Modified Fuzzy Inference Method for Heat Flux Inversion of Geothermal Reservoir Heat Source

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Abstract. The key to determine the performance of fuzzy inference inversion is to select a reasonable domain. However, there is no universal method for selecting domain at present. According to the characteristics of heat flux of geothermal heat source and the research of fuzzy inference inversion process, this paper modified the fuzzy inference method from two aspects of domain setting and iteration termination condition. The recommended domain and selection scheme for solving the problem of geothermal heat flux are given, and the modified fuzzy inference inversion method is applied to Rucheng geothermal field to verify the method. The results showed that the modified fuzzy inference inversion method could overcome the problem that the solution of the traditional method fell into a cycle, and the relative error of the verification term was less than 5%. Compared with the traditional method, the modified method greatly improved the computational efficiency, and the number of iterations was reduced to only 7. This method has a good application prospect for geothermal heat source inversion and resource evaluation.

1 Introduction

Obtaining geothermal source information accurately, especially the flux is crucial for the efficient evaluation and rational utilization of geothermal energy. Currently, the geophysics inversion methods [1-2] are mainly applied, which always need huge drilling and geological information [3]. However, these methods often require lots of drilling to obtain data, which not only needs a lot of cost, but also the obtained geological abnormal area cannot be directly confirmed as a geothermal heat source. Although thermal methods and remote sensing methods can obtain temperature distribution intuitively, due to the self-sealing characteristics of geothermal, it is often only possible to obtain temperature information in shallow areas [4,5]. For geothermal exploration, thermal method is easier and more intuitive than others. Based on inverse heat transfer problem, applying a few data can invert expected parameters, which is more economical and efficient [6].

However, due to the complex structure of geothermal reservoir and the highly ill-posed characteristic of inverse problem, the researches on inverse geothermal heat transfer problem are extremely rare. At present, the methods to solve the inverse heat transfer problems are divided into three major categories: Tikhonov regularization method [7], gradient-based methods (steepest descent method, conjugate gradient method, Levenberg - Marquardt method, etc.) [8], and evolutionary- and artificial intelligence-based methods (genetic algorithm, neural network method, particle swarm optimization algorithm, fuzzy inference method, etc.) [9]. The regularization method can overcome the illposed problem to some extent, but its calculation speed and accuracy are greatly affected by the selection of regularization parameters [10]. There is still no common method of parameter selection. Gradient-based method is often fast and has strong anti-ill-posed ability [11]. However, they are greatly influenced by selection of the initial value and are easily fall into the local optimal solution. In addition, the inversion results are significantly deteriorated when the information is incomplete or the error is large. When the situation is complex or there is a large amount of information, the calculation efficiency will be significantly affected, and even the inversion results may not be calculated. Although there are many researches and methods on the inverse heat transfer problem, the ill-posed problem still cannot be completely overcome, and most of the research is also limited to low-dimensional and simpler problems [12]. Geothermal reservoir is complex, with discontinuous physical parameters, non-uniform and discontinuous anisotropic media, and the existence of seepage. Related studies have shown that the fuzzy inference method has good calculation efficiency and robustness[13]. Compared with the low computational efficiency of genetic algorithms and the need for huge data to train of neural networks, the fuzzy inversion method has a brighter application prospect for solving the inverse geothermal heat sources problem [14].

A modified fuzzy inference inversion method for geothermal heat flux inversion is presented. The application of the modified method to Rucheng geothermal field case verifies the effectiveness and accuracy of the method.

2 Modified fuzzy inference method

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Fuzzy inference method is mainly based on fuzzy set theory, which simulates human thinking for uncertain reasoning, decision-making and calculation [15]. The establishment of fuzzy rules determines the efficiency and accuracy of calculations [16], and the judgment conditions also affect the accuracy of the results and the efficiency of calculations. The main determinant of fuzzy rules is the selection of the domain. Based on expert knowledge, it is still a large human factor and uncertainty factor, and there is still no common rule; the judgment condition is currently generally determined by the sum of squares of errors. Various factors are considered more comprehensively, but according to different situations, there is still improvement for the judgment condition. According to the actual situation of geothermal reservoir heat source heat flux inversion, this paper studied the influence of different domain selections on the efficiency and accuracy of fuzzy inversion.

2.1 Fuzzy inversion for geothermal source

There is still some controversy about the definition of geothermal heat source. The main point is that geothermal heat sources include magma packages, radioactive material decay, fracture friction, and chemical reaction heat. For hydrothermal geothermal system, especially in areas near volcanoes, the magma packet theory is generally accepted [17]. When studying geothermal utilization, more attention is paid to the areas where the heat flow is concentrated and strong in the geothermal reservoir in order to arrange the location of wells reasonably. Therefore, it is assumed that a certain area at the bottom of the reservoir is a heat concentrated area to provide energy to the reservoir.

According to the general characteristics of geothermal reservoirs, a simplified three-dimensional geothermal reservoirs model is established. The simplified model is divided into three layers. The uppermost layer is the surface area, and its main heat transfer way is heat conduction; the second layer is a thermal storage layer, and its main heat transfer ways are heat conduction and heat convection; the third layer is the area below the thermal storage layer , The main heat exchange way is heat conduction. According to the reservoir characteristics and the existence of seepage, each reservoir is set as a porous medium, and the heat exchange mode is controlled by adjusting its parameters. The energy equation of a three-dimensional geothermal reservoir is:

$$\left(\rho C_p\right)_{eff}\frac{\partial T}{\partial \tau}+\rho_f C_f q\nabla T-\nabla q_T=-\Phi_T \tag{1}$$

Where T indicates the reservoir temperature (K), $(\rho C_p)_{eff}$ represents the effective volumetric capacity, defined as $(\rho C_p)_{eff}=\varphi\rho_f C_f+(1-\varphi)\rho_s C_s$, where ρ_s indicates the solid density (kg/m^3), C_f and C_s are fluid and solid heat capacities (J/kg·K) respectively. q_T can be described as follow:

$$q_T=\lambda_{eff}\nabla T \tag{2}$$

Where λ_{eff} represents the effective thermal conductivity, defined as $\lambda_{eff}=\varphi\times\lambda_f+\left(1-\varphi\right)\times\lambda_s$, where λ_f and λ_s are fluid and solid thermal conductivities (W/m·K) respectively.

Because the geothermal heat source heat flux is relatively small, after establishing the reservoir model, twelve kinds of collocations and four initial values are set up to study the influence of the domain on fuzzy inversion. The input domain is set to three cases: PA: [-75,75], PB: [-90,90], PC: [-105,105]. The output universe is set to four cases: OA: [-1.5,1.5], OB: [-3,3], OC: [-4.5,4.5], OD: [-6,6]. The twelve combinations are: Case 1 (PA-OA), Case 2 (PA-OB), Case 3 (PA-OC), Case 4 (PA-OD), Case 5 (PB-OA), Case 6 (PB-OB), Case 7 (PB-OC), Case 8 (PB-OD), Case 9 (PC-OA), Case 10 (PC-OB), Case 11 (PC-OC) and Case 12 (PC-OD). After the direct problem is solved, six temperature measurement points are selected to obtain the data, and the actual measurement point temperature under the true value of the heat flux density is obtained, and the six measurement points are used to calculate the four cases with the initial value of 0.25, 1, 3, and 5 respectively.

According to the actual situation, we select the part in Fig.1 [18] as the research area and select ten wells for research. Among the ten wells, ZK4 and ZK8 are production wells, and the rest are testing wells. Their distribution is shown in Fig.1. The data of six wells for calculation (ZK3, ZK4, ZK5, ZK6, ZK7, ZK9) is selected, and the data of the remaining wells are used to verify the accuracy of the method.

Fig. 1. Location of boreholes.

2.2 Results and discussion

2.2.1 Initial value is 0.25

When the initial value is set to 0.25, the inversion results of the 12 cases are shown in Fig.2 and Fig.3. The first number in the brackets in the figure represents the unilateral length of the input domain, and the second number represents the unilateral length of the output domain. According to the results, it is not difficult to find that the entire inversion process presents two characteristics. One is that a trend is gradually approaching the real value, as shown in Fig.2; the other is that it oscillates at both ends of the real value, as shown in Fig.3. The ideal inversion process is the form shown in Fig.2. The results of Case 6 are the most ideal in various situations in Fig.2, with high calculation efficiency and accurate calculation results. When the input domain is the same and the output domain increases, the number of iterations decreases. However, when the output domain is too large, the adjustment value will increase. In some cases, results can be obtained, such as cases 2, 7, and 11. The domain value matches well. When the ratio of input and output domain values is further changed, accurate inversion results cannot be obtained, which then manifests as equal amplitude oscillations on both sides of the real value. When the output domain is the same, the input domain and the inversion performance do not show a correlation, and it is more important to select the appropriate value. The smaller the input domain, the larger the output domain, the greater the oscillation amplitude, and the larger the output domain of the same input domain, the slower it will reach the same amplitude oscillation. When the input domain is large, when the output domain is small, the second situation is less likely to occur. When the second situation occurs, if the final oscillation occurs, the average of the two repeated values is exactly the result of the inversion. The input-output domain ratio in Fig.2 is over 30, and each case in Fig.3 is less than 30. Of course, 30 is not necessarily a critical value, but it can be explained that the larger the input domain, the better. However, the increase in the ratio will increase the number of iterations, and affect the calculation efficiency. So, selecting a reasonable value and ratio is more appropriate.

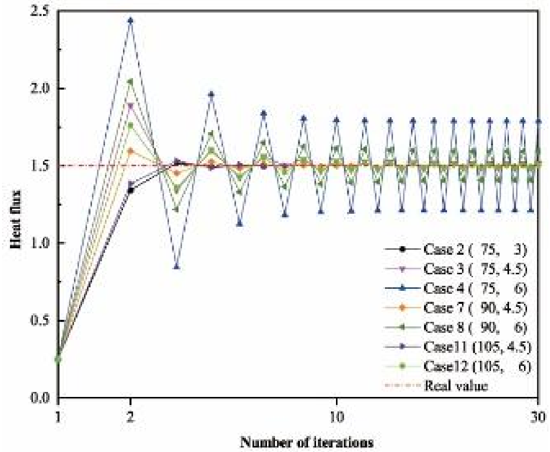

Fig. 2. Inversion of type 1 when initial value is 0.25.

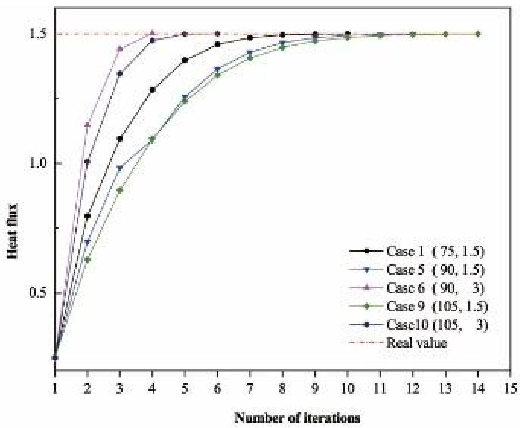

Fig. 3. Inversion of type 2 when initial value is 0.25.

2.2.2 Initial value is 1

When the initial value is 1, the inversion results are shown in Fig.4 and Fig.5. When the initial value is 1, the inversion performance and rules of various cases are basically the same. The set of case 6 is still the best performing group, and the results are still shown in two characteristics. When the inversion process is as in the first situation, the number of iterations is reduced to varying degrees, but the change is not large, and the number of iterations is reduced at most 2. In some cases, the number of iterations does not change.

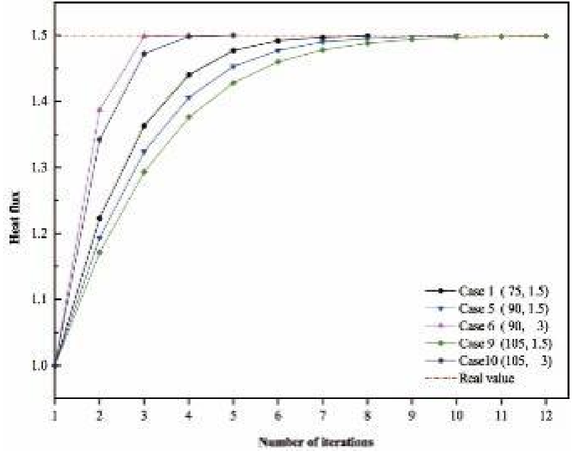

Fig. 4. Inversion of type 1 when initial value is 1.

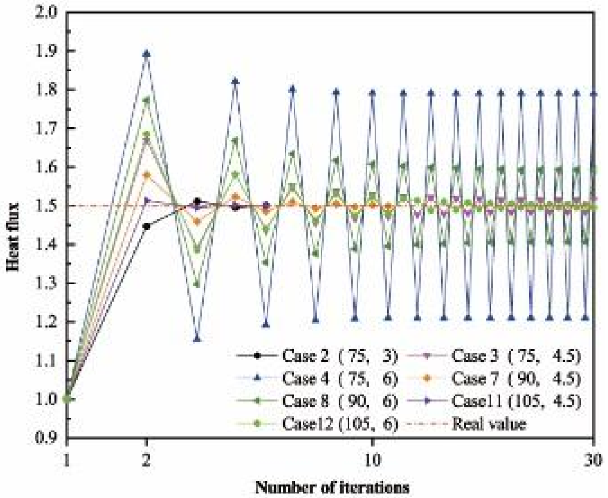

Fig. 5. Inversion of type 2 when initial value is 1.

2.2.3 Initial value is 3

When the initial value is 3, the inversion results are shown in Fig.6 and Fig.7. The performance and law of the inversion process are basically the same as the above two cases, indicating that this method can solve the inversion problem regardless of whether the initial value is greater or less than the real value. At this point, the best-performing case is also case 6.

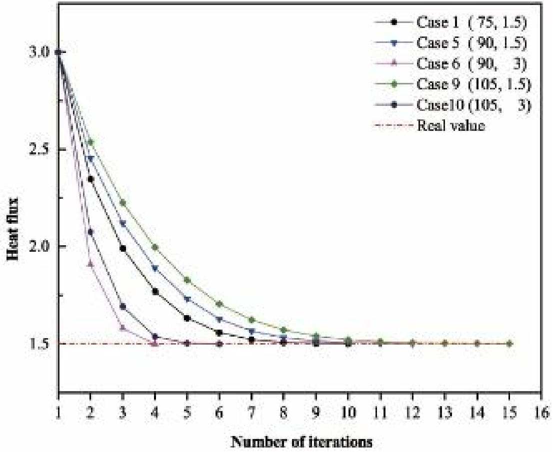

Fig. 6. Inversion of type 1when initial value is 3.

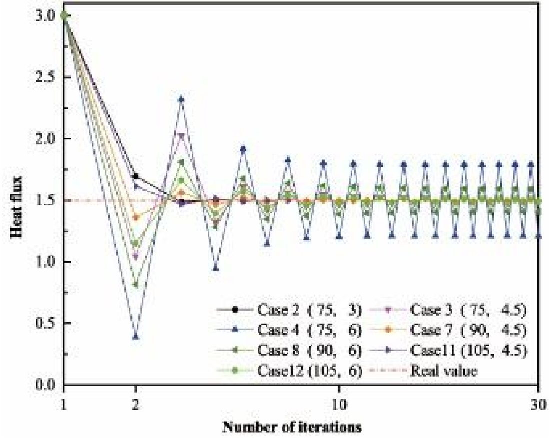

Fig. 7. Inversion of type 2 when initial value is 3.

2.2.4 Initial value is 5

When the initial value is 5, the inversion results are shown in Fig.8 and Fig.9. Through the comparison of the above four cases, the fuzzy inversion is not very sensitive to the selection of the initial value. Although it also shows the characteristics that the closer to the real value, the faster the inversion, but the number of iterations in different cases is almost the same, the maximum difference is 2, the inversion speed is not much different, and the final inversion results are all the same. The performance and rules of the inversion process of each case are consistent. Among the 12 cases set, the performance of case 6 is the best. It is not difficult to find that when the second inversion value and the first inversion value are on the same side of the real value, the inversion result can usually be obtained. When they are different, the situation of equal amplitude oscillation on both sides of the real value will often appear.

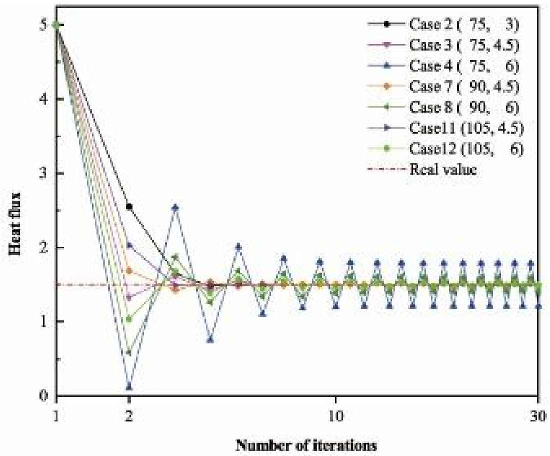

Fig. 8. Inversion of type 1 when initial value is 5.

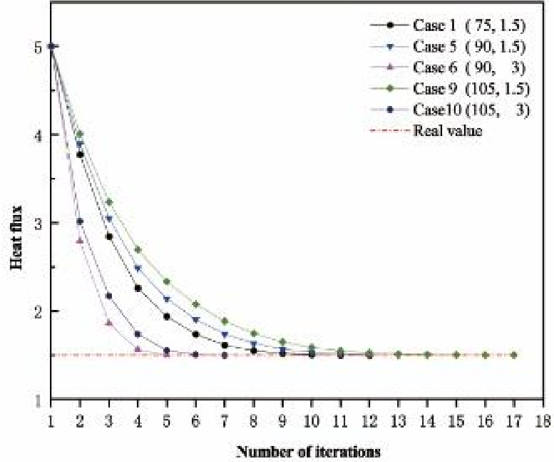

Fig. 9. Inversion of type 2 when initial value is 5.

2.3 Introduction of modified method

According to the characteristics of geothermal and the above research, a modified fuzzy inversion method is proposed. This method is modified in two aspects.

2.3.1 Selection of input and output domains of fuzzy rules

In view of the characteristics of geothermal and the performance of the above-mentioned cases, it is recommended to choose Case 6 (input domain [-90,90], output domain [-3,3]) as the domain choice for inverting geothermal heat flux. It can also adopt another scheme. Select the input and output domains according to the actual situation, take a larger value for the input domain, and set the input-output domain ratio to be greater than 30.

2.3.2 Selection and improvement of iteration termination conditions

According to the results, although the above selection of the domain has been optimized, there may still be unavoidable oscillations. The accurate inversion can also be obtained by selecting and improving the iteration termination conditions. The iterative termination condition generally chooses the method in which the sum of squares of errors is less than a small value.

Although this method can consider the impact more comprehensively, it cannot converge to obtain the inversion result after the oscillation occurs, and it will continue to fall into the loop. Therefore, additional judgment conditions must be added. Because the establishment of the direct problem model has many artificial factors that cannot fully simulate the geological situation. For geothermal problems, if the limit value of the error square is often selected too large, a satisfactory result may not be obtained. If the selection is too small, the termination condition will never be met when applied to the actual situation. When the same value is found consecutively, the calculation is stopped, the result is output and the manual inspection is prompted. When it is determined that there are inversion values with the same or very similar intervals, the average value of the two end points of the oscillation is taken as the input of the next operation until the requirement of the sum of squares of the error is satisfied to obtain the inversion result.

When the fuzzy rules are selected reasonably, the inversion result can be obtained quickly. In addition, when the iteration termination condition is further restricted, the phenomenon of non-convergence can be effectively solved and the inversion result can be obtained. Two improvements to fuzzy inversion can effectively solve the problem of heat flux inversion in geothermal reservoirs.

3 Rucheng verification case

To verify the reliability and superiority of the modified fuzzy inversion method, the modified method is applied to the inversion of heat flux in Rucheng hydrothermal geothermal field. The Rucheng geothermal field is located on the north side of the middle section of the Nanling Mountains, surrounded by a depression basin of high and low in the middle, which belongs to the terrain of middle and low mountains. The geomorphic structure is divided into three types: structural erosion, structural denudation and erosion accumulation type. The geothermal field area belongs to a warm and humid subtropical climate, with an annual precipitation of 1051.9-2303.6 mm, a multiyear average temperature of 289.75 K, and an annual average absolute humidity of 16.8%. In the geothermal area, the width of the hot water riverbed is 20-70 m, and there are deposits of sand and gravel ranging from 0.5-20 m. The highest flood water level of the Reshui River is 342.734 m, the lowest water level is 339.879 m, and the maximum flow is about 300 m^3/s. The exposed stratum in this area is simple, mainly composed of Quaternary alluvium, Sinian sandy slate, shallow metamorphic sandstone and early granite rock mass of Yanshan. Then, compare it with the results of traditional fuzzy inversion. The inversion results are shown in Fig.10. The modeling of the Rucheng geothermal reservoir is also simplified to the general geothermal reservoir model mentioned above. The parameters of each part are set as shown in Table 1 according to the actual exploration results, and the shape and location of the heat source are artificially assumed. The area at the bottom except for the region of the heat source is in an adiabatic condition, and the surrounding area is also in an adiabatic condition with certain initial value. The top is in robin condition. Since the actual situation is natural convection heat exchange with air in the large space, the convective heat transfer coefficient is 5 $W/(m^2 \cdot K)$. The locations of selected measurement points are shown in Table 2.

Using the traditional fuzzy inversion method and the improved fuzzy inversion method, the same heat flux result ($1.361 W/m^2$) was obtained, which is shown in Fig.10. It can be seen that when only the termination condition is improved, the number of iterations is reduced from the original infinite loop at the same value (19) to 15, which improves the calculation efficiency and prevents getting into the loop. If the choice of domain is also improved, the fuzzy inversion does not show an oscillating form, but smoothly inverts to the final value, and greatly reduces the number of iterations to 7. Since there is no accurate result of heat flux obtained from actual exploration, the remaining drilling data is selected and is compared with the data obtained from the inversion value to verify the accuracy of the inversion results. The results are shown in Table 3. The measured temperature id obtained from actual drilling data at selected points. Inversion temperature is calculated by substituting inversion parameters into the established simulation model.

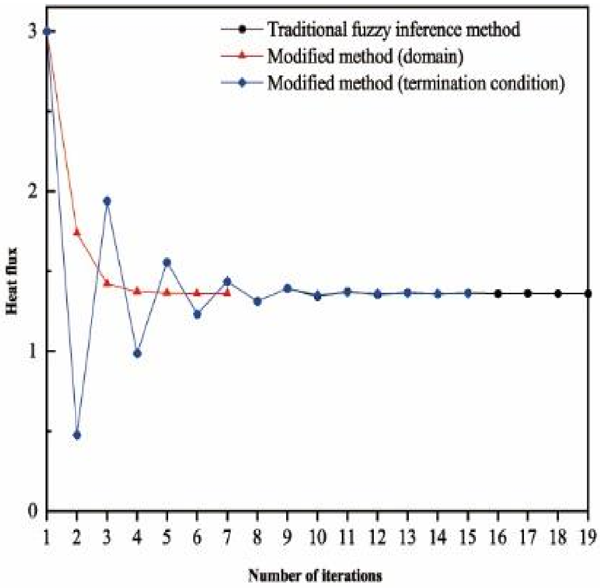

Fig. 10. Inversion results f Rucheng geothermal field.

Table 1. Parameters of the simplified model of Rucheng geothermal field.

Layer	Length	Width	Height	Density
	m	m	m	kg/m^3
1	800	800	40	1730
2	800	800	60	2000
3	800	800	400	2600
Layer	Thermal conductivity		Heat capacity	
	W/(m·K)		J/(kg·K)	
1	0.9		840	
2	2.0		950	
3	2.6		837	

Table 2. Locations of selected measurement points.

Number	Length	Width	Depth
	m	m	m
ZK3	550.54	453.33	100
ZK4	430.11	408.89	100
ZK5	344.09	275.56	100
ZK6	627.96	173.33	100
ZK7	275.27	480	100
ZK8	490.32	591.11	100
ZK9	576.34	777.78	100
ZK10	275.27	533.33	95
ZK12	309.68	177.78	100
ZK18	722.58	635.56	100

The relative errors between the temperature field data obtained by the inversion and the actual measured temperature data are within 5%, which verifies the accuracy of the fuzzy inversion method. Compared with the traditional fuzzy inversion method, the improved fuzzy inversion method greatly improves the calculation efficiency and the problem of loop.

Table 3. Verification of the inversion results of Rucheng geothermal field.

Number of drilling wells	Measured temperature	Inverse temperature	Relative error
	K	K	%
ZK8	318.75	316.31	-0.77
ZK10	312.85	320.55	2.46
ZK12	302.15	313.36	3.71
ZK18	300.95	308.37	2.47

4 Conclusion

By analyzing the characteristics of the fuzzy inversion process, combined with the characteristics of the researched object, the fuzzy inference method is modified in terms of the domain setting and the iterative termination condition. A recommended domain and selection method are given, and the reliability of the modified method and its superiority compared with the traditional fuzzy inversion method are verified through the Rucheng geothermal case. The specific conclusions are summarized as follows:

(1) The results of the fuzzy inversion process under different initial values and different domain settings show that fuzzy inversion can efficiently obtain accurate inversion results. The fuzzy inversion process presents two characteristics: one-sided approximation to the true value and final oscillation with equal amplitude around the real value on both sides.

(2) In view of the characteristics of the traditional fuzzy inversion process, improvements were made in two aspects: the domain setting and the selection of the iterative termination condition. The recommended domain of geothermal heat source heat flux (input domain [-90,90], output domain [-3,3]) and selection methods are presented, and modified iteration termination conditions are proposed.

(3) The modified fuzzy inversion method was applied to Rucheng geothermal reservoir case, and the inversion results were verified based on other drilling data. The results showed that the relative errors of the inversion results were all within 5%, indicating the method’s effectiveness and accuracy. Compared with the traditional method, the modified method overcomes the shortcomings (falling into an oscillation cycle) of the traditional method. At the same time, the number of iterations is greatly reduced to 7.

Acknowledgements

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