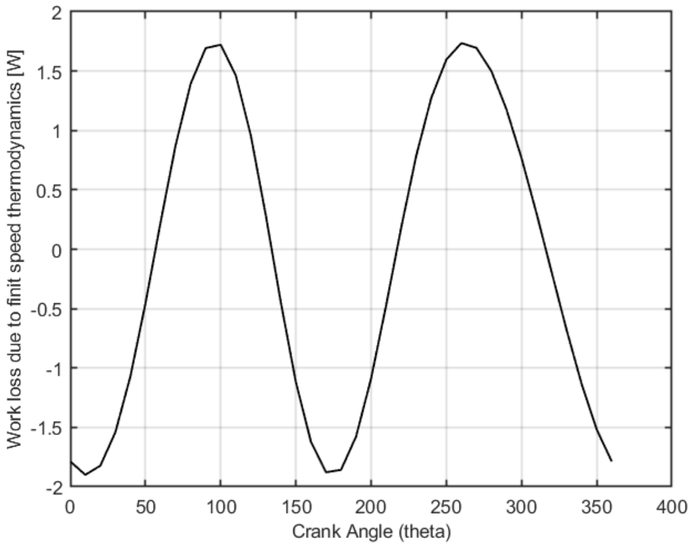

Fig. 7. Work loss due to piston finite speed

The model shows the sum of finite speed piston losses is -4.3 W. Moreover, the summary of the results is shown in table 2.

Table 2. Summary of thermal model results

	Isothermal	Adiabatic	Simple	Experimental
Work $_{in}$ (J)	9.08	9.70	6.30	7.50
Power (W)	133.8	143.75	93.2	111.43
Thermal efficiency (%)	30.70	30.90	21.2	24.70

In the above table, indicated work of the isothermal model is 9.08 J. In this model, the effect of friction losses, heat transfer losses, load losses are not included and therefore heat exchangers are insignificant. Therefore, power and efficiency estimated due to the isothermal model are 133.8 W and 30.70% respectively. The adiabatic model is much accurate because it provides true information on each compartment of the Stirling engine. It is limited because it does not contain any losses of the Stirling engine. The indicated work, power, and efficiency calculated due to this model is 9.7 J, 143.75 W, and 30.90 % respectively. In adiabatic including all the losses and seeing the non-ideal regenerative case, simple analysis is found which depicts the real performance of Stirling engine. The indicated work, power, and efficiency evaluated due to the simple model is 6.30 J, 93.2 W, 21.2%, and experimentally, these are found 7.50 J, 111.43 W, and 24.70 % respectively. The percentage error between the simple model and the experimental result is found -16.7 %, -14.1% for power and efficiency respectively.

5 Conclusion

Throughout this work, it is tried to simulate the design methodology of gamma type double piston cylinder engine for waste heat recovery of exhaust gas of diesel engine. The engine is divided into six compartments (i.e., expansion chamber, heater tubes, regenerator, cooler, cooler connecting pipe, compression chamber). Connecting pipe of cooler is also included for evaluation of real pressure variation in the chamber. Moreover, real piston motion variations are also included in our model for performance of Stirling engine. This research work is purely based on an enhanced adiabatic model. In this work, isothermal, adiabatic, and enhanced simple analysis of double piston cylinder Stirling engine is done. The exhaust gas of the diesel engine is used as a heat source for the Stirling engine. The heater, cooler, mean pressure, and frequency of the engine are 424 K, 294 K, 3.5 bar, and 14.67 Hz with helium as a working fluid inside the Stirling engine respectively. Simulation results show indicated work done by isothermal, adiabatic simple, and experimentally calculated as 9.08 J, 9.7 J, 6.3 J, and 7.50 J respectively and power estimated by these models are 133.82 W, 143.75 W, 93.2 W, and 111.43 W respectively. Moreover, the simulation shows that heater and cooler walls are not conducting very well. The temperature of the wall heater wall is 424 K while the temperature of the gas inside heater tubes is 384 K. In addition, the power losses due to piston finite speed are calculated as -4.3 W in our Stirling engine. The percentage error between experimental and simulation results are found -16.7 %, -14.1% for power and efficiency respectively. The above-mentioned results show a close agreement with simulation and experimental results.

References

1. B.H. Tsai, C.J. Chang, C.H. Chang, Energy, **100** ,416–424, (2016)
2. H. Hachem, R. Gheith, F. Aloui, and S. Ben Nasrallah, Energy Convers. Manag., **171**, 1365–1387, (2018)
3. S. W. B. Kongtragool, Renew. Sustain. Energy Rev., **7**, 131–154, (2002).
4. A. K. Almajri, S. Mahmoud, and R. Al-Dadah, Energy Convers. Manag., **145**, 93–106, (2017)
5. W. L. Chen, K. L. Wong, and H. E. Chen, Energy Convers. Manag., **77**, 118–128, (2014)
6. S. Alfarawi, R. Al-Dadah, and S. Mahmoud, Appl. Therm. Eng., **106**, 1380–1390, (2016)
7. S. Toghyani, A. Kasaeian, S. H. Hashemabadi, and M. Salimi, Energy Convers. Manag., **87**, 521–529, (2014)
8. D. G. Thombare and S. K. Verma, Renew. Sustain. Energy Rev.,**12**, 1–38, (2008)
9. I. Urieli and D. Berchowitz, *Stirling Cycle Engine Analysis*. (1984).
10. P. R. Foster, *Liquid cooled stirling engine with a segmented rotary displacer*, (2019).
11. A. Bagheri, W. C. Mullins, P. R. Foster, and H. Bostanci, Energy Convers. Manag., **201**, 65, (2019)
12. A. S. Nielsen, B. T. York, and B. D. MacDonald, Appl. Energy, **253**, 5, (2019),
13. A. C. Ferreira, M. L. Nunes, J. C. F. Teixeira, L. A. S. B. Martins, S. F. C. F. Teixeira, and S. A. Nebra, Appl. Therm. Eng., **123**, 646–657, (2017),
14. S. Zare, A. R. Tavakolpour-Saleh, and M. H. Sangdani, Energy Convers. Manag., **210**, (2020)
15. C. Çınar, F. Aksoy, H. Solmaz, E. Yılmaz, and A. Uyumaz, Appl. Therm. Eng., **130**, 1373–1379, (2018),
16. F. Aksoy, H. Solmaz, H. Karabulut, C. Cinar, Y. O. Ozgoren, and S. Polat, Appl. Therm. Eng., **93**, 359–367, (2016)
17. A. Sripakagorn and C. Srikam, Renew. Energy, **36**, 1728–1733, (2011)
18. H. S. Yang and C. H. Cheng, Appl. Energy, **200**, 62–72, (2017)
19. H. Damirchi, G. Najafi, S. Alizadehnia, B. Ghobadian, T. Yusaf, and R. Mamat, Energy Procedia, **75**, 137–143, (2015)
20. R. Li, L. Grosu, and D. Queiros-Condé, Energy Convers. Manag., **114**, 28–37, (2016)
21. C. P. Speer, *Modifications to Reduce the Minimum Thermal Source Temperature of the ST05G-CNC Stirling Engine*, (2018).

22. G. T. Reader and C. Hooper, *Stirling engines*. (1983).
23. A. Wegner, *Calculations and Experiments on gamma Stirling engine*, (2018).