Characterization of view in relation to solar-control systems

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Abstract

In this study, we developed a method to identify the affecting parameters that can characterize the view-out in relation to different solar-control systems. We hypothesize that the photometric composition, e.g., contrast, in the visual environment as a result of using solarcontrol systems impacts the subjective assessment of the view out. To test our hypothesis, we conducted user assessment studies where we measured objective photometric measurements and subjective human responses in a real semi-controlled environment. The user-assessment study was done with 51 participants. The participants were randomly allocated to a combination of five views out and six solar shading systems in a work environment where they answered questions related to the indoor environment and view quality. The relation between view and solar-control systems and their impact on the subjective assessment of view out was tested through the development of linear mixed-effects models. The models were developed using forward and backward selection and likelihood ratio test (LRT) to test the effects of adding or removing variables.

Photometric quantities were measured as achromatic contrast and calculated both locally and globally, which are two different algorithms for the calculation of contrast as perceived by humans. Both were found to be significantly associated with respectively view rating and satisfaction and could be used to characterize the viewout quality through commonly used shading devices.

Introduction

Higher-level energy considerations of building envelopes and windows in the last decades have led to increased use of solar-control systems to reduce excess solar gains and to ensure thermal and visual comfort in buildings. However, studies have shown that people tend to compromise not only on visual comfort [1] or acceptance of higher levels of light intensities [2] but also on thermal comfort [3] and [1] for better access to view. Although energy efficiency and, to some extent, the thermal comfort performance of these devices are directly measurable through quantitative parameters, there are fewer known visual parameters to rate their visual quality and performance. The lack of tangible visual quality measures

for such devices is sensitized by the new recommendations for the view out in DS/EN 17037:2018 Davlight in buildings [2] under the CEN/TC 169 "Light and Lighting" scope. The mentioned standard poses requirements for the content and view access through the windows to ensure the quality of the visual environment. The quality of the view out depends on window properties, visual features in the view, and the position of the window and viewer (distance and orientation), among others. Therefore, understanding the effect of shading devices on view in terms of view quality and photometric variations can help the usage of such devices and a basis for better comparing them in relation to view. Moreover, the identified parameters that affect view quality in relation to shading devices can be incorporated in the early design phase for decision-making purposes using simulation tools.

So far, different methods have been made to quantify the indoor view quality. *DS/EN 17037:2018 Daylight in buildings* [4] offers simple geometrical stratification methods of the view out with three different levels of recommendations: minimum, medium, and high. If the minimum level is followed, t is ensured that "*All occupants of a space should have the opportunity for the refreshment and relaxation afforded by a change of scene and focus*" [5]. Mardaljevic [6] advanced the geometrical stratification method, which quantifies the illumination received at the building aperture from a visible external entity.

In an attempt for a more comprehensive framework to address all aspects of view outside the window, Ko et al. [7] sorted the parameters affecting view quality into three guiding categories: view content, view access, and view clarity. The view content revolves around the visual features seen through the window view. View access is a measure of how much view an occupant has access to from a specific position and is particularly dependent on the geometric relationship between the view opening and the occupant. View clarity is a measure of how clear (i.e., without distortion) a view out is perceived by an occupant; thus, it depends on the properties and design of the window and any obstructions of view.

View in relation to shading should consider several aspects of view quality. Considering the importance of view content on the perception of the view [8], the view out quality through shading devices could be defined dependent on the view content, i.e., greenery, surrounding buildings, traffic, sky, and ground [9]. While different views can include different features, the composition of the feature in the view defines the level of complexity. In the experiment study by Oliva et al. in 2004, the perceptual dimensions of visual complexity were found to be the "quantity of objects, clutter, openness, symmetry, organization, variety, and colors" [10]. Zaikina et al. investigated whether luminance-based quantification can be used to evaluate the visibility of the shape and details of real 3D objects observed by people [11]. Here the visual complexity was represented by the sharpness and viability of the scene since an increase in the sharpness as a measure of more visible details. In the study, three luminance measures of an object were tested - the luminance ratio, mean luminance, and the standard deviation. The study showed that the standard deviation of the luminance values calculated using Root Mean Square (RMS) correlates best with the perception of viability. When using the RMS, the luminance of each pixel in the image is compared to the mean luminance of all pixels as an indication of the variability between different pixels.

Clarity of view through shading can be determined by looking at visual acuity, color perception, and contrast sensitivity [7]. Visual acuity measures vision's ability to discern shapes and objects at a certain distance. At the center, 5°, the fovea is placed as the visual axis of the eye. This is the area of the eye with the greatest acuity as it has high spectral sensitivity, and the light reaching the fovea has encountered a minimum of absorption and scattering. This allows for a high resolution of detail and fine discrimination. Form recognition happens within the center 2° of the visual field, also called the foveal vision [12]. Hence, to ensure visual clarity, the available view openings must have an uninterrupted area as a minimum corresponding to the foveal vision.

Visual contrast sensitivity is the amount of contrast the vision needs to discern shapes and objects. The human visual system is constantly adapting to the level of light. The sensitivity to glary sources follows the adaptation, meaning that a glary source is only perceived glary if the surroundings are markedly lower in light level than the specific source. It is, therefore a matter of the contrast in light levels when talking about the perception of the visual environment. The first measure for contrast was The Michelson contrast developed in the second half of the 20th century. This contrast is a global contrast where only the deviation between the maximum and minimum luminance of the entire field of view was considered [13]. Other global contrasts have followed - a more complex one of them is the usage of the RMS contrast by Pavel et al. For this measure, the intensity of all of the fields of view, e.g., pixels of an image representing a view, are considered in relation to the average intensity of the field of view (FOV) [14]. Since global contrasts consider the

whole FOV and not the contrasts between a source and its surroundings, other local metrics have been developed for this purpose. In 2004 Rizzi et al.[15] developed the RAMMG algorithm, a local contrast measure used on images. In this algorithm, the mean contrast for different pixel levels by the local contrast of each pixel and its eight surrounding pixels is found. Local contrast algorithms hence seem closer to human perception of the visual environment as they can easily identify differences in luminous intensity of pixels beside each other than far away from each other. This method has been shown to correlate best to visual perception of different visual compositions in a daylit environment [16].

While several methods have been created to address view quality [7], [9] or with focus on only specific shadings such as fabric screens [17] currently, no metric has been developed to assess the view of a window in full.

Dependent on the view content, the clarity and complexity of the view can vary dramatically depending on the type of solar control being used. Different hypothesis can be set on how the solar control systems, such as shadings or glazing or a combination of them, can change the perception of the view. We hypothesize that the photometric composition of the combined shading and view, measured by using complexity or contrast measures, affects the perception of the view and hence can be used as a measure to characterize the view in relation to the shading devices. We have tested this hypothesis in an experimental study where combined objective and subjective measurements were done to identify the parameters most affecting view out in relation to shading devices.

Methodology

The experimental setup was tested initially in a pilot study. In this pilot study, we explored photometric relations and the amount of view [18]. The scenes were composed of varying contrasts and visual complexity, respectively, calculated as RAMMG and RMS. The scenes were presented to the participants using a VR headset. Two view outs were selected to be investigated. The two views were on each end of the view quality rating (VQR) based on the D&V Analysis Method [9]. The two view types were both presented with four different external solar shading types – big horizontal Venetian blinds, small horizontal Venetian blinds, vertical louvers, screens, and shutters with greenery Figure 1. The blinds were white.

To test the amount of view, the big horizontal blinds and the vertical louvers were presented with different angles for the view out with a high VQR. This setup showed that the variables chromatic contrast, visual complexity, and perception of the amount of view significantly affect the subjective responses [18].

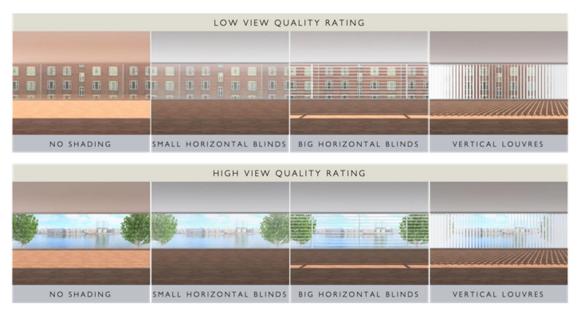


Figure 1: Overview of view-outs and solar shading types used in VR Solar Shading Experiment [7].

It was found that the subjects preferred high visual complexity, a high amount of view available no matter VQR and low chromatic contrast. The amount of interruption of the foveal vision could affect the preferences within the solar shading types. The results also indicated that the subjects preferred the big horizontal blinds followed by the small horizontal blinds and thereafter the vertical louvers.

Experimental Design

Based on the findings from the pilot study, we set up the experimental design to investigate how the independent variables view out, and solar shading affects the dependent subjective responses. The subjective responses were measured using continuous scales through multiple questions using an on-screen questionnaire. During the experiment, the subjects were randomly allocated to the view conditions, and the corresponding subjective responses were measured. In each trial, five participants were seated in each room at different distances and directions toward the window. Each participant saw all the conditions from one view position in the room. Due to the different positions, each of the five subjects had different views through the window. Hence, ten different views were tested. Figure 2Error! Reference source not found. shows the layout of the room and the position of each subject that is marked with a cross. Figure 3 shows the shading types from the participants' five different seating positions and view directions.

The indoor conditions temperature, CO₂, humidity, and light level were measured, and observations of parameters such as weather, noise, and view changes were done to account for a potential influence.

Questionnaire

The questionnaire was split up into two parts – The first part contained demographic information, and the second part contained the evaluation of the view conditions. The questionnaire was developed using the "Guide to good questionnaires" [19]. All questions had a simple structure and were formulated clearly. To accommodate the onscreen work environment, the questionnaire was answered online [20] on the subjects' own laptops. A continuous scale from 0-100 was used throughout for acceptability and satisfaction assessments of view and the shading devices. The questionnaires were prepared in English and Danish.

Physical Conditions & Shadings

Based on the VQRs [5], several buildings were rated, and two identical meeting/office rooms were chosen on the campus of the Technical University of Denmark with west orientation and VQR of high and medium. Four external solar shading types were selected for the experiment based on field research combined with an analysis of the market. The dimensions of the slats were chosen based on existing products from Blendex [21]. Mockups of the external shadings were built and placed outside the windows with the possibility of switching between the different shading types in a randomized sequence. The paint was semi-glossed to make the slats reflect some of the light without causing glare. Figure 4 illustrates the frames created with the different mockups, and in Table 1, the geometry and specifications for the slats can be found.



Figure 2: Setup in room. The "X" marks the placement of the subjects laptop 30 cm from the edge of the table.

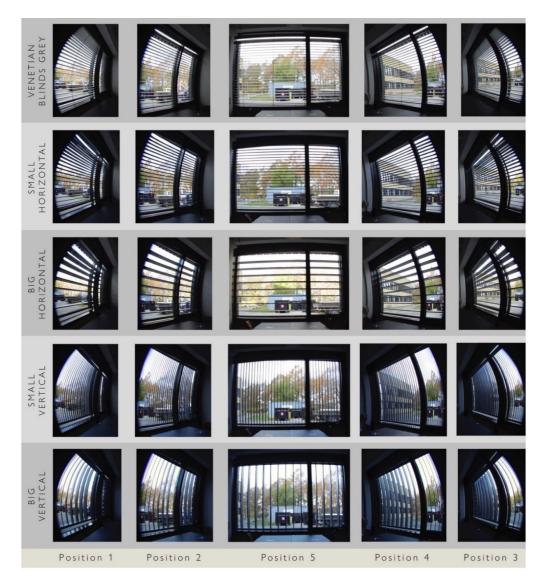


Figure 3: An overview of the different solar shading conditions seen from each subject position in one of the rooms.

Table 1: Solar shading slats specifications in mockups.

	SMALL SLATS	BIG SLATS
Thickness	8 mm	18 mm
Width	8 cm	20 cm
Distance	7 cm	18 cm
Quantity: Horizontal Vertical	22 38	8 14
RAL color	7022	7022



Figure 4: Left: Visualizes the frame with the four solar shadings. Right: the measurements of the frame.

Measurements

The equipment was prepared and tested or calibrated prior to the experiments. Temperature, CO₂, and humidity were measured indoors using HOBO and VAISALA. The photometric measurements were done with photometers with fisheye lenses and HDR imaging. The images were then processed with *Evalglare v. 2.09* [22] to calculate selected photometric parameters. During the experiments, to ensure the images were taken at the exact same spot each time, the photometers were mounted on tripods with leg positions marked on the floor.

Experiment procedure

The experiment was performed over three days with two rounds each day. The exact procedure to follow was described as a checklist to ensure consistency. When the subjects arrived, they were asked to take out their computers and open the questionnaire through a received link. Before starting the experiment, the subjects were given an introduction with the necessary instructions. During the experiment, the artificial lights were off. The experiments started with the subjects sitting at their assigned table in one of the rooms. When the questions were answered, the subjects went out to the hallway with their computers. Then images were taken with a photometer at each position to document the light conditions during the test. Afterward the subjects were directed into the second room.

Simultaneously with the subjects being in one room, the shading conditions of the other room were changed to the next test. After the first 6 tests, 3 in each room, a break of 10 minutes was scheduled for the subjects, and during that time the two frames were switched between the windows

in the two rooms. After the swop of the frames the last 6 tests were done following the same procedure.

Participants

The experiments were performed with 51 participants mainly between the age group of 20-29 years. The gender of the subjects was 45% female, 53% male, and 2% other. 64% of the subjects responded that they were either using glasses or contact lenses, with the majority being near-sighted. Only one subject was color-blind (red/green). Approximately 30% of the participants found themselves sensitive to Light.

Analysis Methods

All data analysis was executed in the statistical computing software R [23][24], and the package lme4 [20] for linear mixed-effects analysis was used.

The general linear mixed-effect model for investigating the relationship between the subjective responses (SR) and selected fixed effects has the following expression:

SR ~ SST _F + Photometric variable _F + Subject _R + ε (1)

Index F denotes the model's explanatory variables selected as fixed effects, while index R signifies the random effect. SST represents Solar Shading Types, i.e., Existing Grey Venetian blinds, Big Horizontal slats, Big Vertical slats, Small Horizontal slats, and Small Vertical slats.

From this general expression, the best-fitting models are found for the responses to the questions Q.A: Assessment of view out and Q.B: Satisfaction with view out. In Q.A. the subjects rated the view on a scale from terrible to excellent. In Q.B. the subjects rated the view from very unsatisfying to very satisfying.

The models were developed using forward and backward selection. Likelihood ratio test (LRT) was used to compare models to determine if a variable should be added to the model or not. In the software R, the LRT were done using ANOVA (Analysis of Variance function based on mean square error MBE calculations) as determined by the procedure.

Results

In this paper, the focus is on the results related to the effects of different solar shading types on the perception of view out. From the general linear mixed effect model, the best fitting models are found for both questions Q.A and Q.B. The models found can be seen below:

$Q.A \sim SST_F + LC_F + VOA_F + Subject_R$ (2	.)		
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$$Q.B \sim SST_F + LC_F + VOA_F + Subject_R$$
(3)

Where SST is the 5 different solar shading types used in the experiments, LC is the calculated achromatic local contrast, and VOA is the view out assessment, meaning how the subject assessed the view without solar shading. Table 2 and 3 show the intercept and coefficients for the linear mixed effect models (2) and (3).

From the first model (2) analysis, negative coefficients are found for all solar shading types except Big Horizontal, which is incorporated in the intercept. This indicates that the Big Horizontal is the most preferred type of solar shading. Hereafter comes Big Vertical, although Small Horizontal and Existing Grey Venetian blind all have coefficients close to Big Vertical. The noticeable difference between Small Vertical and the others shows that this type by far, is the least preferred solar shading. If color is taken as the main difference between the types Small Horizontal (dark grey) and Existing Grey (light grey/metallic), the coefficients indicate that the darker color is most preferred.

The Local achromatic contrast (LC) is negatively correlated to the subjective responses of *Q.A: Assessment of view out,* presumably due to how the LC is calculated, where LC is found for the total field of view. Meaning that, for the subjects placed at a greater distance to the window, the part of the field of view which is window becomes remarkably lower than for those placed close to the window. This will inevitably affect the LC, as the rooms were all quite dark during the experiments. Hence the found correlation for LC is for the subjects' field of view, while the desired LC would be for the view out, which would naturally increase when adding solar shading to the view out. This could be solved in the future by only calculating the LC for the window part of the field of view.

Assessment of view without solar shading (VOA) is seen to have a positive correlation to *Q.A: Assessment of view out of the view with solar shading*. Even though the coefficient is quite small in magnitude, view-out assessment potentially has a substantial influence, meaning a high rating of VOA without solar shading will positively affect the assessment of the same view out with solar shading. The significance of the determining variables for *Q.A*: Assessment of view out is determined by an LRT of a full model and a reduced model in accordance with a null hypothesis. The level of significance is set at 5%. Solar shading types has shown to be statistically significant for *Q.A*: Assessment of view out ($\chi^2 = 47.86$, P-value = 3.79 ·10⁻⁹), as has local achromatic contrast ($\chi^2 = 4.2$, P-value = 0.04). Therefore, solar shading types and local achromatic contrast can be said to have a significant effect on the perception on view out when perception is measured by the subjective assessment of view out.

Generally, the same tendencies regarding solar shading types are seen for Q.B: Satisfaction with view out as for Q.A: Assessment of view out, however, there is a small difference in magnitudes and a slight change in order. Big Vertical is no longer the second most preferred solar shading type, but the second least preferred type, instead, the Small Horizontal is seen to be the second most preferred type after Big Horizontal, which is included in the intercept. The order of the similar types Small Horizontal and Existing Grey Venetian blinds are the same as for Q.A, again indicating that the darker color is being most preferred.

The coefficients of the two seem to differ more for *Q.B:* Satisfaction with view out than they did in *Q.A:* Assessment of view out, indicating that a change from one type to another will have a higher influence for Q.B indicating satisfaction. Local achromatic contrast is again found to be negatively correlated to the subjective responses. However, it has less of an effect on *Q.B:* Satisfaction with view out than on *Q.A:* Assessment of view out. The LRT analysis with level of significance is determined to be 5% ($\chi^2 = 58.54$, P-value = 2.44 ·10⁻¹¹), shows that solar shading types are statistically significant for the linear mixed effect model, whereas local achromatic contrast ($\chi^2 = 0.06$, P-value = 0.81) is not.

(SST: Big Horizontal, Continuous variables = 0)		45.97	
FIXED EFFECT	TYPE	COEFFICIENT	VALID RANGE
SST	Categorical		
Big Vertical slats		- 7.06	
Small Horizontal slats		- 7.55	-
Existing Grey Venetian blinds		- 8.39	
Small Vertical slats		- 22.35	
Local achromatic Contrast	Continuous	- 10.83	[0.85;2.22] * [1.34;8.25] **
View Out Assessment	Continuous	0.52	[7;100]

Table 2: Estimates and intervals for the linear mixed effect model for Q.A.

 INTERCEPT

INTERCEPT (SST: Big Horizontal, Continuous variables = 0)	50.20			
FIXED EFFECT	TYPE	COEFFICIENT	VALID RANGE	
SST Big Vertical slats Small Horizontal slats Existing Grey Venetian blinds Small Vertical slats	Categorical	- 12.23 - 9.93 - 11.18 - 26.77	-	
Local achromatic Contrast	Continuous	- 1.38	[0.85;2.22] * [1.34;8.25] **	
View Out Assessment	Continuous	0.33	[8;100]	

Table 3: Estimates and intervals for the linear mixed effect model for Q.B.

* The interval of the data used in the models i.e., the natural logarithmic function of the measured photometric variable. ** The interval of the measured values of the photometric variable

Conclusion and discussion

Five different solar shading types and their effect on the perception of view-out were investigated in a userassessment experiment. We hypothesized that the photometric composition of the combined shading and view affect view perception and as a measure to characterize the view in relation to the shading devices. The photometric composition of combined view and shading was determined by view complexity and clarity using respectively RMS and RAMMG.

Based on the conducted analyses and the results, several findings can be concluded. The selected solar shading types had a statistically significant impact on the perception of view. This was true both when the perception is defined to be measured by assessment of view out (Q.A) and by satisfaction with view out (Q.B). Hence, the shading devices' selection can be representative to test the initial hypothesis. The most preferred solar shading type was Big Horizontal which has 18 mm thick horizontal slats, with a depth of 200 mm and a slat-to-slat distance of 180 mm. When comparing the other types of solar shading to Big Horizontal, all other types decreased both the assessment and the satisfaction with the view out. The least preferred solar shading type was by far was

Small Vertical has 8 mm thick vertical slats with a depth of 80 mm and a slat-to-slat distance of 80 mm. The intermediate types – Big Vertical, Small Horizontal, and the Existing Grey Venetian Blinds, were in the same preference level. The two types consisting of small horizontal slats were ranked in order of color – going from the darkest being most preferable to the lightest. The photometric parameter achromatic RAMMG contrast [12] was significantly associated to *Q.A: Assessment of view out.* The correlation coefficient found using the mixed effects analysis was negative, meaning that an increase in RAMMG contrast caused a more negative assessment of the view out. The color of the solar shading had a noteworthy impact on the perception of the view-sout.

Based on the knowledge about local contrast this is sensible since a darker color will increase the local contrast for the view out, which in previous studies has been found to give a more positive perception.

It was assumed that the window size, geometry, and division would not impact the results conducted; hence it is necessary to investigate this further to confirm or dismiss this assumption. In relation to this, the impact of interruption of the foveal vision would be an essential aspect to investigate if that is the reason for the preference for solar shading types.

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