Effect of heating on the stability of the three-dimensional boundary layer flow over a rotating disk

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Abstract. The effect of heating on the stability of the laminar three-dimensional boundary layer flow over a rotating disk was experimentally investigated. Local convective heat transfer coefficients were obtained at different running speeds and heating rates by means of an electrically heated disk apparatus placed in a large water tank. The accuracy of the method was assessed by comparison with predictions of the analytical self-similarity solution for laminar flow, and an excellent agreement was found. By means of local heat transfer measurements, the critical Reynolds number corresponding to the onset of vortices was determined as a function of the wall temperature difference and Prandtl number. A substantial increase of the critical Reynolds number with higher wall temperature difference was observed for the three-dimensional flow. The observed stabilizing effect due to heating of three-dimensional water flows was comparable with the predictions of perturbation analyses conducted for two-dimensional flows.

Nomenclature

α	expansion coefficient	[-]
A	area	[m^2]
A	amplitude (Landau model)	[-]
h	heat transfer coefficient	[$W\ m^{-2}\ K^{-1}$]
k	expansion coefficient	[-]
K	correlation constant	[-]
Nu	Nusselt number	[-]
Pr	Prandtl number	[-]
q	heat flux	[$W\ m^{-2}$]
r	radial coordinate	[m]
Re	Reynolds number ($Re_r = \omega r^2/\nu$)	[-]
T	Temperature	[K]
u	radial velocity component	[$m\ s^{-1}$]
v	tangential velocity component	[$m\ s^{-1}$]
w	axial velocity component	[$m\ s^{-1}$]
$\hat{w}$	amplitude of perturbation	[-]
z	axial coordinate	[m]

Greek Symbols

α	radial wave number of perturbation	[-]
β	circumferential wave number of perturbation	[-]
δ	boundary layer thickness	[m]
ε	perturbation parameter	[-]
ν	viscosity	[$m^2\ s^{-1}$]
ρ	density	[$kg\ m^{-3}$]
ω	rotational speed	[$rad\ s^{-1}$]
$\bar{\omega}$	frequency of perturbation	[-]

Subscripts

0	reference
∞	ambient or bulk
abs	absolute (onset of absolute instability)
cr	critical (onset of instability)
r	radial
t	turbulent (end of transition)
w	wall

1 Introduction

It has long been recognized that heating can affect the stability of laminar boundary layer flows significantly [1, 2]. In the late 1960s, a dramatic increase of the critical Reynolds number, i.e., the point at which a specific infinitesimal perturbation is amplified, and hence the laminar flow becomes unstable, was theoretically predicted for two-dimensional laminar water flows overheated surfaces [3, 4]. For instance, Wazzan et al. [4] found by means of solving a modified Orr-Sommerfeld equation, including additional temperature-dependent viscosity terms, that the critical Reynolds number of the

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Blasius flow past a flat plate could increase from its isothermal value 520 up to a level of about 16,000 due to heating. Using an approximation method, Hauptmann [3] predicted independently a somewhat similar effect. In at least some situations, the shift of the critical Reynolds number can be related to changes in the curvature of the involved boundary layer velocity profiles due to variation of viscosity with temperature [1, 2]. Then, it follows that gases are stabilized by cooling and liquids by heating [2].

During the last decades, different methods have been proposed for calculating the effect of heat transfer on the stability of two-dimensional boundary layers [5]. Generally, these calculation methods can be classified into (i) direct methods and (ii) asymptotic methods. Following direct methods, the perturbation equations are solved directly after inserting specific property laws and considering specific boundary conditions. Typical examples are given by Wazzan et al. [4, 6], Lee et al. [7], or Al Musleh and Frendi [8]. In asymptotic methods, all property variables are expanded in Taylor series as a function of a suitable perturbation parameter, and then, after inserting, the resulting equations are solved for each power of the perturbation parameter. This method is also known as property expansion method, and typical examples are given by Herwig and co-workers [5, 9, 10]. So far, only few experimental studies [11−14] are available in the open literature, and their data supported at least qualitatively the predicted trends. However, all investigations mentioned above dealt with two-dimensional flow configurations. So far, no widely accepted work considered the effect of heating on three-dimensional boundary layers[3].

The flow past a rotating disk, illustrated in Figure 1, represents a paradigmatic configuration for studying three-dimensional boundary layer flows and their stability. In a rotating reference frame, the radial velocity component represents a crossflow, and the situation becomes rather similar to flows past swept wings. In accordance to Reed and Saric [16], “much of our knowledge of crossflow has and will continue to develop from study of the disk”.

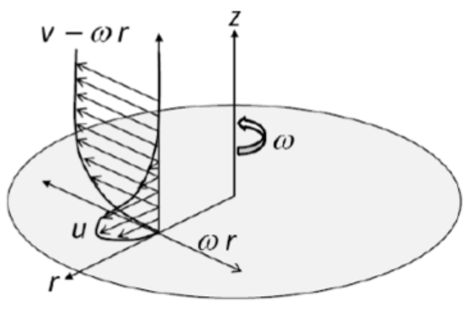

Fig. 1. Mean tangential and radial velocity profiles in the rotating boundary layer as seen in the rotating reference frame

The different flow regimes over a rotating disk are illustrated in Figure 2. For sufficient small local Reynolds numbers $\mathrm{Re}_r = \omega\, r^2/\nu$ with viscosity ν and rotational speed ω, the flow remains laminar. Spiral vortices occur due to the crossflow instability at a radial distance r_{cr}, corresponding to Re_{cr}. The end of transition to turbulence is achieved at r_t. A fully turbulent flow is observed at a Reynolds number of about $\mathrm{Re}_t = 3.2 \times 10^5$ [17, 18]. Interestingly, the rotating disk boundary layer supports an absolute instability at $\mathrm{Re}_{abs} = 2.6 \times 10^5$ [19], a value that is rather close to Re_t. The implication of absolute instability is that laminar flow cannot exist beyond Re_{abs} regardless of the preparation of the experiment. It also offers a natural explanation for the low data scattering (of order 3 % up to 5 %) reported in the literature regarding the observed turbulent Reynolds number Re_t. There exists a noticeable range regarding the critical Reynolds number Re_{cr} in research. Precise hot wire measurements for an isothermal disk indicated a value of about $\mathrm{Re}_{cr} = 8.6 \times 10^4$, which is in good agreement with perturbation calculations [16−19]. Still, several authors reported much higher values for the critical Reynolds number, as listed in [20]. The reason for that is caused by substantial experimental challenges to determine the onset of infinitesimal perturbations.

Although many theoretical and experimental stability investigations are available for the rotating disk (see, for instance, the references cited in [16−20]), the effect of heating on stability was excluded so far. In addition to the somewhat questionable analysis [15], the only exception is a conference contribution [21] in which an unexpected observation (namely the rather low number of spiral vortices found by means of IR thermography on a heated disk rotating in the air) was qualitatively explained on the basis that, due to heating, “the stability characteristics may, therefore, change significantly”.

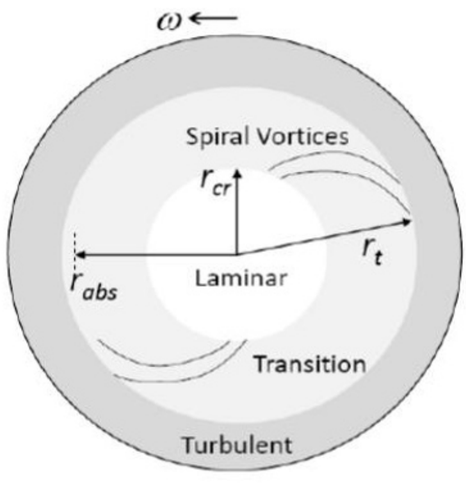

Fig. 2. Flow regimes over a rotating disk

[3] The theoretical stability analysis [15] is not widely accepted because the outcome of this study (which considered an artificial fluid with a temperature- dependency of viscosity of water but with a Prandtl number of dry air) was rather questionable: the thermal boundary layer of the base flow was much thicker than the velocity boundary layer for a fluid with a Prandtl number close to unity, the critical Reynolds number was of order 228 even for a fluid with a very small temperature-dependent viscosity, the impact of heating and velocity profiles were fully in contrast to any experience reported in literature.

The exclusion of the effect of heat transfer on stability on rotating disk flows is somewhat surprising because many technical applications are largely concerned with heated or cooled disk systems [20]. On the other hand side, the effect of heat transfer on stability is comparable small in the case of air as already known from two-dimensional stability analyses [3−5], and it can easily be overseen in typical heat transfer experiments. Hence, a special experimental investigation was planned and performed in order to provide some information regarding this important topic.

2 Theoretical considerations

From an experimentally point of view, the investigation of the effect of heat transfer on the onset of infinitesimal perturbations is rather challenging or nearly impossible in the case of a disk rotating in still air. Since two-dimensional calculations [3−5] demonstrated that the effect should be much higher in water, a disk rotating in a water tank was chosen as approach. In the following, some general theoretical considerations are presented which demonstrate that such an approach offered indeed a good chance for determining the effect of heating on the onset of the instability of the three-dimensional flow over a rotating disk.

2.1 Property expansion method

The strength of the property expansion method is that the general expression for the critical Reynolds number

$$Re_{cr} = Re_0\left(1 + \varepsilon\left(k_\rho\alpha_\rho + k_\mu\alpha_\mu\right) + O(\varepsilon^2)\right) \qquad (1)$$

remains valid for every Newtonian fluid and any base flow with the small perturbation parameter

$$\varepsilon = \frac{T_w - T_\infty}{T_\infty} \qquad (2)$$

and the thermo-physical coefficients

$$k_\rho = \frac{T}{\rho}\frac{\partial\rho}{\partial T}\bigg|_0 \text{ and } k_\mu = \frac{T}{\mu}\frac{\partial\mu}{\partial T}\bigg|_0 \qquad (3)$$

and the coefficients a_ρ and a_μ depending on both Prandtl number Pr and base flow [10]. Hence, equation (1) describes the shift of the critical Reynolds number Re_{cr} for sufficient small normalized wall temperature differences ε not only for two-dimensional but also for three-dimensional boundary layers. The characteristics of specific base flows are covered by the specific values of the coefficients a_ρ and a_μ. These coefficients have to be computed after solving a complex set of flow-specific perturbation equations. For the Blasius flow past a flat plate, information about the (negative) values of these coefficients can be found in [10]. Their order of magnitude is about unity, and they also exhibit a moderate Prandtl number dependency. A similar situation might be assumed for the three-dimensional flow over a rotating disk, too. Then, the effect of heating or cooling on stability is mainly governed by the value of the thermo-physical coefficients k_ρ and k_μ. Some typical values for air and water at 1 bar and 293 K are listed in Table 1.

Table 1. Thermophysical coefficients for air and water at 1 bar and 293K

Fluid	Pr	k_ρ	k_μ
Air	0.717	−1.000	0.775
Water	7.010	0.057	−7.132

Inspection of Table 1 indicates that the shift of the critical Reynolds number is small in the case of air but significant in the case of water. Hence the property expansion method proposed by Herwig and co-workers [5, 9, 10] offers a simple answer to the question why the effect of heat transfer on stability was mainly overseen in the scientific literature about disks rotating in the air so far. The different signs of the coefficients are leading to the conclusion that heating stabilizes the flow in the case of water. For water, the stabilizing effect depends mainly on the change of viscosity with temperature.

2.2 Spiral Vortices and Heat Transfer

For a rotating disk in a fluid otherwise at rest, a natural length scale is given by $(\nu/\omega)^{1/2}$ [1, 2, 20], and all axial dimensions, z, can be normalized by this length scale in order to get a suitable self-similar coordinate. The velocity and temperature profiles can be computed by means of an exact self-similarity solution (also known as von Kármán solution) in the laminar flow regime [20]. The velocity boundary layer thickness is of order $\delta \approx 5\,(\nu/\omega)^{1/2}$. In the case of air with Pr = 0.7, the thermal boundary layer thickness is of comparable size, whereas in the case of water with Pr = 7 a much thinner thermal boundary layer results.

The size of the spiral vortices is governed purely by hydrodynamics. The inviscid analysis by Stuart et al. [22] yielded as dominant term a relation $\sim\exp(-3z(\omega/\nu)^{1/2}/2)$ for the vanishing of the perturbation velocity field as a function of the axial coordinate z. As a result, the spiral vortices are essentially confined to a region close to the disk wall ($z = 0$) up to a value of about $z \approx 2\,(\nu/\omega)^{1/2}$. The viscous flow calculations by Kobayashi et al. [23] yielded a similar result. This implies that the main transport effect due to spiral vortices occurs well within the velocity boundary layer. Figure 3 shows schematically the normalized velocity and temperature profiles on a heated rotating disk for two different Prandtl numbers corresponding to air and water, respectively.

The domain of the spiral vortices is comparable to the laminar radial velocity boundary layer of the base flow caused by the rotating disk. The thermal boundary layer thickness due to the laminar base flow is much thicker than the spiral vortices region in the case of air (Pr = 0.7). In the case of water (Pr = 7.0) the thermal boundary layer is much thinner.

The convective heat transfer caused by the laminar base flow can be correlated by means of

$$Nu_0 = K(\mathrm{Pr})\mathrm{Re}_\mathrm{r}^{1/2} \qquad (4)$$

The correlation constant K is in fact a function of the Prandtl number Pr and can be obtained by means of the known self-similarity solution of the unperturbed laminar flow [20, 24].

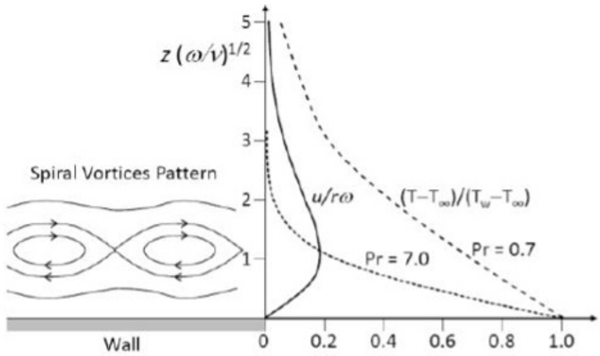

Fig. 3. Velocity and temperature profiles on a rotating disk for two different Prandtl Numbers

Due to the spiral vortices, an additional heat transfer contribution, Δh, results for the unstable flow regime over a rotating disk. Its existence was visualized by IR thermography in [21]. In the case of water, this additional contribution might be substantial because the spiral vortices do effectively transport hot fluid from to wall to the cold region outside of the thin thermal boundary layer (see Figure 3). In the case of air, this mechanism is much weaker because the transport of fluid occurs mainly in the hot region.

2.3 Landau Model and Critical Reynolds Number

The mathematical investigation of stability is exceptionally complicated, but valuable insight can be gained often by means of a simple phenomenological model proposed by Landau [25]. This model assumes that below the critical value of a suitable control variable (e.g., $(\mathrm{Re} - \mathrm{Re}_{cr})/\mathrm{Re}_{cr}$) all perturbations modes of a suitable order parameter A are vanishing. At the onset of instability, at least one mode is growing. As discussed in detail in [25], one gets finally the simple relation

$$\begin{aligned} |\mathrm{A}| &= 0 && \text{for } \frac{Re - Re_{cr}}{Re_{cr}} < 0 \\ |\mathrm{A}| &= k\sqrt{\frac{Re - Re_{cr}}{Re_{cr}}} && \text{for } \frac{Re - Re_{cr}}{Re_{cr}} \geq 0 \end{aligned} \tag{5}$$

corresponding to a Hopf-bifurcation with an empirical constant k. Identifying the order parameter A with the velocity of the perturbation (i.e., the velocity magnitude of the spiral vortices) and assuming a laminar heat transfer mechanism (i.e., $\Delta h \propto \mathrm{A}^{1/2}$), the following heat transfer correlation results:

$$\begin{aligned} \frac{\Delta h}{h_0} &= 0 && \text{for } \frac{Re - Re_{cr}}{Re_{cr}} < 0 \\ \frac{\Delta h}{h_0} &= k\left(\frac{Re - Re_{cr}}{Re_{cr}}\right)^{1/4} && \text{for } \frac{Re - Re_{cr}}{Re_{cr}} \geq 0 \end{aligned} \tag{6}$$

The above equation enables a determination of the stability limit, Re_{cr}, by means of a data fitting procedure for Reynolds numbers Re much larger than Re_{cr} for which measurable deviations $\Delta h = h(\mathrm{Re}) - h_0$ from the undisturbed laminar heat transfer coefficient h_0 can be observed.

3 Experimental method and procedure

The experimental apparatus was essentially the same as employed in a prior investigation [24]. In the following, the experimental method and procedure are briefly described. For further details about the apparatus and data reduction items, the reader is invited to consult reference [24].

3.1 Rotating disk apparatus

The electrically heated disk was placed in a transparent pool of water with a cylindrical shape, as shown in Figure 4. The disk was driven by an electric motor with variable running speed. In order to minimize natural convection contributions, the heated surface was down-side oriented in the pool. The disk device and its design, including dimensions, are shown in more detail in Figure 5, and relevant data are listed in Table 2. The disk device consisted of a main disk including a heating wire and temperature sensors and thermal insulation to minimize parasitic heat losses.

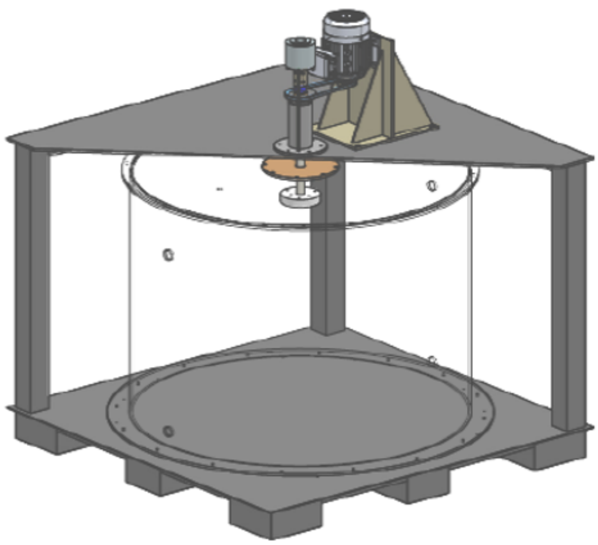

Fig. 4. Experimental test rig

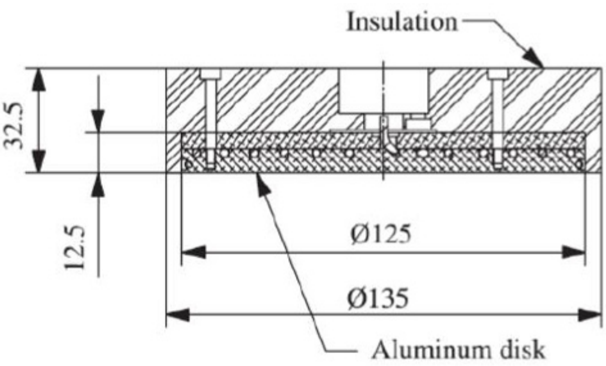

Fig. 5. Disk design and dimensions (mm)

Table 2. Technical data of the test rig

Disk diameter (heated area)	125 mm
Disk thickness (total)	32.5 mm
Heating power range	0 – 720 W
Running speed range	0 – 1450 rpm
Pool radius	575 mm
Pool height	1010 mm

Resistance temperature detectors (RTD-PT1000 high accuracy class 1/10 DIN) were used to measure the temperature at five different radial locations. The RTDs were embedded 0.5 mm beneath the aluminium surface disk. All electrical signals from the disk temperature sensors and heat inputs were supplied by slip rings and recorded during the measurements. The outer disk surface was carefully polished to ensure that the disk surface was smooth in terms of hydrodynamics [1]. The optimization of the heating wire rooting and its dimensions required several numerical heat conduction calculations. For the present purpose, an isothermal surface was required. In the case of laminar flow regime over a rotating disk with constant boundary layer thickness δ, a uniform wall temperature T_w = constant also corresponds to a constant heat flux condition q = constant [20]. Finally, a practically uniform temperature distribution was achieved with a maximum deviation of only $\Delta T_w/T_{w,0} < 0.004$.

Since it was known that free-stream particles could affect the transition [13, 26], clean and degassed water was used in the tank. But even then, it was observed in a preliminary test that small bubbles arising from dissolved air accumulated at the heated disk during long-time measurements. The water in the tank was heated for a few days to avoid this serious disturbance. This procedure worked well, and during the following measurements, no problems caused by bubbles or particles were observed.

3.2 Data reduction and uncertainty analysis

The general heat balance equation for the disk apparatus and the data reduction for obtaining heat transfer coefficients h are discussed in detail in [24]. It was found that radiation heat transfer and the parasitic heat conduction losses through the shaft were essentially negligible (less than 1 %) for a rotating disk in water. The natural convection heat transfer effect was small due to the down-side orientation of the disk in the pool. Still, it contributed in the case of low rotational Reynolds numbers some per cent to the total heat transfer coefficients. Since it was a rather systematic effect, the desired forced convection heat transfer coefficient h was correspondingly corrected [24].

The down-side orientation of the heated disk resulted in the liquid being heated from above which represented an inherently stabilizing effect. However, as shown in [1], laminar boundary layer stability is not affected by the buoyancy effect until the Richardson number is about 0.005. In the present experiment, the actual value of the Richardson number was even smaller, and hence this stabilizing effect was neglected.

The heat transfer augmentation due to the spiral vortices, $\Delta h/h_0$, was related to the measured local temperature, T, and bulk temperature, T_∞, and undisturbed wall temperature, T_w, using the following local heat transfer equation:

$$h_0(T_w - T_\infty) = (h_0 + \Delta h)(T - T_\infty) \tag{7}$$

because q = constant holds for the isothermal disk in laminar flow. Then, the order parameter of the Landau model called

$$\frac{\Delta h}{h_0} = \frac{T_w - T}{T - T_\infty} \tag{8}$$

During the measurements, local temperatures T were measured at different radial locations r for several running speeds of the disk and various ambient temperature levels T_∞ and heating rates q. The undisturbed wall temperature T_w was identical to the wall temperature for purely laminar flow. Its value was determined through local temperature values for sufficiently low Reynolds numbers Re_r, i.e., within the inner laminar flow region (see Figure 2) where the wall temperature remained constant.

Special attention was spent on accounting for temperature-dependent material properties. In the case of water, the thermophysical properties like Prandtl number or viscosity depend on temperature significantly. The evaluation of thermophysical property values at a simple reference temperature (e.g., film temperature, bulk or wall temperature) might introduce a significant error in some heat transfer calculations. Hence the so-called property-ratio method [24] was used assuming the set of exponents for laminar flow.

The tolerances of the disk dimensions were negligible. The uncertainty level of the temperature measurements was not larger than 0.5 K and typically smaller after careful calibration. The uncertainty level of the electric heating power supply was rather low (of order 0.1 %). The running speed of the rotor was frequency-controlled up to an uncertainty of 1.5 rpm. The total uncertainty level of the local rotational Reynolds number $\mathrm{Re}_r = \omega\, r^2/\nu$ was 0.1 up to 3.0 %. The uncertainty regarding the actual Prandtl number Pr was about 1 %. The total uncertainty level regarding the local Nusselt number Nu was 2 up to 5 %.

3.3 Validation

An excellent validation case for assessing the accuracy of the experimental apparatus and procedure is given by a comparison of the laminar heat transfer results (see equation (4)) with the prediction of the exact self-similarity solution [20, 24]. In Figure 6, the experimentally obtained local Nusselt number Nu is plotted against the local Reynolds number Re_r. The agreement between the experimental data and the prediction of the self-similarity solution was excellent over the entire laminar flow regime. The analytical treatment enabled a calculation of K as a function of Prandtl number by means of the self-similarity solution for velocity and temperature.

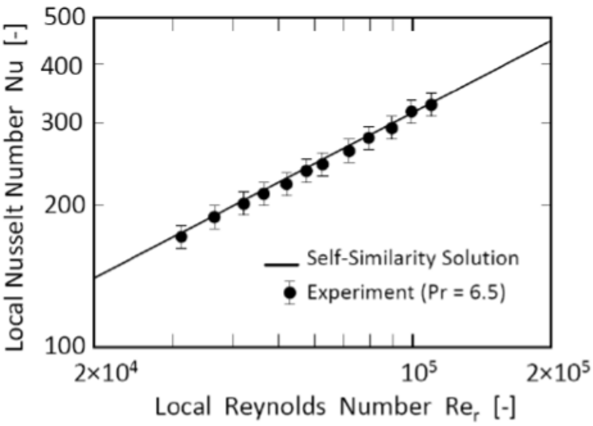

Fig. 6. Laminar convective heat transfer from a rotating disk in still water

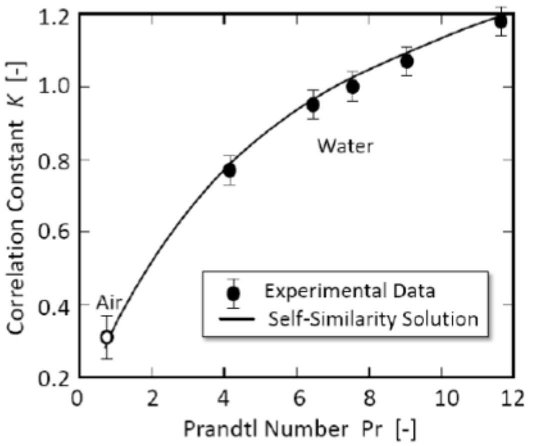

Fig. 7. Laminar convective heat transfer correlation constant K against Prandtl Number Pr

For instance, the analytical expression $Nu = 0.95\ Re_r^{1/2}$ resulted in the case of Pr = 6.5. The same excellent agreement between experimental data and the self-similarity solution prediction was observed in the case of other Prandtl numbers ranging from Pr = 4 up to Pr = 12. The different values of Prandtl number were achieved by different bulk temperatures (ranging from close to 4°C up to more than 44°C).

The result for the experimentally determined laminar heat transfer correlation constant K (equation (4)) is shown in Figure 7 for some Prandtl number values. The value for Pr = 0.7 corresponds to measurements performed with a rotating disk in air. The other values were obtained for a rotating disk in the water at different ambient temperatures.

The agreement between the experimentally obtained values, and the prediction of the self-similarity solution was excellent. Based on this observation, it can be stated that the actual experimental apparatus was capable of enabling an investigation of laminar flow over a heated rotating disk. It was further assumed that the high accuracy of the test apparatus also supported a study of the onset of instability.

4 Results and discussion

In this section, the results of local heat transfer measurements are presented and discussed, especially regarding the effect of heating on the stability of the boundary layer flow over a rotating disk.

4.1 Temperature deviations and heat transfer

For several heating rates, running speeds and bulk temperature levels, the normalized temperature deviation in respect to the laminar flow regime were experimentally obtained. Examples for such a measurement are shown in Figure 8. Despite the noticeable uncertainty level, the laminar flow regime with an essentially constant wall temperature and a dramatic increase of the temperature deviation at a certain local Reynolds number can be noticed in Figure 8. The jump of the wall temperature occurred at different Reynolds numbers of approximately 1.5×10^5 and 3.0×10^5 depending on the wall temperature difference $\Delta T = T_w - T_\infty$, respectively. This observation might be interpreted as an indication that heating can stabilize laminar water flow, but since such temperature jumps are also already known as characteristics for the transition region in the case of heated disks rotating in the air [20, 27], a further analysis was necessary. Since the Landau model is connecting the occurrence of a perturbation mode directly with the Reynolds number and a transition point, the heat transfer data were evaluated in respect to the Landau model (equation (6)).

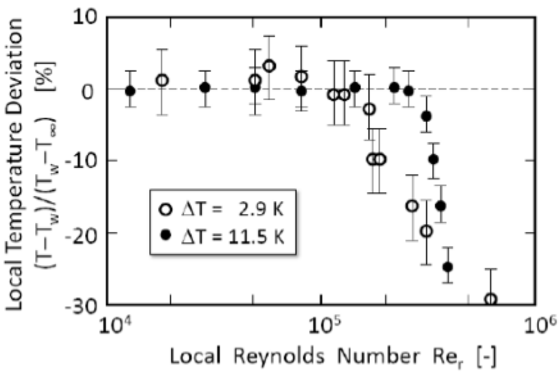

Fig. 8. Normalized temperature profiles for two different wall temperature differences

As an example, the results obtained for a wall temperature difference value of $T = 2.9$ K are shown in Figure 9. The Landau model seemed to be well applicable to the present experimental data, as demonstrated in Figure 9. By means of minimizing the error between the experimental data and the Landau model, the values for the two model parameters k and Re_{cr} were determined. The value $Re_{cr} = 1.5 \times 10^5$ was obtained for $\Delta T = 2.9$ K as illustrated by means of Figure 9. The physical meaning of the Landau model parameter Re_{cr} of Figure 9 became clearer when the local Nusselt number expression Nu_r/K was plotted against Reynolds number Re_r as shown in Figure 10. Since different Prandtl numbers Pr (between Pr = 4 and Pr = 6.5) were involved during the measurements, the local Nusselt number Nu_r was divided by the laminar heat transfer correlation parameter K(Pr). Then, the expression Nu_r/K becomes strictly identical with $Re_r^{1/2}$ (see equation (4)) for any Prandtl number in the laminar regime. This behaviour was indeed observed up to a certain critical Reynolds number in the experiments with water (see the corresponding line denoted by $m = 1/2$ in Figure 10). For sufficient large Reynolds numbers, Re > Re_{cr}, deviations from the laminar line were observed. The same critical Reynolds number values were found in the plots shown in Figure 10 as obtained by the Landau model (see Figure 9).

The critical Reynolds number Re_{cr} depended significantly on the wall temperature difference ΔT. For sufficient large Reynolds numbers of about Re > 3.0 up to 4.0×10^5, the local Nusselt numbers could be correlated by means of a turbulent heat transfer correlation (see the corresponding line denoted by m = 0.8 in Figure 10)

$$Nu_r \propto Re_r^{0.8} \quad (9)$$

In literature, agreement with this correlation is assumed to represent the fully turbulent flow regime [20]. In addition to the new data obtained for a disk rotating in water, the accurate heat transfer data reported by Elkins [27] are schematically plotted in Figure 10.

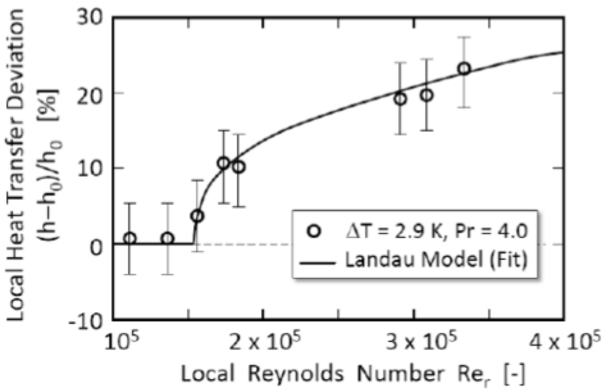

Fig. 9. Application of the Landau model to heat transfer from a rotating disk

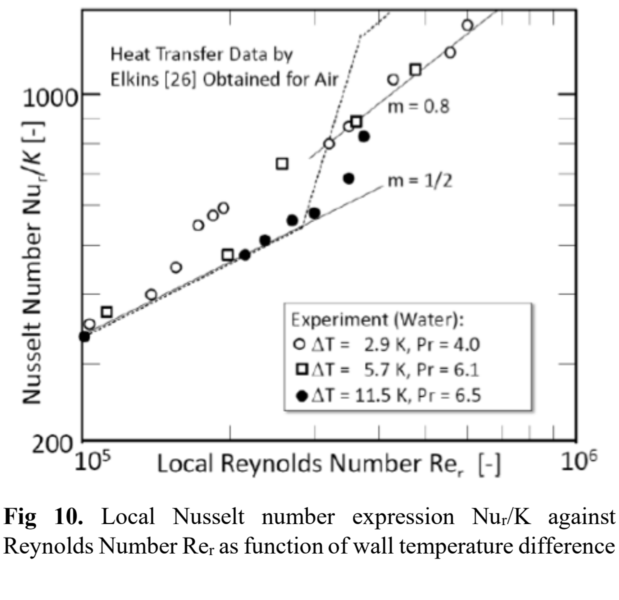

Fig 10. Local Nusselt number expression Nu_r/K against Reynolds Number Re_r as function of wall temperature difference

In the case of air, the significant increase of the local Nusselt number between $Re_r = 2.6 \times 10^5$ and 3.6×10^5 is typically identified with the transition region [20]. This is reasonable, because weak spiral vortices certainly do not contribute substantially to the total heat transfer in the case of air as discussed in section 2.2, and the observed heat transfer augmentation could be better explained by a further developed transitional flow. But in the case of water, even weak spiral vortices seemed to create a measurable heat transfer effect.

4.2 Critical Reynolds number

Based on heat transfer data discussed in section 4.1, the critical Reynolds number Re_{cr} was obtained as function of wall temperature difference $\Delta T = T_w - T_\infty$, Figure 11. In Figure 11, Re_{cr} was defined as the local Reynolds number value for which a first systematic deviation from the laminar heat transfer correlation was observed. In addition to the experimental data, a best-fit line is plotted in Figure 11. Remarkably, this line predicted an isothermal critical Reynolds number $Re_{cr,0} = 8.8 \times 10^{4,}$ which is in excellent agreement with literature data [16−20]. This coincidence might be considered as a further strong indication that the actual critical Reynolds number is indeed a good description variable of the onset of instability of the water flow over a rotating disk.

The observed stabilization of the three-dimensional flow over a rotating disk due to heating was compared with results calculated for the two-dimensional Blasius flow past a flat plate. For the latter case, reliable literature data are available [4, 5, 9, 10]. The property-expansion method developed by Herwig and co-workers (equation (1)) was able to capture the linear trend over the considered temperature range. However, since the base flow over a rotating disk is quite different than the Blasius flow, the good agreement between the new data and the predictions by the direct method (equation proposed by Wazzan et al. in [4] and evaluated as in [9]) for small temperature differences should not be over-interpreted.

A significant effect of Prandtl number Pr on the critical Reynolds number was not observed in the present study. For instance, the same critical Reynolds number $Re_{cr} = 1.5 \times 10^5$ was obtained for water with Pr = 4 and Pr = 7 for the same wall temperature difference $\Delta T = 2.9$ K. Furthermore, the data points plotted in Figure 11 behaved similarly although different Prandtl numbers were involved. Such a weak or moderate Prandtl number dependency is in agreement with predictions of the two-dimensional theory proposed by Herwig and co-workers [10].

4.3 Heating and absolute instability

So far, a rigorous linear perturbation analysis for the rotating disk in a fluid with temperature-dependent viscosity is essentially missing. Still, an interesting item might be suggested using Lingwood's isothermal analysis [19] regarding absolute instability.

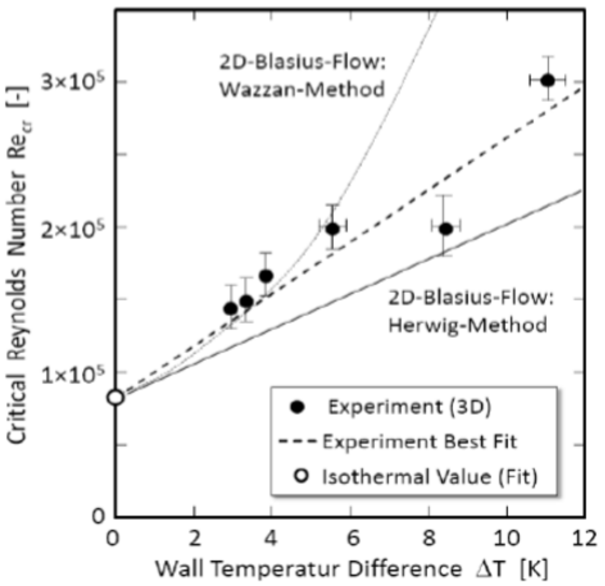

Fig. 11. Critical Reynolds Number Re_{cr} as function of wall temperature difference ΔT for the three-dimensional flow over a rotating disk in water

It is known that inviscid crossflow instability destabilizes boundary layer layers, and her calculations [19] showed that in the case of a rotating disk boundary layer the absolute instability occurring at Re_{abs} is caused by an inviscid mechanism (although the exact value of the corresponding Reynolds number Re_{abs} is affected by viscous flow effects).If Coriolis and streamline curvature effects are neglected, the Rayleigh equation for the rotating disk calls

$$\left(\alpha\frac{u}{r\omega}+\frac{\beta}{\sqrt{Re}}\frac{v}{r\omega}-\bar{\omega}\right)\left(\frac{d^2}{d\left(z\sqrt{\omega/v}\right)^2}-\alpha^2+\frac{\beta^2}{Re}\right)\widehat{w} = \left(\alpha\,\frac{d^2}{d\left(z\sqrt{\omega/v}\right)^2}\frac{u}{r\omega}+\frac{\beta}{\sqrt{Re}}\,\frac{d^2}{d\left(z\sqrt{\omega/v}\right)^2}\frac{v}{r\omega}\right)\widehat{w} \tag{10}$$

In the inviscid Rayleigh equation (10), viscosity is considered to act only in the establishment of the base flow *u* and *v* and for defining the boundary layer thickness and the Reynolds number Re of the flow. Since all of these base flow quantities depend rather weakly on temperature-dependent viscosity terms, it follows that the stability limit predicted by the Rayleigh equation (10) would be the same as for an isothermal configuration. Thus it might be stated that the Reynolds number for the absolute instability, Re_{abs}, is not strongly affected by temperature-dependent viscosity terms.

Since the end of the transition, Re_t, is observed to be in the vicinity of this absolute instability, it might be concluded that the fully turbulent flow state on a heated disk is reached close to the values obtained for an isothermal disk. This conclusion is supported by the present experimental data, see Figure 10. Then, it follows that heating (and cooling) affects practically only the onset of transition but not the end of the transition in the case of such a crossflow instability. This is a remarkable difference to two-dimensional configurations like the Blasius flow past a flat plate. Here, Linke [28] observed in 1942 a systematic effect of heating on the turbulent drag of a flat plate and concluded that heating affects the stability of the boundary layer (that was later confirmed by detailed analysis [1, 2]).

5 Conclusion

In this contribution, the results of an experimental study employing an electrically heated rotating disk placed in a large water tank were reported. Local wall temperatures and heat transfer coefficients were determined. The accuracy of the test apparatus was assessed by comparing actual heat transfer data with predictions of the exact self-similarity solution for laminar flow. The critical Reynolds number was found by wall temperature measurements without the need to employ invasive measurement techniques like hot-wire anemometry in the flow field. Although some data scattering was unavoidable, a substantial increase of the critical Reynolds number due to heating was found. As known to the authors, this represented the first experimental study dealing with the effect of heating on the stability of the three-dimensional flow. The observed increase of the critical Reynolds number was in reasonable agreement with literature results reported for two-dimensional heated water boundary layers. The present approach yielded asymptotically an isothermal critical Reynolds number value that was very close to the literature result obtained for a disk without heating.

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