Mathematical model of water drop trajectory in artificial rainfall

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Abstract. In the article, a mathematical model of the movement of a water drop flying out of a nozzle with a deflector nozzle during artificial rain was built. The resulting equations were solved by an approximate method using a specially developed program at EHM. The parameters affecting the irrigation process were analyzed based on the obtained results.

1 Introduction

Water consumption is increasing worldwide by an average of 10% per year. This is causing global problems. It should be noted that the main bulk of water consumption falls on agriculture. This problem is more urgent in the region of Uzbekistan, with an arid climate. In the following years, several decrees and decisions of the President of the Republic of Uzbekistan aimed at solving the problem were adopted. In agriculture, measures to consistently apply efficient irrigation technologies to the sector are distinguished by the fact that they are aimed at solving the problem of water scarcity. Sprinkler irrigation has its place among economic irrigation technologies [1-16]. In some foreign countries (RF, the Republic of Belarus, Ukraine, etc.), the rain irrigation method accounts for up to 60% of agricultural crop reclamation. The technology of artificial rain irrigation is little studied in the Central Asian region. Today, in Uzbekistan, drip irrigation has been introduced on 198.9 hectares, discrete irrigation systems on 5.9 hectares, 78.8 hectares are irrigated through portable, flexible pipes, and 20.9 hectares are irrigated using film. In comparison, artificial sprinkler irrigation method was introduced on 11.2 thousand hectares[17-35]. This is about 2.1% of the area where efficient irrigation technologies have been introduced.

According to the decree of the President of the Republic of Uzbekistan "On the development strategy of the new Uzbekistan for 2022-2026", it is aimed to save at least 7 billion cubic meters of water annually and increase the area of livestock feed to 722 thousand hectares due to the effective use of water resources. In implementing these tasks, the technology of artificial rain irrigation is one the important directions[1-32].

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2 Materials and Methods

It is necessary to develop irrigation methods suitable for the region of Uzbekistan and optimize the parameters of artificial irrigation devices, considering climatic conditions.

Almost all researchers have noted that the movement of the water flow coming out of the deflector nozzle occurs in 3 sections in the process of artificial rain (Fig. 1).
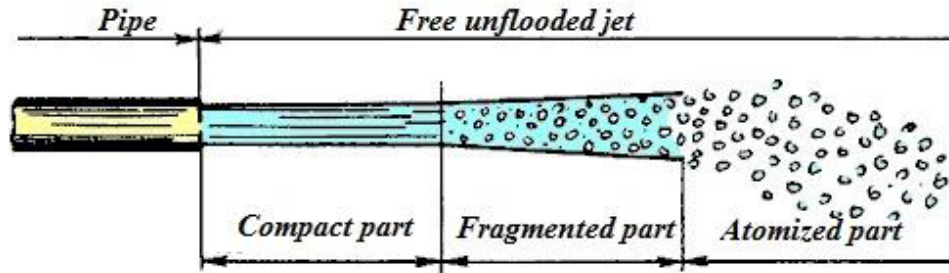

Fig. 1. The behavior of the water jet ejected from the deflector nozzle in different phases.

G.M.Gadjiev, M.K.Mustafaeva, I.D.Fedorenko found that the evaporation during the flight of water drops reaches 20-30% during irrigation with artificial rain. In the studies of V.Ya.Chichasov and V.N.Chernomortseva, this indicator was 10-15%. In the research of A.A.Rachinsky and V.K.Sevryugin in the conditions of Central Asia, it is stated that the evaporation of a drop of water is in the range of 1-2% [1]. The change in the evaporation of a water drop over such a large range is explained by the fact that the researchers used different methods.

The flying of a drop of water under the influence of the wind was studied in the studies of V. E. Khabarov, A. I. Shtangey, and it was determined that the higher the height of the drop of water from the surface of the earth, the greater the water wastage [2]. According to SANIIRI (Uzbekistan), when the rate of irrigation with the Fregat sprinkler is 614 m^3/ha, water loss due to evaporation reaches 30%. According to M.S. Mansurov, in the climatic conditions of Azerbaijan, when the wind speed is 3 m/s, the water droplet is blown away by 7-10%, and evaporation occurs when the ambient temperature is 25-30°C, and the wind speed increases from 1.1 to 3.1 m/s. It reaches from 13.7 to 20.7%.

All studies uniformly emphasize that as the flight time and flight height of the water drop increases, the wind blowing and evaporation of the water drop increases.

In B.M. Lebedev's studies, it was determined that the thickness of the sprayed water curtain is equal to 0.10-1.28 mm when the diameter of the deflector nozzle is 50 mm and the diameter of the nozzle is 4-16 mm [3]. The expansion of the water flow was 3-4 degrees. In research, the length of the 1st plot is 3-4 cm, and the length of the 2nd plot can be found by the following formula proposed by A.P. Isaev [4]:

$$L_2 = 278.7 - 2.1 * 10^{-4} * R_e * D, \quad m. \qquad (1)$$

here D is the diameter of the water droplet, R_e is the Reynolds number.

So, the flight distance of a water drop from a nozzle with a deflector nozzle:

$$L = L_1 + L_2 + L_3 , m. \qquad (2)$$

Several researchers have investigated the flight distance of a water droplet in Section 3. In Nadezhkina's thesis, the flight of a water droplet was calculated by taking into account

the wind speed, and a mathematical model of a water droplet was developed [5]. But when creating the process model, the internal friction force of the water droplet and the upward Archimedean force were not considered. Also, in the scientific article of A.G. Vinogradov and O.M. Yakhno, this process is thoroughly investigated [6]. In the scientific work, the drop of water ejected from the nozzle through the deflector nozzle was taken as a gas, not a liquid. The mathematical model is considered from the point of view of the law of conservation of energy and the law of fluid integrity. In the process, not a drop of water but the movement of a gas mixed with water in an air environment was investigated. Therefore, the obtained mathematical model does not fully represent the movement in the process of artificial rain.

To build a mathematical model of the flight of a water droplet, we take the point of departure from the nozzle of the deflector nozzle of the water droplet as the head of the coordinate system so that the position of the water droplet in the process of artificial raining can be determined in the coordinate system (Fig. 1), and we build a coordinate system on the plane. A drop of water creates a material spherical ball movement in the air (gaseous environment). Let the movement of a water droplet occur in a gas medium with a certain density.

In that case, we use Newton's second law to describe the motion of a drop of water:

$$m\vec{a}(t) = \vec{F}(t). \qquad (3)$$

here m is the mass of the water droplet; $\vec{a}(t)$ is water droplet acceleration vector; $\vec{F}(t)$ is the component of all forces acting on the water droplet.

The following forces act on a water droplet in flight:
- Downward $\overrightarrow{F_T}(t)$ force of gravity;
- Upward $\overrightarrow{F_A}(t)$ anticipatory Archimedean force;

The resistance force of the medium to the body moving in the opposite direction to the velocity vector, consisting of the viscous friction force between the body and the medium $\overrightarrow{F_B}(t)$ and the opposite resistance forces of the medium $\overrightarrow{F_L}(t)$.
Everyone knows the force of gravity determined by the formula.

$$\overrightarrow{F_T}(t) = m\vec{g}$$

Here, m is the force of gravity of the body, g is downward, the vector of the acceleration of free fall, and has a standard absolute value of 9.81 m/s^2.

Archimedean force defined by the expression

$$\overrightarrow{F_A}(t) = -\rho_o V_T \vec{g}$$

Here is the density of the medium, V_T is the volume of the water droplet, and g is the free fall acceleration vector.

The viscous friction force was found experimentally and is called the Stokes formula. For a water droplet moving in the shape of a spherical sphere, it looks like this:

$$\overrightarrow{F_L}(t) = -\frac{1}{2} C_T \rho_c S_T \left|\vec{\vartheta}(t)\right|\vec{\vartheta}(t)$$

Here $\vec{\vartheta}(t)$ is the velocity vector of the water droplet.

Opposite resistance force is found by the formula.

$$\overrightarrow{F_V}(t) = -6\pi\mu_T\, r\, \vec{\vartheta}(t)$$

Here π is a dimensionless coefficient that depends on the shape of the body, and its value is equal to 0.4 for spherical bodies, μ is the density of the medium, r is the cross-sectional surface of the body perpendicular to the direction of movement, and $\vec{\vartheta}(t)$ is the absolute value of the body's velocity vector.

As a result, for a water droplet in the form of a spherical sphere moving in some medium, it looks like Newton's second law in the formula (3):

$$m\vec{a}(t) = m\vec{g} - \rho_c V_T\, \vec{g} - 6\pi\mu_c\, r\, \vec{\vartheta}(t) - \frac{1}{2} C_T \rho_c S_T\, |\vec{\vartheta}(t)|\vec{\vartheta}(t)$$

Calculating the position of a body in motion allows you to express it through its acceleration.

If certain formulas

$$m = \rho_T V_T; \quad V_T = \frac{4}{3}\pi r^3; \quad S_T = \pi r^2$$

taking into account, we get the following equation from Newton's second law

$$\vec{a}(t) = \left(1 - \frac{\rho_c}{\rho_T}\right)\vec{g} - \frac{4.5\mu_C + 0{,}15\rho_c r|\vec{\vartheta}(t)|}{\rho_T r^2}\vec{\vartheta}(t)$$

We write the resulting equation in the form of the projection of vectors on the coordinate axis:

$$a_x(t) = -\frac{4.5\mu_C + 0{,}15\rho_c r|\vec{\vartheta}(t)|}{\rho_T r^2}\vec{\vartheta}(t) \qquad (4)$$

$$a_y(t) = \left(\frac{\rho_c}{\rho_T} - 1\right)\cdot 9.81 - \frac{4.5\mu_C + 0{,}15\rho_c r|\vec{\vartheta}(t)|}{\rho_T r^2}\vartheta_y(t) \qquad (5)$$

The absolute value of the vector of velocities is equal to.

$$|\vec{\vartheta}(t)| = \sqrt{\vartheta_x^2(t) + \vartheta_y^2(t)}$$

To represent the projections of the acceleration $\vec{a}(t)$ of a body in motion

$$K(t) = \frac{4.5\mu_C + 0.15\rho_c r\sqrt{\vartheta_x^2(t) + \vartheta_y^2(t)}}{\rho_T r^2}$$

by introducing a variable coefficient, the expressions (4) and (5) can be simplified in the following way:

$$a_x(t) = -K(t)\vartheta_x(t)$$
$$a_y(t) = \left(\frac{\rho_c}{\rho_T} - 1\right)\cdot 9{,}81 - K(t)\vartheta_y(t)$$

We use the time discretization method to calculate the resulting equation.

For this, we consider the state of the body in motion in separate units of time. Let the initial time be $m_0 = 0$ and let the subsequent states lag each other by the same value, called the time step $m_1, m_2, m_3, ... m_n$

We assume that $\Delta t = 0.1$ s. We also assume that the velocity and acceleration of the body change only between selected times. This is acceptable for small values of the time step. If the values of the velocity vector and acceleration projections in the time unit ti are denoted by $\vartheta_x(t_i), \vartheta_y(t_i), a_x(t_i), a_y(t_i)$ respectively, then the projections of the velocity vector at the same time t_{i+1} is calculated by the formula.

$$\vartheta_x(t_{i+1}) = \vartheta_x(t_i) + a_x(t_i)\Delta t \qquad (6)$$
$$\vartheta_y(t_{i+1}) = \vartheta_y(t_i) + a_y(t_i)\Delta t \qquad (7)$$

If we put the projection of the acceleration vector obtained above in (6), (7):

$$\vartheta_x(t_{i+1}) = (1 - K(t_i)\Delta t)\vartheta_x(t_i)$$
$$\vartheta_y(t_{i+1}) = \left(\frac{\rho_c}{\rho_T} - 1\right) \cdot 9.81\Delta t + (1 - K(t_i)\Delta t)\vartheta_y(t_i)$$

where $K(t_i)$ is the resistance coefficient of the environment obtained above.

The coordinates of a drop of water at a given time are calculated by the following formula:

$$x(t_{i+1}) = x(t_i) + \vartheta_x(t_i)\Delta t \qquad (8)$$
$$y(t_{i+1}) = y(t_i) + \vartheta_y(t_i)\Delta t \qquad (9)$$

Let's consider the calculation of formulas (8) and (9).

Let the initial position of the water droplet be given by the equation $x(0)=(L_1+L_2)\cos\alpha$, $y(0)=L_1 +L_2) \sin\alpha$
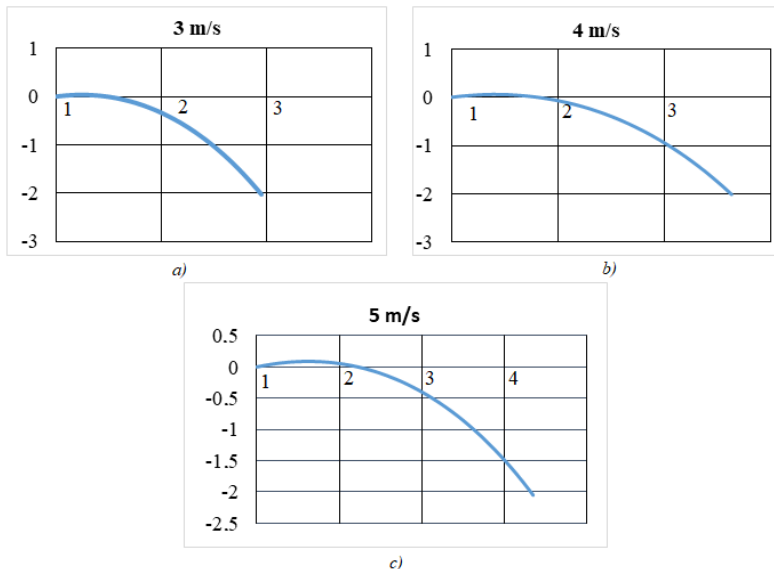

Fig. 2. The graph of the trajectory of a water drop is created based on mathematical modeling: *a)* The initial speed of the water droplet when it leaves the nozzle with a deflector nozzle is 3 m/s, the diameter of the water droplet is 2 mm, and the angle of departure is 15 degrees; *b)* Also, the initial speed is 4 m/s; *c)* Also, the initial velocity of the water droplet is 5 m/s.

The initial velocity $\vec{\vartheta}(0)$ consists of the components $\vartheta_x(0)$, $\vartheta_y(0)$ oriented by the angle alpha according to the condition $|\vec{\vartheta}(0)|$ given the absolute value:

$$\vartheta_x(0) = |\vec{\vartheta}(0)| \cos(u\pi/180)$$
$$\vartheta_y(0) = |\vec{\vartheta}(0)| \sin(u\pi/180)$$

The initial data also serve as the values that determine the parameters of the air and environment. $\rho_c = 1.205$ kg/m^3 at air temperature 20°C; $\mu_c = 18.1 \cdot 10^{-6}$ Pa·s.

Figure 2 shows the graph of the results obtained on the computer based on the initial data.

Figure 3 shows the graph of the time it takes for a drop of water to fall from a height of 2 meters (corresponds to the position where the deflector is installed at the height of 2 meters) and the time it travels along the X-axis.

Graph of dependence on the distance traveled on the X-axis: the diameter of the water droplet is 2 mm, the flight angle is 15 degrees, and the installation height of the deflector nozzle is 2 m.
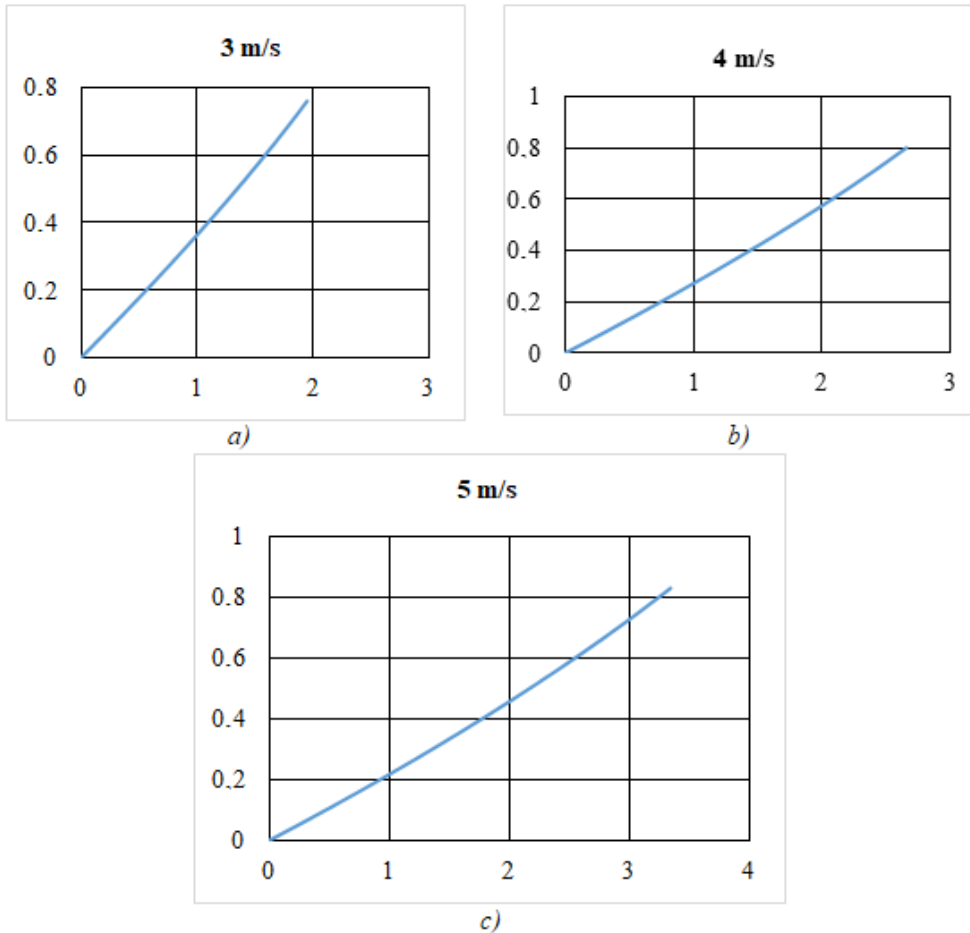

Fig. 3. The time it takes for a drop of water to fly

3 Results and Discussion

The analysis of the obtained data shows that the greater the initial velocity of the water drop, the greater the distance traveled along the X-axis. When the initial speed of a water drop is 3 m/s, its flight distance L_3 is 1.95 meters; when the initial speed of the water drop is 5 m/s, L_3 is 3.35 meters. Also, the flight time is 0.76 and 0.83 seconds, respectively. The water drop rises to a height of 0.034 m above the nozzle height in 0.08 seconds of the flight, with an initial velocity of 3 m/s. At this time, the water drop travels a distance of 0.3 m along the X-axis. Also, these figures expect a height of 0.088 m in 0.14 seconds of flight with an initial speed of 5 m/s and a distance of 0.66 m along the X-axis. The speed of a drop of water moving from a height of N=2 m with an initial speed of $V_o(t)$=3 m/s at the time of meeting the surface of the mass is equal to $V_x(t)$= 2.15 m/s and $V_y(t)$= 5.87. These figures are $V_x(t)$= 3.94 m/s and $V_y(t)$= 3.03 m/s at $V_o(t)$=5 m/s.

4 Conclusions

In sprinkler irrigation technology, the data obtained based on the mathematical model of a drop of water ejected from a deflector nozzle serves to justify the selection of optimal modes of irrigation and the parameters of sprinkler devices, taking into account climatic conditions.

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