

# Pricing and inventory Decision Optimization under subscription mode in Supply chain management

Kefan Hu\*

School of Economics and Management, Beijing Jiaotong University, China

**Abstract.** This paper investigates the current situation of subscription mode, applies it to the joint management of pricing and inventory in supply chain management, builds the product pricing and inventory model under subscription mode, and verifies the model derivation and results based on the joint decision model of pricing and inventory, and simulates the decision-making process of the model by computer programming. By comparing the difference between the theoretical model of joint decision of pricing and inventory under uncertain supply and demand conditions and the model of joint decision of pricing and inventory under subscription mode, analyzing the influencing factors and exploring the optimization method of product pricing and inventory model algorithm under subscription mode. The results show that: (1) in supply chain management, it is necessary to solve the problem of supply and demand balance through effective joint decision of pricing and inventory, and effectively improve the competitiveness of logistics enterprises.(2) The introduction of subscription mode in the joint management of pricing and inventory can reduce the uncertainty of both sides, so as to better solve the problem of inventory replenishment strategy and pricing decision under the condition of supply and demand uncertainty than the traditional decision-making model, so as to achieve the coordination of supply chain under the uncertain environment.

## 1 Introduction

### 1.1 Research Background

In today's world, the rapid development of economic globalization has brought new challenges to enterprises: (1) the increasing uncertainty in all aspects of the market; (2) the advance of the purchase of products is gradually lengthened;(3) There are more and more factors affecting the cost and pricing strategy of different kinds of products; (4) More and more factors affect inventory control decisions; (5) the overall supply chain management of enterprises is becoming more and more difficult. Enterprises are increasingly concerned about how to ensure the supply of products faster and better meet the demand for products, especially in the service and manufacturing industries. Many enterprises have adopted new supply chain management strategies, such as rapid response strategy, differential delay strategy, etc., in order to solve the balance need in the supply chain -- the problem of product supply and demand balance<sup>[1,2]</sup>. Through the use of pricing and inventory joint decision, in order to solve the balance between supply and demand, is a lot of measures of supply chain management, a very effective way and method, has been recognized by various industries.

### 1.2 Literature Review

Many domestic scholars have carried out researches on the application of subscription mode. Shi Ruisheng (2013) believes that it is very important to apply the publishing and subscription technology to the system architecture design, and has conducted a series of in-depth studies on this issue, and found a series of key technical problems: data intelligent distribution and routing technical problems in a large-scale distributed environment<sup>[3]</sup>; Liu Xujun, Ma Yue and Yu Dong (2010) constructed an abstract model based on the publish-subscribe model<sup>[4]</sup>. In the process of building the model, the program language of the model detection tool was used to convert the established abstract model into the corresponding PRISM program. Finally, the queue theory was applied and the detection tool was used to investigate the performance of the publish-subscribe communication model. And analyzed.

At present, there are also some researches on the joint decision of product pricing and inventory. Zhang Mingyang (2018) believes that the promotion of sales profit is the most concerned aspect of enterprises. Therefore, by exploring the joint decision between inventory and pricing, the probabilistic sales model and the general presale model, where consumers are strategic consumers, are studied respectively. Based on the background of e-commerce pre-sale of customer to

\* Corresponding author: 3136714761@qq.com

business [5], Gao Limin (2020), based on different pricing mechanisms and joint decision-making between pricing and inventory. It is found that the optimal two stages of the product: the pricing decision and the order quantity decision are crucial, especially in the pre-sale mode [2]. In general, the goals of e-commerce are as follows: lowest inventory level and maximum expected total profit. Therefore, the author mainly considers perishable products and new products, and establishes the price commitment mechanism and the two-stage expected profit model under the dynamic pricing mechanism.

### 1.3 Research significance

Theoretical significance: At present, the application of subscription mode in supply chain management is not perfect, domestic and foreign scholars have little research on the joint decision-making of pricing and inventory of products under the subscription mode. This paper constructs the pricing and inventory model under the subscription mode, not only can study the relevant theories of the joint decision-making model of the product pricing and inventory under the uncertain supply and demand condition, but also analyzes the uncertain supply and demand condition and the subscription mode condition with specific examples based on the dynamic pricing and inventory joint decision-making model [6]. Further, the optimization scheme of the pricing and inventory decision model under the subscription mode is proposed.

In reality, with the increasing volatility of market environment, the uncertainty of demand and supply has become an important factor in global supply chain management. Production uncertainty is mostly caused by unexpected events that are not predicted, such as the damage of production and processing machines or parts, major economic events at home and abroad, etc. At the same time, the globalization trend of supply chain procurement has introduced more possible impacts on procurement and supply, such as fluctuations in the global financial industry, and various possible supply delays. Therefore, the study of product pricing and inventory joint management in supply chain management under the subscription mode is an important dimension of supply chain supply and demand coordination under the uncertain environment.

## 2 Theoretical model of dynamic pricing and inventory joint decision making

### 2.1 Problem Description

In a complete supply chain, a downstream supplier needs to purchase a certain product in quantity from an upstream supplier.  $q(t)$  Due to the influence of various uncertain factors, the quantity of products that an upstream supplier can provide after receiving an order is, where the influence factor  $y$  of supply uncertainty is a random variable on, its density function is, and its distribution function is.  $yq(t)[0,1]f(y)F(y)$  It is assumed that under normal circumstances, these conditions are known to the

upstream and downstream suppliers [8].

At the same time, it is assumed that the downstream supplier is unable to meet the upstream supplier's order, resulting in the shortage cost  $h_2$  per unit; The cost of inventory held by downstream suppliers is  $h_1$  per unit.

Subtract the quantity demanded from the quantity purchased to represent the change in inventory level. Downstream suppliers can price and purchase in the hope of maximizing profits over a period of time. The model for this problem actually consists of a dynamic pricing and the newsboy model combined.

### 2.2 Variable definitions and assumptions

Assume that the downstream supplier makes the decision during the time period  $T$ ; Assume that the demand function is;  $d(t) = a - bp(t)$  Assuming an initial inventory of 0 indicates the end of the previous phase and the beginning of the next. The model parameters are as follows:

Table 1 Model parameters and definitions

Model parameters and definitions	
$d(t)$	Demand for time $t$
$q(t)$	Quantity purchased at time $t$
$c$	Unit purchase cost
$y$	Random variable with uncertain supply, density, distribution function $f(y), F(y)$
$r$	The mean of a random variable $y$ with uncertain supply
$h_1$	Carrying cost per unit of inventory
$h_2$	Loss cost per unit out of stock
$I(t)$	Inventory level at time $t$
$I_1(t)$	$I_1(t) = \max\{0, I(t)\}$
$I_2(t)$	$I_2(t) = \max\{0, -I(t)\}$

### 2.3 Establish a dynamic pricing and inventory joint decision model

Firstly, the model is built with the goal of maximizing the overall profit of the enterprise, and the constraint conditions such as the change amount and the change interval of variables are written out as follows:

$$\max_{p(t), q(t)} E \int_0^T [p(t)d(t) - cyq(t) - h_1I_1 - h_2I_2] dt \quad (1)$$

$$\frac{dI(t)}{dt} = yq(t) - d(t) \quad (2)$$

$$0 \leq q(t) \leq \bar{q} \quad (3)$$

$$I(0) = 0 \quad (4)$$

The objective function represents the expected profit maximization of the firm. In the integrand function, the first part  $p(t)d(t)$  is the sales revenue of the enterprise, the second part  $cyq(t)$  is the purchase cost paid when purchasing, the third part  $h_1I_1$  is the inventory cost when holding inventory, and the fourth part  $h_2I_2$  is the shortage loss when the supply requirements cannot be met [9]. The rest is the differential equation for the change in inventory, and define the initial inventory as 0.

In order to make the problem meaningful, suppose that

the demand is greater than 0, then  $p(t) \in (0, \frac{a}{b})$ . Since the target expression contains random variable  $y$ , in order to facilitate analysis and solution, let  $D(t) = \int_0^t d(s)ds$ , then  $Q(t) = \int_0^t q(s)ds$ ,

there is  $\frac{dD(t)}{dt} = d(t)$   
 $\frac{dQ(t)}{dt} = q(t)$

Then formula can be converted to definite equivalent form, as follows:

$$\max_{p(t), q(t)} \int_0^T [p(t)d(t) - crq(t) - h_1 \int_w^\infty [yQ(t) - D(t)]f(y)dy - h_2 \int_0^w [D(t) - yQ(t)]f(y)dy] dt \quad (5)$$

Where  $w(t) = \frac{D(t)}{Q(t)}$ , the original problem in the model can be converted into a definite equivalent form composed of above formula.

### 2.4 Principle of maximum value in optimal control theory

According to the maximum principle in optimal control theory, we can write its corresponding Hamiltonian function (Hamilton) as follows:

$$H = p(t)d(t) - crq(t) - h_1 \int_0^w [yQ(t) - D(t)]f(y)dy - h_2 \int_0^w [D(t) - yQ(t)]f(y)dy + \lambda(t)q(t) + \mu(t)d(t) \quad (6)$$

$$\dot{\lambda} = -\frac{\partial H}{\partial Q} = h_1 \int_w^\infty yf(y)dy - h_2 \int_0^w yf(y)dy = h_1 - (h_1 + h_2) \int_0^w yf(y)dy \quad (7)$$

$$\dot{\mu} = -\frac{\partial H}{\partial D} = -h_1 \int_w^\infty yf(y)dy + h_2 \int_0^w yf(y)dy = -h_1 + (h_1 + h_2) \int_0^w yf(y)dy, \mu(T) = 0 \quad (8)$$

The  $\dot{\lambda}$  and  $\dot{\mu}$  is  $Q$  and  $D$  association state variables respectively.  $w(t) =$  demand rate/rate of adoption. if the rate of demand < the rate of purchase, then having inventory will result in the cost of carrying inventory, if the rate of demand > the rate of purchase, then there is the loss of inventory.

When the system is in a stationary state,  $\lambda(0) = \lambda(T)$

$$\dot{\lambda} = \begin{cases} h_2 \int_0^w yf(y)dy & \text{if } w(t) > 1 \\ [h_2 \int_0^w yf(y)dy, h_1 + \int_w^\infty yf(y)dy] & \text{if } w(t) \leq 1 \end{cases} \quad (9)$$

The following formula is given by  $\frac{\partial H}{\partial q} = -cr + \lambda$ :

$$q(t) = \begin{cases} \bar{q} & \text{if } \lambda > cr \\ (0, \bar{q}) & \text{if } \lambda = cr \\ 0 & \text{if } \lambda < cr \end{cases} \quad (10)$$

The following formula is given by  $\frac{\partial H}{\partial p} = a - 2bp - b\mu = 0$ :

$$p(t) = \begin{cases} \frac{a}{b} & \text{if } a - b\mu(t) > 2a \\ \frac{a - b\mu(t)}{2b} & \text{if } 0 \leq a - b\mu(t) \leq 2a \\ 0 & \text{if } a - b\mu(t) < 0 \end{cases} \quad (11)$$

### 2.5 Summary of this chapter

This chapter studies the relationship between purchasing, inventory and pricing of a single supplier in the whole supply chain under the condition of supply uncertainty. By combining pricing strategy and purchasing decision, the supplier can maximize its profit. Suppose that in a supply chain, a downstream supplier needs to purchase from an upstream supplier. When an upstream supplier receives an order, the upstream supplier cannot fully supply the product, and there is a random factor  $y$ . These conditions are known to the upstream and downstream supplier, and the downstream supplier can maximize its profit through pricing and purchasing in a period of time.

## 3. Comparative analysis of the subscription mode and the supply and demand uncertain model

### 3.1 Example analysis

#### 3.1.1 Example analysis under uncertain supply and demand conditions

When supply and demand are uncertain, assume that  $y$  is uniformly distributed on  $[0, 1]$ .

$$f(y) = \begin{cases} 1, & 0 \leq y \leq 1 \\ 0, & \text{other condition} \end{cases} \quad (12)$$

$$F(y) = \begin{cases} 0, & y < 0 \\ y, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases} \quad (13)$$

its expected number  $r = \frac{1-0}{2} = 0.5$ , Suppose  $h_1 = 1$ ,  $h_2 = 3$ ,  $p = 5$ ,  $T = 5$ ,  $c = 2$ , The demand function is  $d(t) = 6 - p(t)$ , The calculated solution is:  $p(t) = 5.5 - 0.5t$   $q(t) = t + 1$

#### 3.1.2 Example analysis under subscription mode

By introducing the subscription mode into the pricing and inventory model, the supply uncertainty, namely the random variable of supply uncertainty, can be reduced to some extent  $y$  The value probability of is biased to 1, then there are three hypotheses as follows:

Hypothesis I: Random variable  $y$ , The value probability of is linearly increasing;

Hypothesis II: Random variable  $y$ , The value probability of is rising more and more sharply;

Hypothesis III: Random variable  $y$ , The value probability of is rising steadily;

Verify the above three hypotheses:

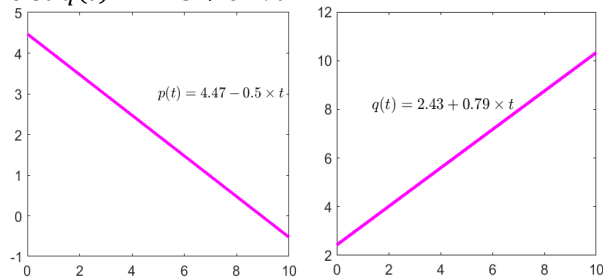
Hypothesis I: suppose, again, then, solution. Set  $f(y) = Ay$ , because  $\int_0^1 f(y)dy = 1$ , then there is  $\frac{A}{2}y^2 = 1$ , get  $f(y) = 2y$

Hypothesis II/III: Suppose, again, solved, therefore: set  $f(y) = By^2 + Cy$ , because  $\int_0^1 f(y)dy = 1$ , then there is  $f(1) = 1, B = -3, C = 4$ , get  $f(y) = -3y^2 + 4y$

Since hypothesis I is more able to present the stability of random variable with uncertain supply, which is more consistent with the assumptions of this paper, this paper carries out an example analysis based on hypothesis I, Under a subscription model supply uncertain random variable  $y$ ,  $f(y) = 2y$ ,  $F(y) = \int_0^y (2y)dy = y^2$  for example analysis.

Its expected value:  $r = \int_0^1 yf(y)dy = \int_0^1 (2y^2)dy = \frac{2}{3} \approx 0.67$

Also assume that the demand function:  $h_1 = 1h_2 = 3p = 5T = 5c = 2$ , The calculated solution is  $d(t) = 6 - p(t)$ , as shown in the figure below:  $p(t) = 4.47 - 0.5t$ ,  $q(t) = 2.43 + 0.79t$ .



**Figure 1.** Price and purchase curve corresponding to the example.

### 3.2 Connection and difference between model algorithm under subscription mode and uncertain supply and demand

#### 3.2.1 Connection between subscription mode and model under supply and demand uncertainty

In the case of subscription mode and supply and demand uncertainty, the premise assumption is the relationship between purchasing, inventory and pricing of a single supplier in the whole supply chain<sup>[10]</sup>. By combining the pricing strategy with the purchasing decision, the supplier is expected to maximize its profit. Suppose that in a supply chain, a downstream supplier needs to purchase from an upstream supplier. Due to various uncertain factors, the upstream supplier cannot fully provide the product after receiving the order. There is a random factor  $y$ , and these conditions are known by the upstream and downstream suppliers. In order to maximize profits over a period of time.

In the dynamic pricing and inventory model, the extended model of the deterministic storage EOQ model learned in operations research and the idea of the indeterminate storage newsboy model are applied in the condition of uncertain supply and demand or subscription mode. In the process of analyzing the constructed model, the maximum principle of the optimal control theory is adopted<sup>[11]</sup>. With the help of Hamiltonian function, The

goal of the maximum profit of the supplier is constructed and solved. Finally, based on the constructed model, the same set of concrete case data is used for example analysis.

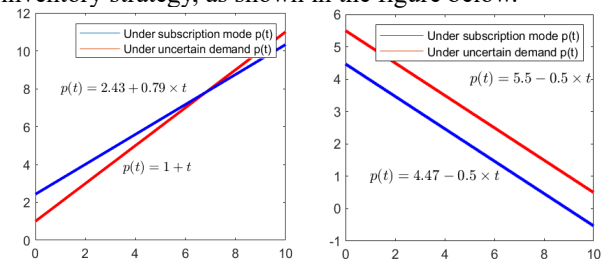
Based on the established model of dynamic pricing and inventory joint decision, according to the maximum principle of optimal control theory, using Hamiltonian function, the sufficient and necessary conditions for obtaining the optimal solution are obtained. At the same time, it gives the pricing strategy and purchasing decision of single supplier in the whole supply chain under the zero inventory strategy, and shows that under certain conditions, in the process of realizing the profit maximization of the enterprise, it is very important to control the purchase and pricing, so as to keep the inventory at a minimum.

#### 3.2.2 Differences between the subscription mode and the supply and demand uncertain model

The most significant effect of introducing the subscription mode into the joint decision model of product pricing and inventory is as follows: to some extent, the uncertainty of supply and demand among enterprises at each node of the supply chain can be reduced<sup>[5]</sup>, so that the probability of random variable  $y$  can be changed from the uniform distribution in the case of uncertainty to the continuous monotonously increasing distribution in the case of subscription mode, so as to calculate.

Based on the joint decision model of dynamic pricing and inventory of products built above, the case analysis is carried out. In the specific case analysis of subscription mode, the mathematical analytic formula method and computer function image analysis method are introduced, and the probability density function is determined by the method of hypothesis verification, and the solution is carried out.

Through the probability density function determined by the hypothesis verification method, a set of case values are taken for example analysis, and the computer function image analysis method is used to compare. It is found that using the subscription mode for pricing is lower than the pricing under the uncertain condition, and the price is more conducive to product sales. The subscription mode is more stable than the purchase quantity under the uncertain condition, and the purchase quantity is more conducive to the enterprise to determine the ordering and inventory strategy, as shown in the figure below.



**Figure 2.** Comparison between subscription mode and pricing and procurement under uncertainty.

## 4. Research conclusions and prospects

This paper studies the technological development status of subscription mode at home and abroad and the application status of various industries, as well as the joint decision management status of product pricing and inventory, builds the joint decision model of product dynamic pricing and inventory, and based on the established dynamic pricing and inventory joint decision model, carries on the case analysis under the condition of uncertain supply and demand conditions and subscription mode. By comparing the two, the following conclusions are obtained through model solving and example analysis, as well as the comparison of the two cases:

(1) First of all, this paper is consistent with the conclusions of classical literature on the joint decision of product pricing and inventory, that is, it is not feasible to focus only on the single aspect of product pricing or product inventory in the management measures of the whole supply chain<sup>[12]</sup>. By applying the joint decision of price and inventory, the balance between supply and demand in the supply chain can be solved. It is a feasible management measure. In the current competitive market, the application of pricing and inventory joint management to achieve supply chain management is an important means of competition for the survival and development of logistics enterprises and enterprises.

(2) Secondly, this paper builds a product dynamic pricing and inventory joint decision model, combines dynamic pricing with the classic newsboy model, realizes the algorithm simulation in the production, procurement and pricing of enterprises, and introduces the subscription mode into the product pricing and inventory joint decision model to build the information sharing channel of the ordering and delivery channels in the supply chain. The upstream supplier and downstream demander are subscribers and publishers of each other. By sharing ordering and delivery information, the uncertainty of supply and demand of upstream and downstream enterprises in the supply chain can be reduced, and the transaction completion rate of both sides can be improved, so as to maximize the target profit and achieve the optimal purpose of the whole supply chain.

(3) Through the analysis of concrete examples, the model constructed in this paper is calculated, and the uncertain supply and demand conditions and subscription mode conditions are analyzed numerically and by computer simulation. It is found that the introduction of subscription mode into the product pricing and inventory joint decision-making model can be better than the traditional pricing and inventory control decision-making model. It can better solve the problem of inventory replenishment strategy and pricing decision under uncertain conditions, so as to achieve coordination of supply chain under uncertain conditions.

In this paper, there are still many shortcomings in the research on the joint decision of product pricing and inventory under the subscription mode. Therefore, the following prospects are proposed for the follow-up research and future research directions:

(1) In the study of this paper, when constructing the

pricing and inventory joint decision model, the initial inventory is assumed to be zero, and the pricing, procurement and inventory strategies of a node enterprise in the supply chain are too high. It is hoped that the assumptions can be relaxed in the future research, and the research can be carried out under more commonly applicable conditions. To increase the universality of the pricing and inventory joint decision model and algorithm.

(2) Based on the established pricing and inventory joint decision model, this paper only calculates a node enterprise in the whole supply chain when carrying out concrete example analysis. In future studies, it should further consider how to make joint decision of product pricing and inventory in the fierce market competition with multiple supply chains, multiple suppliers and multiple products.

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