Study on Recycling Scheme of Lake Mead Wastewater

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Abstract. There is a sever draught happening in the Mead Lake that the observation of the lake elevation is decreasing dramatically. With increasing water demand in people's life, people need to find more supply for them to meet their needs. There is a good way that recycling the wastewater from sinks, toilets and showers. For some people, they may think it impossible and even disgusting but this kind of technology exists and developing. Before conducting the wastewater recycling program, we need to investigate more about the impact of drought on reservoirs like the volume and elevation change of the Lake Mead and consider wastewater recycling as a solution to water shortages, Introduced by the environmental issue, the wastewater recycling program can be conducted. We consider some factors that may influence the plan like the changes in the water level of Lake Mead, demand of water and cost. We also prioritize the decisions that local leaders need to make and their impact. Finally we give our plan for this wastewater recycling program into three stages.

Keywords: Lake Mead; Draught Period; Curve fitting; ARIMA; Wastewater recycling.

1. Background

The Colorado River Reservoir Lake Mead, located on the border of Nevada and Arizona, is the largest reservoir in the United States (Fig. 1). In the summer of 2021, Lake Mead dropped to its lowest level since it first filled up in the 1930s. The drought caused by climate change and the increasing demand for water by the 25 million people served by Lake Mead have reduced the capacity of the reservoir to about 36% of its full capacity. On August 16, 2021, the Bureau of Reclamation announced the first ever water shortage statement for the Colorado River [1]. Initially, this first-level water shortage statement led to a reduction in water supply in Arizona, Nevada, and New Mexico, with agricultural communities being the first to feel.



Figure 1. Lake Mead Overview (National Park Service [2])

2. The volume of Lake Mead

2.1 The factor of the Lake Mead volume change

We all know that the volume of Lake Mead is determined by the inflow, outflow and loss. The relationship is Volume change=|inflow-outflow-loss|

The inflow factors are mainly the Colorado River providing more than 96% of the inflow, additional water from the other three tributaries (specific names), and direct precipitation on the lake. Because the Colorado River provides a large amount of inflow, we can only consider this source. Of course, direct precipitation on the lake will also affect the volume of Lake Mead, but normal precipitation has little effect on the volume of Lake Mead. The outflow is directly consumed by releasing water (for example, through the dam) and directly from the lake. The outf also passed through Colorado's dam release, and the amount of water consumed directly from the lake and can be ignored.

If the lake level drops another more, the water will no longer flow through the Hoover Dam. Loss occurs through evaporation. The climate change makes the situation worse.

As the temperature rises, the snowmelt supply to the river decreases and more water evaporates.

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Figure 2. Sources of Lake Mead [5]

Fig. 3 shows the main factors that influences the water volume in Lake Mead. The climate can change the evaporation and force people to use more water. Meanwhile, the runoff also decreases, which worse the situation and people suffer the draught when there is not enough water in the Lake Mead.

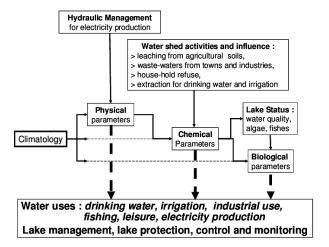


Figure 3. Main factors influencing the volume of water in Lake Mead.[6]

2.2 The relationship among elevation, surface area and volume

Assuming that underwater lake is a common lake basin, such as conical lake basins (water-surface and water variations) or cup-shaped lake pool (water and water depth are non-linear, non-linear), lake's underwater terrain can be considered Continuation of the water. Have similar features. The terrain feature parameters within a certain range above the lake surface (such as terrain height difference, average elevation, and slope, etc.) can be used as indicators of underwater terrain. Therefore, by analyzing and constructing the functional relationship between lake elevation, plane area, and volume increment above the water surface, a numerical model of the relationship between the amount of water below the water surface and the corresponding water surface area can be derived (Fig. 4).

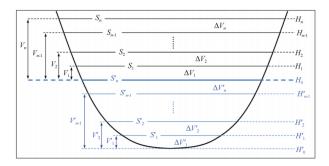


Figure 4. Lake water volume estimation model based on lake basin similarity [7]

The calculation steps are as follows:

Based on the data of the lake Mead, calculate the area S_i ($i=1,\ 2,...,n$) of different elevation planes at equal intervals above the lake surface H_0 , and establish the functional relationship between the area S_i and the elevation H_i .

Calculate the volume V_i (i=1,2,...,n) between the different elevation planes at equal intervals above the lake surface and the lake surface, and use

$$\Delta V_i = V_i - V_{i-1}, i = 1, 2, \dots, n,$$
 (1)

Calculate the volume increment corresponding to the unit elevation increase ΔV_i , $i=1,2,\cdots,n$, establish the functional relationship between the area S_i and the volume increment ΔV_i .

Assuming that the area corresponding to the volume increase of 0 is the lake bottom area, the lake bottom elevation $H_{\ 0}^{'}$ can be approximated by the functional relationship between area and elevation.

Taking the lake bottom elevation as the starting elevation, using the elevation-area function relationship and the area-volume increment function relationship to divide the lake surface H_0 at equal intervals to obtain the lake area and volume increment data pairs corresponding to different water levels below the water surface $(S_i, \Delta V_i)$, $i=1,2,\cdots,n$.

$$i=1,2,\cdots,n$$

Accumulate the volume increment step by step to obtain the lake volume V_i , $i = 1, 2, \dots, n$ corresponding to different water levels below the water surface, and the calculation formula is

$$V_{i}' = V_{i}' + \Delta V_{i-1}', i = 1, 2, \dots, n,$$
 (2)

Using the water surface area and volume data pairs (S'_i, V'_i) corresponding to different water levels, $i=1,2,\cdots,n$, the functional relationship between the lake water surface area S'_i and the volume V'_i can be established. And Table 1 shows the relationship between

the elevation, area and volume of Lake Mead calculated by the Bureau of Reclamation in 2010 [4].

Table 1. Area and Volume of Lake Mead by Elevation Level

Elevation (feet)	Area of Lake (acres)	Volume of Lake (acre-feet)
1229.0	159,866	29,686,054
1219.6	152,828	28,229,730
1050.0	73,615	10,217,399
895.0	30,084	2,576,395

- The water level of Lake Mead is measured by elevation in feet above mean sea level.
- The area of Lake Mead is measured in acres.
- The amount of water in Lake Mead is measured in acre feet.

We need to correspond to the horizontal plane corresponding to the bottom of the lake as the start reference plane, and the statistical analysis of the water level area of different lakes in Table 2 shows that these two have strong linear relationships (R2 = 0.9842) which is shown in Figure 5, and the expression is

$$S = 393.8409H - 328496 \tag{3}$$

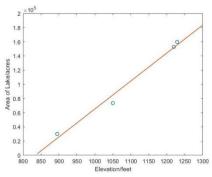


Figure 5. Area of Lake Mead Vs. Elevation. Functional relationship between elevation and surface area in different location.

If we have volume increment data, the same method is used to calculate the area and the functional relationship between the volume increments corresponding to different lake elements, and there should be strong linear relationships between the two parameters.

According to the area-volume increment discussion, we need to know the horizontal surface area S_0 when the volume increment is 0, and this area is the estimated area of the lake bottom of Mead. Using the elevation-area function (Eq. (3)) to calculate the area tends to a certain value, the corresponding elevation H_0 is the estimated lake bottom elevation.

Starting from the estimated lake bottom elevation and the height difference of 1 feet as the interval, calculate the horizontal plane area Si' (i=1,2,...,n) to the lake bottom elevation (lake elevation) corresponding to different underwater elevations. Find different water depth corresponds to the area data of the plane. Use the relationship between volume increment and elevation to obtain the volume increment $\Delta Vi'(i=1,2,...,n)$

corresponding to different underwater depth planes, and use equation (2) to add up gradually to obtain the water storage corresponding to different water surface areas in the Lake Mead Vi' (i =1,2,...,n). Using (Si', Vi') (i =1,2,...,n) data to perform regression analysis, it is found that the relationship between the lake area and the lake water volume in Lake Mead is a standard quadratic function (Figure 7), the model formula is

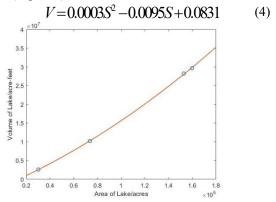


Figure 6. The relationship between area and volume for Lake Mead.

3. The water level of Lake Mead

3.1 Data Sources

The water level is the most important indicator to measure the water regime. The data used here is the monthly average water level from 1937 to 2021, as well as the annual maximum and minimum elevations. Finally, the water level results from 1937 to 2021 are obtained for subsequent simulation and prediction.

3.2 Simulation and prediction methods

From the perspective of analyzing the evolution law of hydrological elements, the time series decomposition method is selected to establish the medium and long-term forecast model of hydrological series. The annual water level time series is divided into periodic (harmonic) components, trend components and random components, namely

$$X_t = T_t + P_t + R_t \quad (t = 1, \dots, N)$$
 (5)

Where X_t is annual water level time series, T_t is trend term, P_t is period term, R_t is random term and N is sequence length.

The trend component is assembled by polynomial assembly, and the periodic component is assembled by harmonic analysis. Trends and period components are deterministic components. After removing the deterministic components, the remaining sequence is the random component, and the random component is simulated and predicted by the ARMA model. The ARMA model order is determined according to the autocorrelation graph, combined with the minimum information criterion AIC [7] and Bayes criterion BIC [8].

Here mainly analyzes the determination of the actual harmonic component by the variance contribution rate, and the variance calculation formula explained by each harmonic component is:

$$C_j^2 = \frac{A_j^2 + B_j^2}{2s^2} \tag{6}$$

Where C_j^2 is The variance contribution rate of the jth harmonic, A_j and B_j are Fourier coefficient, s^2 is total variance of the sequence $X_t^{'}$ (see Eq. (7)). The decomposition model here is:

$$X_{t} = T_{t} + X_{t}^{'}, \ X_{t}^{'} = P_{t} + X_{t}^{"}, \ X_{t}^{"} = R_{t}$$
 (7)

Therefore, the general form of the model is:

$$X_{t} = \left(a_{0} + a_{1}t + a_{2}t^{2} + \dots + a_{p}t^{p}\right) + \left(A_{0} + \sum_{j=1}^{m} \left[A_{j}\cos\left(\frac{2\pi j}{N}t\right) + B_{j}\sin\left(\frac{2\pi j}{N}t\right)\right]\right) + ARMA$$
 (8)

The prediction model is:

$$T_{N+z} = a_0 + a_1(N+z) + a_2(N+z)^2 + \dots + a_p(N+z)^p$$
 (9)

$$P_{N+z} = A_0 + \sum_{j=1}^{m} \left\{ A_j \cos \left[\frac{2\pi}{T_j} (N+z) \right] + B_j \sin \left[\frac{2\pi}{T_j} (N+z) \right] \right\}$$
 (10)

The process of establishing a model is a process of extracting each component from a known sequence. The extraction order is the trend component First, the periodic component seconds and the random component are final. Once the mathematical model of each component is established, the decomposition model in the form of Eq. (8) is obtained. Use the above time sequence decomposition model to simulate and predict the trend, cycle, and random components, and combine the result to obtain final prediction results.

3.3 Analysis of water level characteristics

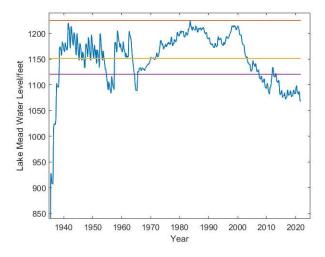


Figure 7. Water level of Lake Mead with time

As we seen from the Fig. 8, we know the maximum water level is 1225.4 feet, in 1983; and the average of water level is 1151 feet. We standardized the data and choose the 1120.7 feet as the water level of draught in Lake Mead because the most of time there is enough water provide. So we can four obvious draught period in history, 1954-1957, 1965-1966, 2008-2011 and since 2012. Previously,

this period is very short, there is no big drop, but the lake is experiencing the longest period that has been seen.

During 1935 - the mid-1960s, the water elevation in the Lake Mead chart above changed rapidly, and the water level became more consistent in a short period of time thereafter. When the water level in the lake dropped and was expected to fall below 1145 feet that the Water Authority announced drought monitoring. Once the water level drops below 1145 feet, the watch will switch to the drought alert-the state in May 2003, when the Landsat image of the latter was acquired. If the top of the lake is below 1145 feet, a drought emergency will take effect. Each of these water level alerts triggered a variety of water limitations and practices in the region, from limiting watering gardens, car wash, running fountains in municipal parks and public places, and improving water fees to encourage protection.

3.4 Water level prediction

Time series analysis is a powerful tool for data processing of ordered data, seeking rules, predicting the future, etc., especially for predicting random data with strong regularity. The results are satisfactory. This study uses the time series decomposition method to establish the annual characteristic water level prediction model of Lake Mead. The results show that its fitting accuracy and prediction accuracy are both high, the determination of the effective harmonic number and the order of the ARMA model is the key to modeling. Variance contribution rate analysis is used to determine the number of harmonics. In the determination of the order of the ARMA model, comprehensive consideration of AIC and BIC criteria and residual randomness test, etc., achieved a relatively ideal effect. Therefore, simulation the time decomposition model based on trend changes, periodic changes and random change analysis are ideal for simulation and predicting the characteristics of Lake

For model 1, we choose the low and the high elevation data since 2012 to see the trend of water level change.

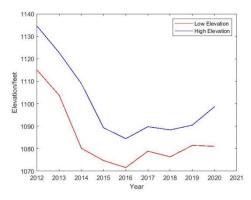


Figure 8. The elevation Vs. time in recent draught period

As we can seen from Fig. 8, the elevation decreases and then starts to increase, so we consider to use polynomial fitting. We use the MATLAB curve fitting toolbox to fit the curves of high elevation and low elevation, find when the fitting effect of the function f(x)=

 $a1*x^2+a2*x+a3+b1*\sin(w1*x)+b2*\sin(w2*x)+A0+c$ is good, and the fitting curve is also smooth.

The Coefficients for high elevation are as follows in Table 2.

Table 2. Coefficients for High Elevation

A0	al	a2	a3	b 1	b2	c	w1	w2
0.06	0.0002	0.95	0.41	2	7.3	0.29	0.43	1.1
092	021	01	73	4	92	24	69	02

And the corresponding goodness of fit is: R-square: 0.975

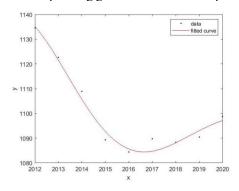


Figure 9. Curve fitting for high elevation

Meanwhile, the coefficients for fitting result of low elevation are as follows in Table 3:

Table 3 Coefficients for High Elevation

A0	a1	a2	a3	b1	b2	c	w1	w2
0.2	0.0002	5.4	0.77	8.6	4.9	0.09	0.51	0.65
48	418	15	07	31	65	627	65	27

And the corresponding goodness of fit is: R-square: 0.9752

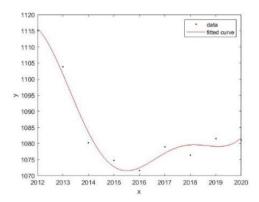


Figure 10. Curve fitting for high elevation

Then we can predict the elevation in 2025, 2030 and 2050 using the fitting equations, the results are as follows:

Table 4. Year and Predicted High or Low Elevation/Feet(1)

Year	Predicted high elevation/feet	Predicted low elevation/feet		
2025	1112.1	1049.3		
2030	1074.9	1026.5		
2050	1108.2	940.7		

For model 2, we choose the data from 2005 to 2020. And draw the curves to see the characters of Lake Mead elevation trend.

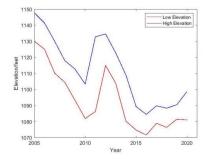


Figure 11. The elevation Vs. time from 2005 to 2020.

As we seen from Fig. 10, the total elevation trend is decreasing but fluctuation, so the linear fitting and polynomial fitting do not work. We consider combining the trigonometric function as we mentioned above in part 6.2.

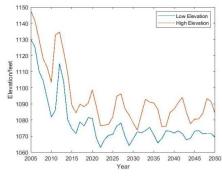


Figure 12. The ARIMA model prediction for high elevation and low elevation

Table 5. Year and Predicted High or Low Elevation/Feet(2)

Year	Predicted high elevation/feet	Predicted low elevation/feet
2025	1081.9	1071.1
2030	1078.1	1068.4
2050	1084.1	1069.3

Compared with these two models, the difference between high elevation and low elevation for model 1 is higher than model 2. As we can see from Fig. 11, the fluctuation is getting smaller because of climate change. So, we need to take measure to save water and recycle the wastewater.

4. Wastewater recycling program

4.1 Factors in the wastewater recycling program

We think there are some considerations when taking measures for the wastewater recycling:

(1) Changes in the water level of Lake Mead

As we all know, with climate change, the water level will reduce the total trend. However, due to fluctuations, the level sometimes increases, which may result in a decrease in wastewater recirculation. We can take measures in three stages (see more details in conclusion).

(2) Demand of water

The demand for water is mainly from agriculture and infrastructure. At the same time, with the development of technology and society, people will use more water to force rapidly. We also need to combine water demand with the level of lake Mid to make up for a certain degree of shortage.

(3) Pollution

The environment is getting worse caused by the mankind activities. We need to pay more attention for the environment pollution especially the water pollution. The reason is we cannot easily recycle the polluted water.

(4) Cost

The waste recycling is high tech that not each government, family, and individual can use the machine because of the expensive cost. With the development, it will be easily come true although it is not for the normal person to use yet.

(5) Wastewater recycling awareness

People need a consciousness that saving wastewater when using water. When people are in the face and hand, people throw dirty water is very common. If people have a strong understanding, the efficiency of the plan will increase faster.

4.2 the decisions that local leaders need to make

- (1) The government need to provide some supports to the companies that works for the wastewater recycling. The companies have the high-tech, but they need money to speed up the research and development of the wastewater recycling.
- (2) The local education administration should be strong aware of the children wastewater recycling education. It is beneficial for us in the long term. The children will form the good habit and may cause a good society.
- (3) The Ministry of Finance can invest more money on this program, as this program is good for the local people and people would like to see the wonderful infrastructure.
- (4) The Ministry of Environmental Protection can edge people to notice the collection of wastewaters, it is a good source from the family, industries and agriculture.

4.3 Wastewater recycling program conduction

- (1) Stage 1 (Present-2025): Unsustainable production and consumption methods are generating large amounts and various types of waste that can exist in the environment for a long time at an unprecedented rate. If this trend continues, by the end of this century, the amount of waste will increase substantially, and by 2025, it will increase four to five times. Preventive waste management methods that focus on changing lifestyles and production and consumption patterns are the best way to reverse current trends.
- (2) Stage 2 (2025-2030): Collect data from various sources: surveys of wastewater treatment utility companies, national and regional databases, and reviews of local regulations and international literature. Treatment configuration, composition of wastewater and byproducts, potential demand, identification of stakeholders,

and local legislation were discussed in depth on resource recovery decisions.

(3) Stage 3 (2030-2050): By 2050, the government plans to create a "network zero society." In order to achieve this ambitious goal, the deployment of low-carbon technologies is a top priority.

5. Evaluation of Model

5.1 Strength

It is easy to operate, and the data of the original time series can be fully utilized, with a fast calculation speed, and the model parameters can be dynamically determined, and good accuracy can be dynamically determined.

It is best to use combined time series or combine time series with other models

5.2 Weakness

Can't reflect the inner connection of things Can't analyze the correlation between two factors

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