A reduced-order method with PGD for the analysis of misaligned journal bearing

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Abstract. In recent years, machine component design has been a major concern for researchers. Emphasis has been placed especially on the analysis of bearing systems in order to avoid detrimental contact. The shaft misalignment is one of the most problems that affects directly the operating conditions of these components. In this context, the present study proposes a reduced-order method "Proper Generalized Decomposition" (PGD) using the separation technique through the alternating direction strategy to solve the modified Reynolds equation, taking into account the presence of misalignment in the shafting system. The solution shows the representation of two types of misalignment geometry, especially axial and twisting. A comparison of the results between the proposed approach and the classical method, through several benchmark examples, made it possible to highlight that the new scheme is more efficient, converges quickly and provides accurate solutions, with a very low CPU time expenditure.

1 Introduction

Hydrodynamic bearings are important components of rotating machines, which are considered to be the best technological solution currently available in various industrial fields like thermal engines, turbomachines , alternators, compressors, etc,. In the ideal case, the axes of both shaft and bush are parallel but in practice this condition rarely exists and the shaft, under several factors, tends to tilt from some angle during its rotation inside the bush, this misalignment is generally caused by : assembly errors, thermal distortion , asymmetric bearing loading, elastic deflection, etc. It is well known that misalignment directly affects the operating conditions of plain bearings, causing vibration and wear, reducing bearing life and bearing performance , resulting in system failure.

The theoretical bases of hydrodynamic lubrication was developed by O. Reynolds in 1886 [1], who was inspired by the experimental results of N.P. Petrov [2] and B. Tower[3], where it has been found that the load carrying capacity of a bearing is due to the high pressures developed in the fluid film, and that the viscosity is the most important properties of the fluid film. The Reynolds equation is an second-order partial differential equation, which is derived from the Navier–Stokes and continuity equations for incompressible flows [4], whose solution provides the pressure distribution in lubricating fluid.

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Several numerical methods have been proposed to solve the Reynolds equation for misaligned bearing analysis, generally we use the finite difference method (FDM) [5–8], we can also find the finite element method (FEM) in some studies [9–11], these methods are known to be accurate but with a very high computational cost and this is not really useful for dealing with certain problems such as optimization problems where multiple solutions are required. In the last decades, group of method called reduced-order models(ROM) was proposed to resolve problems of fluid mechanics , the most popular of this method is an aposteriori methods named the proper orthogonal decomposition (POD) which was used in [12–16], this category of method often require some snapshots of the flow, which mat takes significant computation time.

To circumvent this problem, apriori methods have been developed, which consist in building a reduced base without the "apriori" knowledge of the solution. In this paper we are interested in an apriori model reduction method known as proper generalized decomposition (PGD).The PGD has applied in various engineering problems. In terme of fluid mechanics, Dumon et al [17] have demonstrated the ability of the PGD to deal with the problem of lid-driven cavity with considerable time savings, In [18] PGD was coupled with spectral descritisation to solve Darcy, Taylor-Green and Lid-driven cavity problems. In [19] L. Tamellini et al was proposed a method based on the PGD combined with stochastic Galerkin approximation for solving the steady incompressible Navier–Stokes equation. In [20] PGD was coupled with immersion boundary (IB) to solve fluid structure interaction (FSI) , in [21] Charbi et al have shown the ability of PGD to solve the Reynolds equation for aligned bearing analysis considering the Sommerfeld boundary conditions with very low cost .

In this work the PGD is used for the analysis of misaligned journal bearings with consideration of Swift-Steiber boundary conditions, in order to reduce the computational costs, Section2 describes the governing equations for misaligned hydrodynamic journal bearing, section3 details the application of the PGD on the Reynolds equation, section 4 presents a comparison between the results obtained by the PGD and other methods, section5 concludes the work.

2 Physical and mathematical models

2.1 Governing equations for misaligned hydrodynamic journal bearing

The dimensionless form of the Reynolds equation for a journal operating at steady state regime (in Cartesian coordinates), can be written as :

$$\frac{\partial}{\partial\theta}\left(\overline{H}^3\,\frac{\partial\overline{P}}{\partial\theta}\right)+\eta^2\,\frac{\partial}{\partial\overline{z}}\left(\overline{H}^3\,\frac{\partial\overline{P}}{\partial\overline{z}}\right)=6\,\frac{\partial\overline{H}}{\partial\theta} \tag{1}$$

where: $\overline{P}$ is the dimensionless lubricant pressure, $\overline{H}$ is the dimensionless film thickness, and η is ratio of width to Radius of journal ,$\eta = \frac{R}{L}$.

Based on the approach of Maspeyrot and Frene [22], with consideration of misalignment in both circumferential and axial directions as shown in Fig (1):

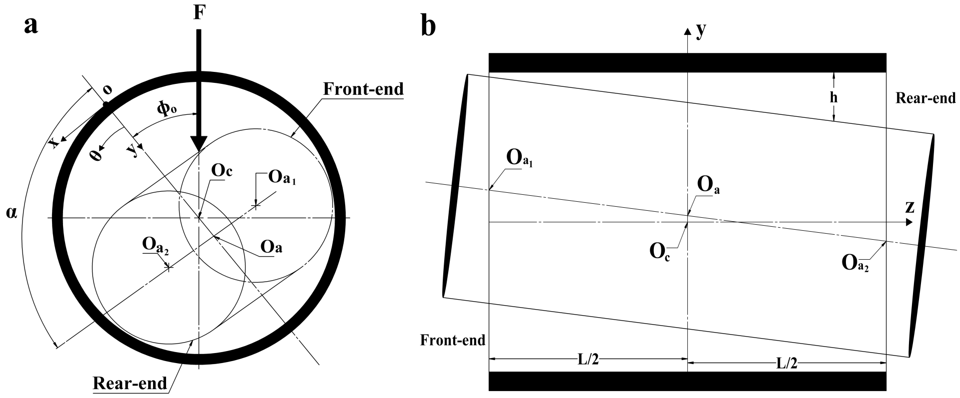

Figure 1: Representative scheme of a misaligned journal bearing.

$$\overline{H} = 1 + \varepsilon\,(1 + cos(\theta)) + \varepsilon'\,(\overline{z} - \frac{1}{2})\,cos(\theta - \alpha) \tag{2}$$

where: ε is the ratio of eccentricity to radial clearance at the bearing mid-plane ($\varepsilon = \frac{e_0}{C}$), α is the misalignment angle between the line of centers and the rear center of the misaligned journal, $\varepsilon'$is the misalignment eccentricity which can be computed from :

$$\varepsilon' = \frac{e'}{C} = D_m\,\varepsilon'_{max} \tag{3}$$

D_m represents the degree of misalignment in value from 0 to 1, $\varepsilon'_{max}$is the maximum possible of ε' is given by :

$$\varepsilon'_{max} = 2(\sqrt{1 - \varepsilon^2\,sin^2(\alpha)}) - \varepsilon\,|cos(\alpha)| \tag{4}$$

The performance characteristics of misaligned journal bearing are computed using the relations below.

The dimensionless load components in the circumferential and the axial directions can be written as follows :

$$\overline{W_\theta} = \int_{\Omega\theta\Omega\overline{z}} \overline{P} * cos(\theta) d\theta d\overline{z} \tag{5}$$

$$\overline{W_z} = \int_{\Omega\theta\Omega\overline{z}} \overline{P} * sin(\theta) d\theta d\overline{z} \tag{6}$$

The total bearing load capacity and the load attitude angle are:

$$\overline{W}_c = \sqrt{\overline{W}_\theta^2 + \overline{W}_y^2} \tag{7}$$

$$\phi = artg(\frac{W_\theta}{W_y}) \tag{8}$$

The dimensionless components of the moment in circumferential and axial,are given by :

$$\begin{cases} \overline{M}_\theta = \int_0^1 \int_0^{2\pi} \overline{P}(\overline{z} - \frac{1}{2}) sin(\theta) d\theta d\overline{z} \\ \overline{M}_{\overline{z}} = \int_0^1 \int_0^{2\pi} \overline{P}(\overline{z} - \frac{1}{2}) cos(\theta) d\theta d\overline{z} \end{cases} \tag{9}$$

Then the total misalignment moment is :

$$\overline{M} = \sqrt{\overline{M}_\theta^2 + \overline{M}_{\overline{z}}^2} \tag{10}$$

2.2 Boundary Conditions

The Reynolds boundary conditions are:

$$\begin{cases} \overline{P}(\theta = 0, \overline{z}) = 0 \\ \overline{P}(\theta = \theta_s, \overline{z}) = 0 \end{cases} \qquad \begin{cases} \dfrac{\partial \overline{P}}{\partial \theta}(\theta = \theta_s, \overline{z}) = \dfrac{\partial \overline{P}}{\partial \overline{z}}(\theta = \theta_s, \overline{z}) \\ \overline{P}(\theta_s < \theta < 2\pi) = 0 \end{cases} \tag{11}$$

3 Proper generalized decomposition for the resolution of the Reynolds equation:

For all suitable test functions $\overline{P}^*$, the weighted residual form of Reynolds equation is :

$$\int_{\Omega\theta\Omega\overline{z}} \overline{P}^* \left[\overline{H}^3 \frac{\partial^2 \overline{P}}{\partial \theta^2} + \frac{\partial \overline{H}^3}{\partial \theta} \frac{\partial \overline{P}}{\partial \theta} + \eta^2 \, \overline{H}^3 \frac{\partial^2 \overline{P}}{\partial \overline{z}^2} + \eta^2 \frac{\partial \overline{H}^3}{\partial \overline{z}} \frac{\partial \overline{P}}{\partial \overline{z}} \right] d\theta \, d\overline{z} - \int_{\Omega\theta\Omega\overline{z}} \overline{P}^* \left[6 \frac{\partial \overline{H}}{\partial \theta} \right] d\theta \, d\overline{z} = 0 \tag{12}$$

In misalignment problem the terms ($\overline{H}^3, \dfrac{\partial \overline{H}^3}{\partial \theta}, \dfrac{\partial \overline{H}^3}{\partial \overline{z}}, \dfrac{\partial \overline{H}}{\partial \theta}$) are non uniforms, so to apply the PGD, these terms must be represented in a separated form as follows :

$$\begin{cases} \overline{H}^3 = A(\theta, \overline{z}) = \sum_{j=1}^F A_j^\theta(\theta) \, A_j^{\overline{z}}(\overline{z}) \\ \dfrac{\partial \overline{H}^3}{\partial \theta} = B(\theta, \overline{z}) = \sum_{j=1}^F B_j^\theta(\theta) \, B_j^{\overline{z}}(\overline{z}) \\ \eta^2 \, \overline{H}^3 = C(\theta, \overline{z}) = \sum_{j=1}^F C_j^\theta(\theta) \, C_j^{\overline{z}}(\overline{z}) \end{cases} \qquad \begin{cases} \eta^2 \dfrac{\partial \overline{H}^3}{\partial \overline{z}} = D(\theta, \overline{z}) = \sum_{j=1}^F D_j^\theta(\theta) \, D_j^{\overline{z}}(\overline{z}) \\ 6 \dfrac{\partial \overline{H}}{\partial \theta} = E(\theta, \overline{z}) = \sum_{j=1}^F E_j^\theta(\theta) \, E_j^{\overline{z}}(\overline{z}) \end{cases} \tag{13}$$

The details on how to obtain the separated form are developed in [23].

For each iteration q of the enrichment n, applying the alternating direction strategy, which consists in computing $X_n^q(\theta)$ from $Z_n^{q-1}(\overline{z})$ and $Z_n^q(\overline{z})$ from $X_n^q(\theta$, we obtain:

$$\left(\sum_{j=1}^F \bar{\alpha}_j^\theta A_j^\theta \right) \frac{\partial^2 X_n^q}{\partial \theta^2} + \left(\sum_{j=1}^F \bar{\alpha 2}_j^\theta B_j^\theta \right) \frac{\partial X_n^q}{\partial \theta} + \left(\sum_{j=1}^F \bar{\beta}_j^\theta C_j^\theta \right) X_n^q + \left(\eta_j^\theta D_j^\theta \right) X_n^q$$
$$=$$
$$-\sum_{i=1}^{n-1} \left[\left(\sum_{j=1}^F \bar{\gamma}_{i,j}^\theta A_j^\theta \right) \frac{\partial^2 X_i}{\partial \theta^2} + \left(\sum_{j=1}^F \bar{\gamma 2}_{i,j}^\theta B_j^\theta \right) \frac{\partial X_i}{\partial \theta} \left(\sum_{j=1}^F \bar{\delta}_{i,j}^\theta C_j^\theta \right) X_i + D_j^\theta X_i \vartheta_{i,j}^\theta + \left(\sum_{j=1}^F \xi_j^\theta E_j^\theta \right) \right] \tag{14}$$

where:

$$\begin{cases} \bar{\alpha 2}_j^{\theta} = \int_{\Omega_{\bar{z}}} \left(Z_n^{q-1}\right)^2 B_j^{\bar{z}} \, d\bar{z} \\ \bar{\alpha}_j^{\theta} = \int_{\Omega_{\bar{z}}} \left(Z_n^{q-1}\right)^2 A_j^{\bar{z}} \, d\bar{z} \\ \bar{\beta}_j^{\theta} = \int_{\Omega_{\bar{z}}} Z_n^{q-1} C_j^{\bar{z}} \frac{\partial^2 Z_n^{q-1}}{\partial \bar{z}^2} \, d\bar{z} \\ \eta_j^{\theta} = \int_{\Omega_{\bar{z}}} D_j^{\bar{z}} Z_n^{q-1} \frac{\partial Z_n^{q-1}}{\partial \bar{z}} \, d\bar{z} \\ \xi_j^{\theta} = \int_{\Omega_{\bar{z}}} Z_n^{q-1} E_j^{\bar{z}} \, d\bar{z} \end{cases} \quad \begin{cases} \bar{\gamma 2}_{i,j}^{\theta} = \int_{\Omega_{\bar{z}}} Z_n^{q-1} Z_i B_j^{\bar{z}} \, d\bar{z} \\ \bar{\gamma}_{i,j}^{\theta} = \int_{\Omega_{\bar{z}}} Z_n^{q-1} Z_i A_j^{\bar{z}} \, d\bar{z} \\ \bar{\delta}_{i,j}^{\theta} = \int_{\Omega_{\bar{z}}} Z_n^{q-1} \frac{\partial^2 Z_i}{\partial \bar{z}^2} C_j^{\bar{z}} d\bar{z} \\ \vartheta_{i,j}^{\theta} = \int_{\Omega_{\bar{z}}} D_j^{\bar{z}} . Z_n^{q-1} \frac{\partial Z_i}{\partial \bar{z}} \, d\bar{z} \end{cases} \tag{15}$$

The resolution of the equation (14) allows us to calculate $X_n^q(\theta)$

$$\left(\sum_{j=1}^{F} \bar{\beta}_j^{\bar{z}} . A_j^{\bar{z}}\right) Z_n^q + \sum_{j=1}^{F} k_j^{\bar{z}} D_j^{\bar{z}} \frac{\partial Z_n^q}{\partial \theta} + \left(\sum_{j=1}^{F} \bar{\alpha}_j^{\bar{z}} C_j^{\bar{z}}\right) \frac{\partial^2 Z_n^q}{\partial \bar{z}^2} + \left(\sum_{j=1}^{F} \bar{\eta}_j^{\bar{z}} B_j^{\bar{z}}\right) Z_n^q$$
$$=$$
$$-\sum_{i=1}^{n-1} \left[\left(\sum_{j=1}^{F} \bar{\delta}_{i,j}^{\bar{z}} A_j^{\bar{z}}\right) Z_i + \left(\sum_{j=1}^{F} B_j^{\bar{z}} \vartheta_{i,j}^{\bar{z}}\right) Z_i + \left(\sum_{j=1}^{F} \bar{\gamma}_{i,j}^{\bar{z}} C_j^{\bar{z}}\right) \frac{\partial^2 Z_i}{\partial \bar{z}^2} + D_j^{\bar{z}} \varrho_i^{\bar{z}} \frac{\partial Z_i}{\partial \bar{z}} + \left(\sum_{j=1}^{F} \xi_j^{\bar{z}} E_j^{\bar{z}}\right) \right] \tag{16}$$

Where:

$$\begin{cases} \bar{\alpha}_j^{\bar{z}} = \int_{\Omega_{\theta}} C_j^{\theta} \left(X_n^q\right)^2 d\theta \\ k_j^{\bar{z}} = \int_{\Omega_{\theta}} \left(X_n^q\right)^2 D_j^{\theta} \, d\theta \\ \bar{\beta}_j^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q A_j^{\theta} \frac{\partial^2 X_n^q}{\partial \theta^2} d\theta \\ \eta_j^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q B_j^{\theta} \frac{\partial X_n^q}{\partial \theta} \, d\theta \\ \xi_j^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q E_j^{\theta} \, d\theta \end{cases} \quad \begin{cases} \bar{\gamma}_{i,j}^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q X_i C_j^{\theta} \, d\theta \\ \varrho_i^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q X_i D_j^{\theta} \, d\theta \\ \bar{\delta}_{i,j}^{\bar{z}} = \int_{\Omega_{\theta}} X_n^q \frac{\partial^2 X_i}{\partial \theta^2} A_j^{\theta} \, d\theta \\ \vartheta_{i,j}^{\bar{z}} = \int_{\Omega_{\theta}} B_j^{\theta} X_n^q \frac{\partial X_i}{\partial \theta} \, d\theta \end{cases} \tag{17}$$

After the resolution of the equation (16), we obtain $Z_n^q(\bar{z})$.

The procedure of the formulation of the Reynolds equation by the PGD and the convergence criteria are detailled in [21]

4 Result and discussion :

This section presents the comparison between the result obtained by PGD and those obtained by Gauss seidel iterative method using the finite difference method with successive over relaxation (FDM_{sor}), where (Ω_{sor}=1.8).

4.1 Performance analysis of misaligned journal bearing :

Figures 2 and 3 show the variation of a number of bearing performance parameters as a function of the degree of misalignment and eccentricity ratios, respectively, used PGD, FDM_{sor} and the results obtained from [24].

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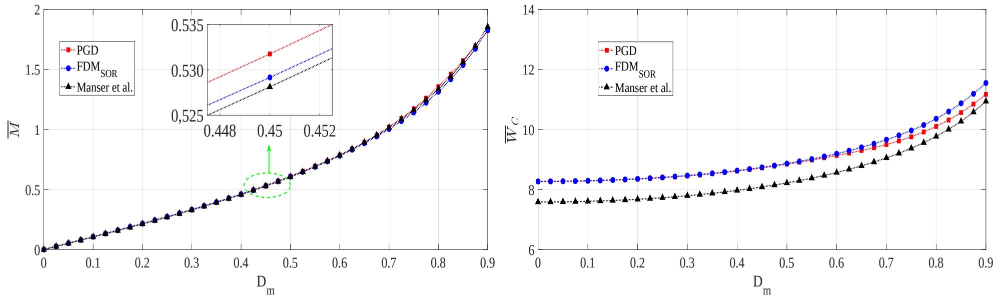
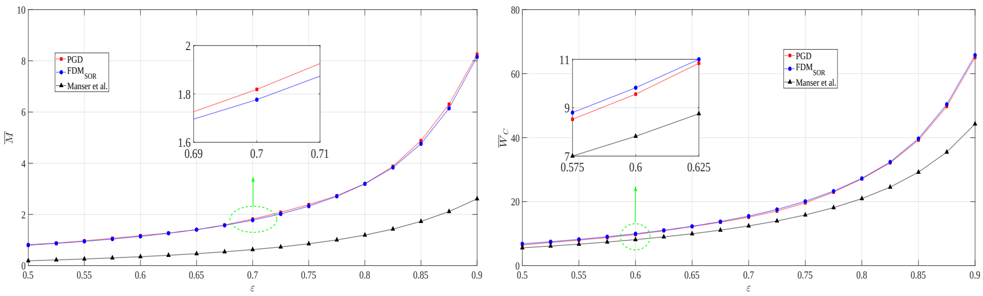
By analyzing theses figures, it can be noted that the results obtained by the PGD are in good correspondence with those obtained by FDM_{sor} .

The difference between the results obtained in [24] and those computed by PGD and FDM_{sor} are justified by the use of the conservative model in [24].



(a) (b)

Figure 2: Variation in bearing performances with the degree of misalignment for : $\frac{L}{D} = 1$, $\alpha = 130°$ and $\varepsilon = 0.6$

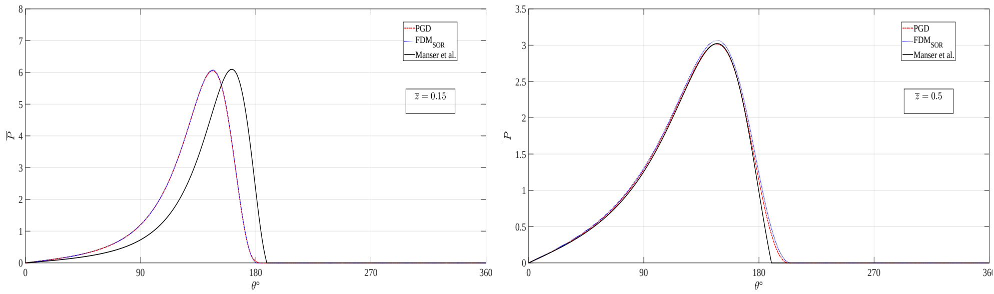


(a) (b)

Figure 3: Bearing performances versus eccentricity for : $\frac{L}{D} = 1$, $Dm = 0.75$ and $\alpha = 130^0$

4.2 Pressure distribution

Figure 4 illustrates the pressure distribution allong the circumferential direction at ($\overline{z} = 0.15$ and $\overline{z} = 0.5$) obtained by PGD and compared with the results obtained by FDM_{sor} and those reported in [24], it can be observed that the pressure curves obtained by PGD and FDM_{sor} are very close, the difference of the results given in [24] compared to other two are also explained by the use of the conservative method.



(a) (b)

Figure 4: Pressure distribution along the circumferential direction at: (a) $\overline{z}$ = 0.15 and (b) $\overline{z}$ = 0.5 for $\frac{L}{D}$ = 1, α = 130°, ε = 0.6 and Dm = 0.75

4.3 CPU time :

Figure 5 illustrates the computational time when increasing the number of nodes using PGD and FDM$_{sor}$, we can clearly see through figure 5 that PGD is more efficient than FDM$_{sor}$ in term of calculation time. We draw the reader's attention that the computations are performed on Intel Core i7-7820HQ CPU @ 2.90 GHz (48GB RAM, 64 bit) using Matlab.

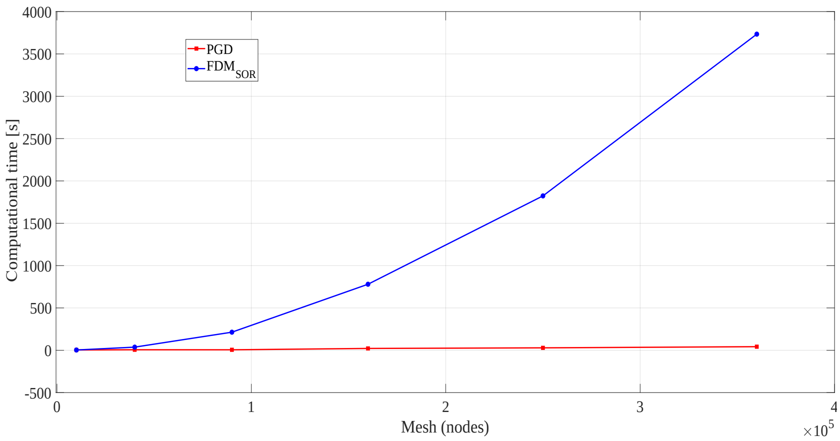


Figure 5: Computational time when increasing the number of nodes for $\frac{L}{D}$ = 1, ε = 0.6, Dm = 0.75 and α = 130^0

5 Conclusion

In this paper we have shown the ability of the PGD to simulate the behavior of a misaligned journal bearing with the consideration of Reynolds boundary conditions. Through a comparative study we have shown that the results obtained by the PGD are as accurate as those obtained by Gauss-Seidel iterative technique, but in terms of computational cost, the PGD is largely more efficient.This work should be extended further by including other journal bearing model parameters like geometry of sliders and the number of grooves as extra-coordinates of the problem.

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