Development of a mathematical model of the heating nuclear power unit for optimization studies of autonomous electric power systems

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Abstract. A two-stage method for creating mathematical models of thermal power nuclear power units for studying autonomous electric power systems is presented in the article. A detailed model of a nuclear power unit is being developed as the first stage. The model has sufficient accuracy of description of technological processes. Optimization calculations are carried out for a large number of operating modes. At the second stage, based on the results of optimization calculations of the first stage, energy characteristics and dependencies are constructed to determine the boundaries of the region of feasible solutions in the form of polynomials. A model of a nuclear power unit is created based on polynomials. A simplified model can be used in optimization studies of electric power systems. The authors propose a two-step approach for solving the polynomial search problem. At the first step, the coefficients of the polynomial are selected, which correspond to the minimum of the maximum value of the modulus of the difference between the function obtained using the polynomial and the function obtained using the detailed model of the power unit. At the second step, the deviation modules are limited to the value found at the first step, and the sum of the deviation modules is minimized at all points.

1 Introduction

There are large autonomous electric power systems (EPS) in the Northern and North-Eastern regions of the Russian Federation (RF). EPS are with rather weak links with the unified energy systems (IPS), in which they are included. External electrical connections can be fixed when optimizing the operating modes of power systems, since they are small or equal to zero. The supply of electrical and thermal energy to consumers is typical for such electric power systems. Often the delivery of organic fuel to such combined heat and power plants (CHP) is carried out along rather complicated routes, which increases the price of fuel. Therefore, the construction of nuclear power plants (NPP) with cogeneration nuclear power units is promising for such power systems [1-6]. NPPs make it possible to increase the economic efficiency and energy security of EPS.

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2 Formulation of the problem

It is necessary to carry out calculations of a significant number of operating modes of autonomous EPS during their study. The tasks of distributing electrical and thermal loads between the equipment of the system must be solved taking into account all technical limitations and fuel costs. The solution of these problems can be carried out with the help of mathematical modeling of power facilities and coordinated optimization of the characteristic modes of their operation. When conducting optimization studies of autonomous EPS, the fuel costs of power plants are determined depending on electrical and thermal loads, as well as restrictions on operating parameters. At the same time, mathematical models of generating equipment are needed, in particular, cogeneration nuclear power units intended for operation. Such a power unit is a rather complex technical object and consists of a large number of equipment elements. Its detailed mathematical model cannot be used to solve the optimization problem of load distribution in the power system due to its complexity. Therefore, there is a need to create a mathematical model of a cogeneration nuclear power unit with a fairly accurate description of the physical processes and suitable for power system optimization studies. In this paper, we propose a technique for creating such models, which includes two stages. At the first stage, a detailed mathematical model of a cogeneration nuclear power unit is developed, which will provide a satisfactory accuracy of the description of the processes, and optimization calculations are performed for a large number of operating modes. At the second stage, the energy characteristics and dependencies are built based on the results of optimization calculations of the first stage. The boundaries of the region of feasible solutions are defined as polynomials. A simplified mathematical model of a nuclear power unit is created on their basis.

The first stage of the methodology for creating mathematical models of cogeneration nuclear power units. A detailed mathematical model is developed on the basis of technological schemes for the elements of a cogeneration nuclear power unit and technical documentation. Optimization studies are carried out according to the criterion of the maximum supply of useful electrical energy for a given thermal load of consumers and the thermal power of the reactor, taking into account physical and technical restrictions on the installation parameters. The solution of the problem makes it possible to evaluate the efficiency of operating modes and obtain a sufficient set of parameters for constructing energy characteristics. The mathematical formulation of the problem has the following form.

$$\max N^{elec}(x, y, Q_d^C, Q_h, P^H, t^{air}), \quad (1)$$

under conditions

$$H(x, y, Q_d^C, Q_h, P^H, t^{air}) = 0, \; G(x, y, Q_d^C, Q_h, P^H, t^{air}) \geq 0, \; x^{\min} \leq x \leq x^{\max}, \quad (2)$$

where x – vector of independent parameters to be optimized (steam flow rates to the turbine condenser, steam from controlled turbine extractions, steam pressure in front of control diaphragms, etc.); y – vector of dependent calculated parameters (flow rates of live steam, steam from unregulated turbine extractions, turbine electric power, pressure in controlled turbine extractions, etc.); N^{elec} – useful electric power of the power unit; Q_d^C – calculated heat load (the design calculation, the nominal mode); Q_h – heat load of the consumer in the characteristic operating modes; P^H – heat power of the reactor; t^{air} – outdoor air temperature in the characteristic operating modes; H – equality constraint vector (includes equations describing: technological connections between circuit elements, material and energy balances, heat transfer, etc., serves to determine the vector y); G – vector of constraint-inequalities (includes constraints on dependent parameters: live steam flow rate, electric power of the turbine, etc.); $x^{\min}$, $x^{\max}$ – vectors of lower and upper bounds of x.

The second stage of the methodology for creating mathematical models of cogeneration nuclear power units. At the second stage, the energy characteristics are constructed in the form of polynomials based on the results of optimization calculations obtained using a detailed mathematical model of a nuclear power unit. When solving similar problems, other researchers often use the least squares method. The method controls only the average deviation between calculations (in this case, between calculations performed using a detailed mathematical model and calculations by polynomials). The maximum deviation is not controlled by this method [7-10].

In this work, the methodical approach is used to approximate complex nonlinear dependencies described by detailed mathematical models by fairly simple dependences. This methodological approach allows you to track the average and maximum deviation. A function of *N* type parameters has the form.

$$f = f(x_1,...,x_N). \qquad (3)$$

A detailed mathematical model is used to determine the value of *f* for given values $x_1,..., x_N$. The ranges of each parameter [$x_i^{\min}, x_i^{\max}$]are set, where $x_i^{\max}$ is the upper limit of the range of x_i, $x_i^{\min}$ is the lower limit of the range of x_i. M_i points are set in the range [$x_i^{\min}, x_i^{\max}$] (as a rule, they are evenly spaced in the range). Point $x_i^1 = x_i^{\min}$ and point $x_i^{M_i} = x_i^{\max}$. The distance (step) between the points will be equal to $\Delta x_i = (x_i^{\max} - x_i^{\min})/(M_i - 1)$.

The coordinates of the *j*-th point are determined from the expression

$$x_i^j = x_i^{\min} + \Delta x_i\,(j-1). \qquad (4)$$

In this case, the total number of points will be (for all possible combinations of parameters x_i, $i = 1,..., N$)

$$M_\Sigma = M_1 \cdot M_1 \cdot ... \cdot M_N. \qquad (5)$$

The calculation is carried out according to a detailed mathematical model for each possible combination of parameters. As a result, for $k = 1,..., M_\Sigma$, the values of the function f^k found with known values of the parameters $x_1^k,..., x_N^k$ are determined. The initial dependence (3) will be represented by a simpler dependence – a polynomial of the *S*-th degree in *N* variables. An *S*-th degree polynomial in *N* variables is the sum of *L* terms of the following form

$$f^p = \sum_{l=1}^{L} \alpha_l \prod_{i=1}^{N} x_i^{k_i^l}, \qquad (6)$$

where *L* is the coefficient of the polynomial at the *l*-th term, *K* is the exponent of the *i*-th parameter in the *l*-th term (the exponents can take the values 0, 1, ..., *S*), *L* is the total number of terms in the polynomial, equal to all possible sets exponents of *N* parameters that meet the conditions:

$$\begin{aligned} &k_1^l = \{0,...,S\}, \\ &k_2^l = \{0,...,S\},..., k_N^l = \{0,...,S\}, \qquad (7) \\ &k_1^l + k_2^l + ... + k_N^l \le S. \end{aligned}$$

Each term of the polynomial corresponds to a set of integer exponents for *N* variables and the coefficient of the polynomial. Polynomial (6) can be represented as a function depending on the coefficients of the polynomial α_l and the parameters x_i of the form $f^p = f^p(\alpha_1,...,\alpha_2, x_1,..., x_N)$. f^p is a linear function of the coefficients (for given values of the parameters x_i) and a non-linear function of the parameters (for given values of the polynomial coefficients). The task of finding a polynomial is to find its coefficients. The following two-step approach is used in this work.

The polynomial coefficients are selected at the first step such that the minimum of the maximum value of the modulus of the difference (deviation) of the function determined using the polynomial and the function determined using the detailed mathematical model is reached. The maximum value of the modulus is determined by M_Σ points. The found minimum of the maximum deviation module is fixed. Part of the deviation modules takes on the maximum value, and the remaining modules have a random value, which is smaller in the general case.

At the second step, the deviation modules are limited to the value found at the first step and the sum of the deviation modules at all M_Σ points is minimized. The polynomial coefficients (providing the solution of the second step problem) are taken as the coefficients that provide the best approximation to the original dependence. They guarantee the minimum of the maximum deviation modulo and the minimum of the sum of the absolute deviations of all deviations (or the minimum of the average deviation modulo) for a given maximum deviation modulus.

Optimization problems (of the first and second steps) are linear programming problems.

Task I (first step)

$$\min_{a_1,...,a_L, x_h} x_h \,, \tag{8}$$

under conditions

$$x_h - \left(f_t - f_t^p\left(a_1,...a_L, x_1^t,...,x_N^t\right)\right) \geq 0 \,,\; x_h + \left(f_t - f_t^p\left(a_1,...a_L, x_1^t,...,x_N^t\right)\right) \geq 0 \,, \tag{9}$$

$$t = 1,..., M_\Sigma \,,\; \underline{a} \leq a_i \leq \overline{a}_\Sigma, i = 1,..., L \,,\; 0 \leq x_h \leq \overline{x}_h \,, \tag{10}$$

where $\underline{a}, \overline{a}$ - minimum and maximum value of the polynomial coefficients, $\overline{x}_h$ – maximum value of the parameter x_h. The value of x_h will be denoted by x_h^* at the point of solution of problem I.

Task II (second step)

$$\min_{a_1,...,a_L, x_h} \sum_{t=1}^{M_\Sigma} x_h^t \,, \tag{11}$$

under conditions

$$x_h^t - \left(f_t - f_t^p\left(a_1,...a_L, x_1^t,...,x_N^t\right)\right) \geq 0 \,,\; x_h^t + \left(f_t - f_t^p\left(a_1,...a_L, x_1^t,...,x_N^t\right)\right) \geq 0 \,, \tag{12}$$

$$t = 1,..., M_\Sigma \,,\; \underline{a} \leq a_i \leq \overline{a}_\Sigma, i = 1,..., L \,,\; 0 \leq x_h^t \leq x_h^* \,. \tag{13}$$

Auxiliary parameters will be denoted by x_h^{t*} at the point of solution of problem II, and the coefficients of the polynomial by $a_1^*,..., a_L^*$. The average module of deviations will be equal to $\Delta^{cp} = \left(\sum_{t=1}^{M_\Sigma} x_h^{t*} \right) \Big/ M_\Sigma$. The optimal polynomial will look like

$$f_{opt}^{p} = f^{p}\left(a_1^*,...,a_L^*,x_1,...,x_N\right). \quad (14)$$

The optimal polynomial provides the maximum modulo deviation x_h^* and the average deviation modulo Δ^{cp} on a set of M_Σ points.

If the accuracy of the approximation is insufficient, then the degree of the polynomial should be increased and the problem solved again. If the accuracy of the approximation is too high, then the degree of the polynomial should be reduced and the problem solved again. In this paper, the practical implementation of the methodology for creating mathematical models of thermal power nuclear power units of power plants intended for optimization studies of autonomous EPS is demonstrated using the example of a thermal power nuclear power unit with a capacity of 55 MW. According to [11, 12], the possibility of operating this power unit is considered in one of the regions of the Far North.

Description of the heating atomic power unit. The power unit under study has a reactor plant (RP) (thermal power 175 MW) and a STU (maximum power 55 MW). The pressurized water nuclear reactor is made according to a two-loop scheme with four direct-flow steam generators (SG) integrated into the core vessel [13]. There is no information about PTU for joint operation with such a reactor. Therefore, in the framework of this work, the design calculation of this turbine plant was performed. Scheme PTU TK-35 / 38-3.4 of the Kaluga Turbine Plant (KTZ) was taken as the basis for the technological scheme of the studied PTU. TK-35/38-3.4 was previously operated as part of a power unit with a KLT-40S reactor plant [14]. The turbine plant under study is conditionally called TK-50/55-3.4. The scheme of the power unit is shown in fig. 1. PTU TK-50/55-3.4 has a heat extraction for hot water supply and heating, two unregulated steam extractions for heating feed water. The schemes for connecting equipment and distributing water and steam flows for this PTU are the same as for TK-35 / 38-3.4. Heat exchangers TK-50/55-3.4 have an area of heat-exchange heating surfaces and coolant flow rates required to generate electrical power up to 55 MW and thermal power up to 30 Gcal/h.

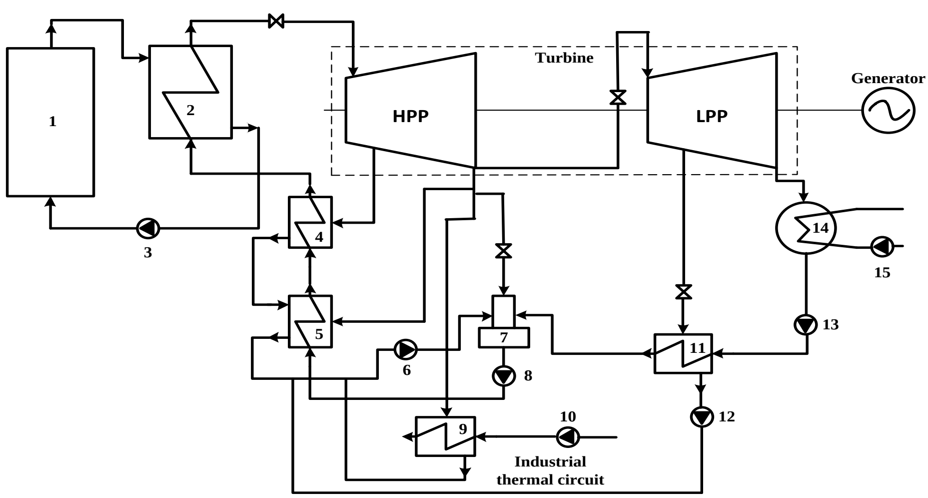

Fig. 1. Simplified process flow diagram for vocational schools TK-50/55-3.4.
1 - reactor plant, 2 - steam generator (4 pcs.), 3 - circulation pumps of the first boiler, 4, 5 - high pressure heaters, 7 - deaerator, 8 - feed pumps, 9 - industrial circuit network heater, 10 - industrial circuit pumps, 11 - low pressure heater, 6, 12, 13 - condensate pumps, 14 - condenser, 15 - cooling water circulation pumps, HPP - high pressure part, LPP - low pressure part.

Mathematical models of the power unit with RP and PTU TK-50/55-3.4 are designed to perform calculations at nominal parameters and calculations when changing electrical and thermal capacities. The models of the PTU equipment elements are based on design calculations in the nominal mode and the geometric dimensions of the high and low pressure heaters (HPE, HDPE), the network water heater and the turbine condenser (the areas of the heat exchange heating surfaces, the diameters and pitches of the pipes of the heat exchange heating surfaces), the nominal flow rates of the water steam, inlet and outlet steam pressures in the steam turbine compartments. Verification calculations of plant equipment elements underlie the mathematical model for partial load calculations, which are performed with given design parameters to determine the parameters of water and steam. The creation of mathematical models of the power unit and the performance of optimization calculations were carried out using the software-computer complex (PCC) of the Computer Programming System (SMPP), created by the staff of the Thermal Power Systems Department of the ESI SB RAS [15]. The stepwise optimization method was used [16-18]. The method was developed at ESI SB RAS. The number of optimized parameters of the verification mathematical model of the power unit is 43, the number of inequality constraints is 155.

3 The results of the calculation

The first stage of the methodology. A series of optimization calculations of the operating modes of the power unit was carried out at calculated thermal loads of consumers equal to 10 Gcal/h, 20 Gcal/h, 30 Gcal/h, with thermal loads of the reactor in the range from 175 MW to 75 MW with a step of 20 MW, at ambient temperatures in the range from -55 ^{0}C to +8 ^{0}C (heating period) and the average outdoor temperature in the non-heating period is +15 ^{0}C.
As an example, in Table. 1 the main results of the optimization calculations of the operating modes of the power unit are shown at the design heat load of consumers of 30 Gcal/h., at reactor heat loads of 175 MW, 135 MW, 95 MW and outdoor air temperatures of -55 ^{0}C, -20 ^{0}C, +1,5 ^{0}C and +15 ^{0}C (non-heating period).

Table 1. The main technical characteristics of the operating modes of the power unit with the calculated thermal load of consumers equal to 30 Gcal/h.

Main indicators	Outdoor air temperatures, ^{0}C			
	-55	**-20**	**+1,5**	**Non-heating period**
Heat load of consumers, Gcal/h	30,0	18,8	11,92	4,8
Heat load of the SG, MW	**175**			
Electric power of the NPP, MW	51,56	53,57	54,79	56,09
Electric power of own needs, MW	0,70	0,80	0,78	0,86
Useful electrical power, MW	50,86	52,77	54,01	55,22
Steam consumption in the last section of the turbine, t/h	157,9	178,1	190,8	203,1
Fuel consumption, tce/h (kg/h)	21,5 ($1,68\times10^{-2}$)			
Heat load of the SG, MW	**135**			
Electric power of the NPP, MW	36,90	40,40	41,69	42,76
Electric power of own needs, MW	0,50	0,59	0,57	0,69
Useful electrical power, MW	36,40	39,81	41,12	42,07
Steam consumption in the last section of the turbine, t/h	109,4	127,3	139,2	151,7
Fuel consumption, tce/h (kg/h)	16,5($1,29\times10^{-2}$)			
Heat load of the SG, MW	**95**			
Electric power of the NPP, MW	21,62	26,56	27,76	28,93

Electric power of own needs, MW	0,26	0,41	0,40	0,47
Useful electrical power, MW	21,36	26,15	27,36	28,46
Steam consumption in the last section of the turbine, t/h	64,8	81,1	92,7	105,0
Fuel consumption, tce/h (kg/h)	11,6(9,1×10^{-3})			

The second stage of the methodology. Based on the calculations of the first stage, data were obtained, with the help of which a simplified mathematical model of a nuclear power unit was created and subsequently used to optimize the EPS. A simplified mathematical model of a nuclear power unit is presented in the following form.

$$G^S_{Con} = f^S_{Con}(N^\Sigma_{El}, Q^C_T, t^{air}),\ P^H = f^H(N^\Sigma_{El}, Q^C_T, t^{air}), \tag{15}$$

$$N^{AN} = f^{AN}(N^\Sigma_{El}, Q^C_T, t^{air}),\ G^S_{Con} \ge G^{S_{\min}}_{Con},\ N^{elec} = N^\Sigma_{El} - N^{AN}, \tag{16}$$

$$P^H_{\min} \le P^H \le P^H_{\max},\ N^\Sigma_{El_{\min}} \le N^\Sigma_{El} \le N^\Sigma_{El_{\max}},\ U^F = k^N P^H S^N \tau_{MD}, \tag{17}$$

where G^S_{Con} – steam flow to the turbine condenser, N^Σ_{El} – total electric power of the power unit, Q^C_T – calculated thermal load of the consumer, N^{AN} – electric power of auxiliary needs, $G^{S_{\min}}_{Конд}$ – minimum allowable steam flow to the turbine condenser, $P^H_{\min}$, $P^H_{\max}$ – minimum and maximum value of the heat power of the reactor, $N^\Sigma_{El_{\min}}$, $N^\Sigma_{El_{\max}}$ – minimum and maximum values gross electric power, U^F – fuel costs, k^N – nuclear fuel burnup factor, S^N – nuclear fuel price, τ_{MD} – operating mode duration. Dependences f^S_{Con}, f^H, f^{AN} are represented as polynomials of the fourth degree in three variables ($N^\Sigma_{El}, Q^S_{Con}, t^{air}$).

The values of the maximum, average deviation, the number of terms of the polynomial (depending on the value of its degree) for the polynomial for determining the steam flow rate in the steam turbine condenser are given as an example in Table. 2.

Table 2. The number of terms of the polynomial, the maximum and average deviation depending on the value of its degree for the option of determining the steam flow in the steam turbine condenser.

Polynomial degree	Number of terms of the polynomial	Maximum deviation	Average deviation
2	10	1,340	0,578
3	20	0,863	0,410
4	35	0,457	0,230
5	56	0,343	0,197
6	84	0,243	0,157
7	120	0,190	0,128

The polynomials obtained at the second stage are drawn up as subprograms and are used when creating a mathematical model of an autonomous EPS.

The presented methodical approach can be used to create mathematical models of other sources of electrical and thermal energy (cogeneration steam turbine, gas turbine and combined cycle plants, etc.) operating in an autonomous EPS.

4 Conclusion

A description of the two-stage methodology for creating mathematical models of cogeneration nuclear power units intended for optimization studies of autonomous EPS is given in this paper. The first stage of the methodology includes the development of a detailed model of a nuclear power unit and optimization calculations for a sufficiently large number of operating modes. At the second stage, based on the results of the optimization calculations of the first stage, energy characteristics and dependencies are constructed that determine the boundaries of the region of feasible solutions in the form of polynomials and, on their basis, a simplified mathematical model of the power unit is created, applicable for optimization studies of autonomous EPS. A two-step approach is proposed for solving the problem of finding the coefficients of a polynomial. The technique is demonstrated on the example of a cogeneration nuclear power unit for operation in the climatic conditions of the Far North.

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