Modeling of effective elastic-plastic properties of layered composites with a periodic structure in the framework of the anisotropic flow theory

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Abstract. The article is devoted to the development of a method for constructing theoretical strain diagrams. The method is based on the use of a model of effective constitutive relations for approximating the deformation diagrams of layered composites obtained using the asymptotic averaging method. To find the elastic constants of the model of a transversally isotropic composite, the method of minimizing the deviation of the approximation deformation diagrams from the diagrams obtained by the asymptotic homogenization (AH) method is used for a series of standard problems of deformation at small deformations. Minimization problems were solved using the Hooke-Jeeves method. The results of numerical simulation by the proposed method for layered composites are presented, which showed good approximation accuracy, which is achieved due to the proposed method for separating the coupled problems of micro- and macroscopic deformation.
Key words: layered composites, transversally isotropic, asymptotic homogenization, numerical simulation, elastic constants.

1 Introduction

Currently, there are many works devoted to modeling the effective mechanical characteristics of composite materials. For practical purposes, the problem of determining the effective elastic characteristics of composites based on information about the microstructure and properties of the constituent phases is of great importance. There are quite a few methods for this problem, but most of them are inapplicable for composites with small deformations [1, 2].

To calculate the effective characteristics of composites, the most promising method is the homogenization method (AH), proposed by N.S. Bakhvalov, G.P. Panasenko, E. Sanchez-Palencia. The method of asymptotic averaging is well developed at present and has been successfully implemented numerically for various problems in mechanics, but mainly for linear problems [3−12].

To solve the problems of macroscopic deformation of structural elements made of composites with small deformations, it is necessary to apply constitutive relations for composites, taking into account anisotropy. The direct use of the asymptotic averaging

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method for constructing constitutive relations is possible and was done in [13], the method is based on solving special problems of microscopic deformation on periodicity cells (PC). Calculations of macroscopic deformation, for example, by the finite element method, when at each node of the grid it is necessary to solve the problem on a periodicity cell, lead to very large amounts of calculations.

The purpose of this work is to develop a method that will allow us to separate the problems of macro- and microscopic deformation of linear elastic composites without using the asymptotic averaging method.

2 Model of transversely isotropic elastic –plastic media with small deformations

Since the solution of local problems on the PC requires large amounts of calculations, we will move from the exact solution of these problems in each specific case of loading the PC with a system of averaged stresses to the construction of analytically effective constitutive relations for the composite as a whole. The constants included in these constitutive relations will be found by approximating the deformation diagrams of composites for particular deformation problems, while the deformation diagrams themselves are calculated with a limited number of options for solving problems on the PC of the composite [14, 15].

The proposed method is in fact a numerical experiment, in which, instead of finding deformation diagrams experimentally, a numerical solution of problems on the PC is used. In the future, both in a numerical and in a real physical experiment, the most successful analytical version of the constitutive relations is selected, which best describe the maximum number of deformation diagrams in a standard verification set of experiments.

Let us consider an elastic-plastic layered composite material, which we will assume to be a transversally isotropic medium. We accept the basic model about the additivity of elastic and plastic deformations

$$\varepsilon = \varepsilon_e + \varepsilon_P. \tag{1}$$

The elastic strain tensor will be proposed with a tensor density

$$\varepsilon_e = \sum_{\gamma=1}^{z} \varphi_\gamma J_{\gamma T}^{(s)}, \tag{2}$$

where φ_γ is the scalar function of the joint invariants $J_\gamma^{(s)}(\sigma, \varepsilon_p)$ of the stress tensor σ and the plastic strain tensor ε_p

$$\varphi_\gamma = \varphi_\gamma\left(J_\alpha^{(s)}\right) = -\rho \frac{\partial \zeta}{\partial J_\gamma^{(s)}}, \quad J_{\gamma\sigma}^{(s)} = \frac{\partial J_\gamma^{(s)}}{\partial \sigma}, \ \gamma = 1,\ldots,z. \tag{3}$$

Here $\zeta\left(J_\alpha^{(s)}\right)$ – elastic potential (Gibbs free energy), ρ – density, $J_{\gamma\sigma}^{(s)}$ – derivative tensor. The functional basis of joint invariants $J_\gamma^{(s)}(\sigma, \varepsilon_p)$ for a transversely isotropic medium consists of 11 invariants, which can be chosen as the following [16–19]:

$$
\begin{aligned}
&J_{\gamma}^{(3)}=I_{\gamma}^{(3)}(\sigma),\quad \gamma=1,...,5 \qquad J_{5+\gamma}^{(3)}=I_{\gamma}^{(3)}(\varepsilon_p),\quad \gamma=1,...,4,\\
&J_{10}^{(3)}=\left(E-\hat{c}_3^2\right)\cdot\sigma\cdot\cdot\left(\hat{c}_3^2\cdot\varepsilon_p\right),\quad J_{11}^{(3)}=\sigma\cdot\cdot\varepsilon_p-2J_{10}^{(3)}-J_2^{(3)}J_8^{(3)}.\\
&I_1^{(3)}(\sigma)=\left(E-\hat{c}_3^2\right)\cdot\cdot\sigma,\qquad I_2^{(3)}(\sigma)=\hat{c}_3^2\cdot\cdot\sigma,\quad I_5^{(3)}(\sigma)=\det(\sigma),\\
&I_3^{(3)}(\sigma)=\left(E-\hat{c}_3^2\right)\cdot\sigma\cdot\cdot\left(\hat{c}_3^2\cdot\sigma\right),\qquad I_4^{(3)}(\sigma)=E\cdot\cdot\sigma^2-2I_3^{(3)}(\sigma)-(I_2^{(3)}(\sigma))^2.
\end{aligned}
\tag{4}
$$

Here E – metric tensor, $\hat{c}_3^2=\hat{c}_3\otimes\hat{c}_3$, and $\hat{c}_3$ – unit vector directed along the transversal isotropy axis, $\hat{c}_\alpha, \alpha=1,2,3$ – basis of the transversal isotropy axes.

The derivative tensors $J_{\gamma T}^{(s)}$ in this case have the following form:

$$
\begin{gathered}
J_{1\sigma}^{(3)}=E-\hat{c}_3^2,\quad J_{2\sigma}^{(3)}=\hat{c}_3^2,\ J_{3\sigma}^{(3)}=\frac{1}{2}\left(O_1\otimes O_1+O_2\otimes O_2\right)\cdot\cdot\sigma,\ J_{4\sigma}^{(3)}=2\,{}^4O_3\cdot\cdot\sigma,\\
J_{5\sigma}^{(3)}=\sigma^2-I_1(\sigma)\sigma+EI_2(\sigma),\qquad J_{6\sigma}^{(3)}=J_{7\sigma}^{(3)}=J_{8\sigma}^{(3)}=J_{9\sigma}^{(3)}=0,\\
J_{10\sigma}^{(3)}=\frac{1}{4}\left(O_1\otimes O_1+O_2\otimes O_2\right)\cdot\cdot\varepsilon_p,\ J_{11\sigma}^{(3)}={}^4O_3\cdot\cdot\varepsilon_p,
\end{gathered}
\tag{5}
$$

where $I_2(\sigma)$ – principal invariants of the tensor [19], $O_1=\hat{c}_2\otimes\hat{c}_3+\hat{c}_3\otimes\hat{c}_2$, $O_2=\hat{c}_1\otimes\hat{c}_3+\hat{c}_3\otimes\hat{c}_1$, and the expression for the tensor ${}^4O_3$ is given in [19].

Substituting these expressions into the formula of the constitutive relations presented in the tensor basis and grouping them by tensor powers, we obtain the representation of the constitutive relations of a transversally isotropic medium in the tensor basis:

$$
\varepsilon_e=\tilde{\varphi}_1E+\tilde{\varphi}_2\hat{c}_3^2+\left(O_1\otimes O_1+O_2\otimes O_2\right)\cdot\cdot\left(\tilde{\varphi}_3\sigma+\tilde{\varphi}_{10}\varepsilon_p\right)+\tilde{\varphi}_4\sigma+\varphi_2\sigma^2+\varphi_{11}\varepsilon_p,
\tag{6}
$$

where

$$
\begin{aligned}
&\tilde{\varphi}_1=\varphi_1+\varphi_5I_2^{(3)}(\sigma),\qquad \tilde{\varphi}_2=\varphi_2-\varphi_1-2\varphi_4I_2^{(3)}(\sigma)-\varphi_{11}I_2^{(3)}(\varepsilon_p),\\
&\tilde{\varphi}_3=\frac{\varphi_3}{2}-\varphi_4,\qquad \tilde{\varphi}_{10}=\frac{\varphi_{10}}{4}-\frac{\varphi_{11}}{2},\qquad \tilde{\varphi}_4=2\varphi_4-\varphi_5I_1(\sigma).
\end{aligned}
\tag{7}
$$

Let us further consider a model in which the elastic potential ζ does not depend on the implementation of the invariants $J_{10}^{(3)}$ and $J_{11}^{(3)}$, and also does not depend on the cubic invariant $I_5^{(3)}(\sigma)$, then $\varphi_5=\varphi_{10}=\varphi_{11}=0$, and then relation (6) takes the form:

$$
\begin{gathered}
\varepsilon_e={}^4\Pi\cdot\cdot\sigma,\\
{}^4\Pi=l_1E\otimes E+l_2\hat{c}_3^2\otimes\hat{c}_3^2+l_3(E\otimes\hat{c}_3^2+\hat{c}_3^2\otimes E)+l_4\left(O_1\otimes O_1+O_2\otimes O_2\right)+2l_5\Delta,\\
l_1=-\frac{\nu_{12}}{E_1},\ l_2=\frac{1+2\nu_{31}}{E_3}-\frac{\nu_{12}}{E_1},\ l_3=\frac{\nu_{12}}{E_1}-\frac{\nu_{31}}{E_3},\ 2l_4=\frac{1}{2G_{13}}-\frac{1}{2G_{12}},\ 2l_5=\frac{1}{2G_{12}},
\end{gathered}
\tag{8}
$$

where Δ – unit tensor of rank 4, ${}^4\Pi$ – elastic compliance tensor, $E_1, E_3, \nu_{12}, \nu_{13}, G_{12}$ and G_{13} – elastic constants.

For the plastic strain tensor, we accept the associated plastic flow model [19], according to which

$$\dot{\varepsilon}_p = h\sum_{\beta=1}^{k}\dot{\aleph}_\beta\left(\partial f_\beta / \partial\sigma\right), \tag{9}$$

where

$$f_\beta = f_\beta(J_\gamma^{(s)}(\sigma,\varepsilon_p)) = 0, \quad \beta = 1,...,k, \tag{10}$$

plastic potentials depending for composites on joint invariants $J_\gamma^{(s)}(\sigma,\varepsilon_p)$, $\dot{\aleph}_\beta$ – loading parameters, h – indicator function that determines active plastic loading $(h=1)$ and unloading $(h=0)$. Equations (10) determine the position of the plasticity surface.

Then, substituting the derivative tensors (5) into (9), we obtain the following expression for the plastic strain rate tensor $\dot{\varepsilon}_p$:

$$\dot{\varepsilon}_p = \psi_1 E + (\psi_2 - \psi_1)\hat{c}_3^2 + \frac{1}{2}(O_1\otimes O_1 + O_2\otimes O_2)\cdot\cdot\left(\psi_3\sigma + \frac{\psi_{10}}{2}\varepsilon_p\right) + {}^4O_3\cdot\cdot\left(2\psi_4\sigma + \psi_{11}\varepsilon_p\right); \tag{11}$$

$$\psi_\alpha = h\sum_{\beta=1}^{k}\dot{\aleph}_\beta\left(\partial f_\beta / \partial J_\alpha^{(3)}\right). \tag{12}$$

For a transversely isotropic medium, we will assume that there are only 2 plastic potentials $(k=2)$ f_1 and f_2

$$f_1 = f_1\left(Y_{1H}, Y_{4H}\right), \qquad f_2 = f_2\left(Y_{2H}, Y_{3H}\right). \tag{13}$$

where

$$Y_{\alpha H} = I_\alpha^{(3)}\left(\sigma - H_\alpha\varepsilon_p\right), \quad \alpha = 1,...,4; \quad H_\alpha = H_\alpha^0\left(I_\alpha^{(3)}\left(\varepsilon_p\right)\right)^{n_\alpha^0}, \tag{14}$$

– joint invariants, where H_α^0, n_α^0 – constants. This model generalizes the well-known Huber-Mises model for isotropic media.

Calculating the derivatives of f_β and substituting them into (12), we obtain

$$\begin{gathered}\psi_1 = \dot{\aleph}_1 f_{11}h, \quad \psi_2 = \dot{\aleph}_2 f_{22}h, \quad \psi_3 = \dot{\aleph}_2 f_{23}h, \quad \psi_4 = \dot{\aleph}_1 f_{14}h, \\ \psi_{10} = -2\dot{\aleph}_2 f_{23}H_3 h, \quad \psi_{11} = -2\dot{\aleph}_1 f_{14}H_4 h.\end{gathered} \tag{15}$$

where $f_{\beta\alpha} = \partial f_\beta / \partial Y_\alpha^{(3)}$ .Then the constitutive relation (11) for plastic deformation takes the form

$$\dot{\varepsilon}_p = \dot{\aleph}_1 h P_{1H} + \dot{\aleph}_2 h P_{2H}, \tag{16}$$

where

$$\begin{aligned} P_{1H} &\equiv f_{11}\left(E - \hat{c}_3^2\right) + 2 f_{14}\, {}^4O_3 \cdot\cdot \left(\sigma - H_3 \varepsilon_p\right), \\ P_{2H} &\equiv f_{22}\hat{c}_3^2 + \frac{f_{23}}{2}\left(O_1 \otimes O_1 + O_2 \otimes O_2\right) \cdot\cdot \left(\sigma - H_4 \varepsilon_p\right). \end{aligned} \tag{17}$$

From (16) we obtain expressions for the loading parameters $\dot{\aleph}_1$ and $\dot{\aleph}_2$.

$$\dot{\aleph}_1 = \pm\sqrt{\frac{\dot{\varepsilon}_p \cdot\cdot P_{1H}}{P_{1H} \cdot\cdot P_{1H}}}, \qquad \dot{\aleph}_2 = \pm\sqrt{\frac{\dot{\varepsilon}_p \cdot\cdot P_{2H}}{P_{2H} \cdot\cdot P_{2H}}}. \tag{18}$$

Functions f_β are chosen in a quadratic form similar to the Mises model:

$$\begin{aligned} 2 f_1 &= \frac{Y_{4H}}{\sigma_{4s}^2} + \left(\frac{|Y_{1H}| + Y_{1H}}{2\sigma_{1s}^+}\right)^2 + \left(\frac{|Y_{1H}| - Y_{1H}}{2\sigma_{1s}^-}\right)^2 - 1, \\ 2 f_2 &= \left(\frac{|Y_{2H}| + Y_{2H}}{2\sigma_{2s}^+}\right)^2 + \left(\frac{|Y_{2H}| - Y_{2H}}{2\sigma_{2s}^-}\right)^2 + \frac{Y_{3H}}{\sigma_{3s}^2} - 1. \end{aligned} \tag{19}$$

The functions $\sigma_{1s}^\pm$ are called the yield strengths in longitudinal tension and compression, respectively, and σ_{4s} are the shear yield strengths in the plane of transversal isotropy. The functions $\sigma_{2s}^\pm$ are called the yield strength in transverse tension and compression, and σ_{3s} – the yield strength in interlayer shear. These functions are usually determined experimentally. For anisotropic media, the difference between the tensile and compressive yield strengths is usually quite significant, so the $\sigma_{\alpha s}^+$ and $\sigma_{\alpha s}^-$ functions can differ significantly [19].

Explicit formulas for invariants (14) have the form

$$Y_{1H} = \left(\sigma_{11} - H_1^0\left(I_1\left(\varepsilon_p\right)\right)^{n_1^0} \varepsilon_{11}^p\right) + \left(\sigma_{22} - H_1^0\left(I_1\left(\varepsilon_p\right)\right)^{n_1^0} \varepsilon_{22}^p\right), \quad I_1\left(\varepsilon_p\right) = \varepsilon_{11}^p + \varepsilon_{22}^p, \tag{20}$$

$$Y_{2H} = \left(\sigma_{33} - H_2^0\left(I_2\left(\varepsilon_p\right)\right)^{n_2^0} \varepsilon_{33}^p\right), \quad I_2\left(\varepsilon_p\right) = \varepsilon_{33}^p, \tag{21}$$

$$Y_{3H} = \left(\sigma_{13} - H_3^0\left(I_3\left(\varepsilon_p\right)\right)^{n_3^0} \varepsilon_{13}^p\right)^2 + \left(\sigma_{23} - H_3^0\left(I_3\left(\varepsilon_p\right)\right)^{n_3^0} \varepsilon_{23}^p\right)^2, \; I_3\left(\varepsilon_p\right) = 2\left(\varepsilon_{13}^p\right)^2 + 2\left(\varepsilon_{23}^p\right)^2, \tag{23}$$

$$Y_{4H} = \left(\sigma_{11} - H_4^0\left(I_4\left(\varepsilon_p\right)\right)^{n_1^0} \varepsilon_{11}^p\right)^2 + \left(\sigma_{22} - H_4^0\left(I_4\left(\varepsilon_p\right)\right)^{n_2^0} \varepsilon_{22}^p\right)^2 + 4\left(\sigma_{12} - H_4^0\left(I_4\left(\varepsilon_p\right)\right)^{n_4^0} \varepsilon_{12}^p\right)^2, \tag{24}$$
$$I_4\left(\varepsilon_p\right) = \left(\varepsilon_{11}^p\right)^2 + \left(\varepsilon_{22}^p\right)^2 - 4\left(\varepsilon_{12}^p\right)^2.$$

3 Method for identification of the model parameters for transversally isotropic composites

Using the method of asymptotic averaging for a linearly elastic layered composite material, it is possible to construct constitutive relations based on those for individual layers. However, these relations do not have an explicit analytical expression; they are calculated in the form of a numerical algorithm for solving a local problem in PC. This method is very accurate from a mathematical point of view, but leads to high costs for calculating.

Let us consider another method for constructing deformation diagrams, when the constitutive relations are given in the form of explicit analytical relations (9), and the constants $E_3\ \nu_{12}\ \nu_{13}\ G_{12}\ G_{13}, H_2^0, H_3^0, H_4^0,$ and $n_2^0, n_3^0, n_4^0, \sigma_{\alpha s}^{\pm}, \ \alpha = 2,...,4$ entering into these relations are found from the condition of the best approximation of the deformation curves $\bar{\sigma}_{ij} = F_{ij}\left(\bar{\varepsilon}_{mn}\right)$ obtained by direct numerical solution of the problem in PC for some standard problems of macroscopic deformation, in which a homogeneous stress-strain state with $\bar{\sigma}_{ij}$ and $\bar{\varepsilon}_{ij}$ independent of coordinates is realized. A numerical method for solving a local problem in PL for a linearly elastic layered composite was implemented in [13].

As standard problems of macroscopic deformation, we consider the class of problems of uniaxial tension-compression of a plate in the form of a parallelepiped, the faces of which are parallel to the coordinate axes Oe_α. The formulation of these problems can be represented as a system with different boundary conditions

$$\begin{cases} \bar{\sigma}_{ij,j} = 0, & \in V \\ \bar{\sigma}_{ij} = C_{ijkl}\bar{\varepsilon}_{kl}^e, & \in V \cup \Sigma \\ \bar{\varepsilon}_{kl}^e = \bar{\varepsilon}_{kl} - \bar{\varepsilon}_{kl}^p, & \in V \cup \Sigma \\ \dot{\varepsilon}_{kl}^p = \dot{\aleph}_1 hP_{1H,kl} + \dot{\aleph}_2 hP_{2H,kl}, & \\ 2\bar{\varepsilon}_{ij} = \bar{u}_{i,j} + \bar{u}_{j,i}, & \end{cases} \tag{25}$$

where C_{ijkl} – components of the elastic modulus tensor, inverse to the elastic compliance tensor ${}^4\Pi$.

4 Uniaxial transverse tension-compression of the composite

Consider the case of tension-compression of the plate along the axis Oe_3. Under such loading $\bar{\sigma}_{11} = 0, \ \bar{\sigma}_{22} = 0, \ \bar{\sigma}_{33} \neq 0$. Then we look for the solution of this problem in the following form:

$$\bar{\sigma}_{33} = \bar{\sigma}_{33}^0 = const, \tag{26}$$

where $\forall \vec{x} \in V$ – uniaxial stress-strain state,

$$\bar{\varepsilon}_{11} = \bar{\varepsilon}_{22} \neq 0,\ \bar{\varepsilon}_{33} \neq 0,\ \text{other}\ \ \bar{\varepsilon}_{12} = \bar{\varepsilon}_{13} = \bar{\varepsilon}_{23} = 0. \tag{27}$$

Let us express $\bar{\varepsilon}^{e}_{33}$ in terms of $\bar{\sigma}_{33}$ using the system (25)

$$\bar{\varepsilon}^{e}_{33} = \frac{\bar{\sigma}_{33}}{E_3}\ . \tag{28}$$

Let us now find the dependence of the plastic strain tensor $\bar{\varepsilon}^{p}_{33}$ on the stress tensor $\bar{\sigma}_{33}$ according to the Huber-Mises model (18), in which the cases of plastic tension and plastic compression should be considered separately.

$$\bar{\varepsilon}^{p}_{33} = \begin{cases} 0, & \text{if } -\sigma^{-}_{2s} < \bar{\sigma}_{33} < \sigma^{+}_{2s}, \\ \left(\dfrac{\bar{\sigma}_{33} - \sigma^{+}_{2s}}{H^{0}_{2}}\right)^{\frac{1}{n^{0}_{2}+1}}, & \text{if } \bar{\sigma}_{33} \geq \sigma^{+}_{2s}, \\ -\left(\dfrac{-\bar{\sigma}_{33} - \sigma^{-}_{2s}}{H^{0}_{2}}\right)^{\frac{1}{n^{0}_{2}+1}}, & \text{if } \bar{\sigma}_{33} \leq -\sigma^{-}_{2s}. \end{cases} \tag{29}$$

From (28) and (29) we find the relationship between $\bar{\varepsilon}_{33}$ and $\bar{\sigma}_{33}$ by the formula $\bar{\varepsilon}_{33} = \bar{\varepsilon}^{e}_{33} + \bar{\varepsilon}^{p}_{33}$.

There are five unknown constants in this dependence: E_3, H^{0}_{2}, n^{0}_{2}, σ^{+}_{2s}, σ^{-}_{2s}. Their search is carried out by comparison with the experimental deformation diagram $\bar{\sigma}^{(э)}_{33}(\bar{\varepsilon}^{(э)}_{33})$, obtained as a result of solving local problems by the method of asymptotic homogenization, arising during transverse uniaxial tension-compression of a layered composite material.

5 Composite interlayer shear

Let us consider the case of interlaminar shift of the plate between the axes Oe_1 and Oe_3. Under such loading $\bar{\sigma}_{13} \neq 0$, the rest $\bar{\sigma}_{ij} = 0$. Then we look for the solution of this problem in the following form:

$$\bar{\sigma}_{13} = \bar{\sigma}^{0}_{13} = const, \tag{30}$$

where $\forall \vec{x} \in V$ – uniaxial stress-strain state,

$$\bar{\varepsilon}_{13} \neq 0,\ \text{other}\ \ \bar{\varepsilon}_{ij} = 0. \tag{31}$$

Dependence of the elastic strain tensor $\bar{\varepsilon}^{e}_{13}$ on the stress tensor $\bar{\sigma}_{13}$:

$$\bar{\varepsilon}_{13}^{e}=\frac{\bar{\sigma}_{13}}{2G_{13}}. \tag{32}$$

From the plasticity function $f_2\left(Y_{2H}, Y_{3H}\right)$, we express the plastic strain tensor $\bar{\varepsilon}_{13}^{p}$ in terms of the stress tensor $\bar{\sigma}_{13}$

$$\bar{\varepsilon}_{13}^{p}=\left(\frac{\bar{\sigma}_{13}-\sigma_{3s}}{H_3^0}\right)^{\frac{1}{2n_3^0+1}}. \tag{33}$$

There are only four unknown constants in this dependence: G_{13}, n_3^0, H_3^0, σ_{3s}. Their search is carried out by comparison with the experimental deformation diagram $\bar{\sigma}_{13}^{(э)}(\bar{\varepsilon}_{13}^{(э)})$, obtained as a result of solving local problems with the help of AH method, arising from the interlayer shear of the composite.

6 Shear in the plane of the composite layer

Let us consider the case of a shift in the plane of the plate between the axes Oe_1 and Oe_2. Under such loading $\sigma_{12}\neq 0$, the rest $\sigma_{ij}=0$. Then we look for the solution of this problem in the following form:

$$\bar{\sigma}_{12}=\bar{\sigma}_{12}^{0}=const, \tag{34}$$

where $\forall\vec{x}\in V$ – uniaxial stress-strain state,

$$\bar{\varepsilon}_{12}\neq 0,\ \text{other}\ \ \bar{\varepsilon}_{ij}=0. \tag{35}$$

The elastic strain tensor $\bar{\varepsilon}_{12}^{e}$ can be expressed in terms of the stress tensor $\bar{\sigma}_{12}$ by the formula

$$\bar{\varepsilon}_{12}^{e}=\frac{\bar{\sigma}_{12}}{2G_{12}}. \tag{36}$$

Expressing the plastic strain tensor $\bar{\varepsilon}_{12}^{p}$ in terms of the stress tensor $\bar{\sigma}_{12}$ according to the Huber-Mises model, we obtain

$$\bar{\varepsilon}_{12}^{p}=\left(\frac{\bar{\sigma}_{12}-\sigma_{4s}/2}{H_4^0\left(4\right)^{n_4^0}}\right)^{\frac{1}{2n_4^0+1}}. \tag{37}$$

There are only four unknown constants in this dependence: G_{12}, n_4^0, H_4^0, σ_{4s}. Their search is carried out by comparison with the experimental deformation diagram $\bar{\sigma}_{12}^{(э)}(\bar{\varepsilon}_{12}^{(э)})$, obtained as a result of solving local problems with the help of AH method, arising from shear in the plane of the composite layer.

7 Formulation of the problem of searching for model parameters

Let us now combine the obtained solutions of four problems: (29), (33) and (37). From the solution of the corresponding local problems with the help of AH method, we have several curves. For each specific set of constants E_3, G_{12}, G_{13}, H_2^0, H_3^0, H_4^0 and $n_2^0, n_3^0, n_4^0, \sigma_{\alpha s}^{\pm}$, $\alpha = 2,...,4$ it is possible to construct curves for a transversally isotropic material, which will be called theoretical.

The unknown parameters E_3, G_{12}, G_{13}, H_2^0, H_3^0, H_4^0, $n_2^0, n_3^0, n_4^0, \sigma_{\alpha s}^{\pm}$, $\alpha = 2,...,4$, can be calculated by fitting experimental tensile and compressive strain curves. To do this, the problem of minimizing the functional of the standard deviation of the experimental and theoretical curves at N points is solved:

$$\Delta = \left(\frac{1}{N} \sum_{i=1}^{N} \sum_{\alpha,\beta} \left| 1 - \frac{\bar{\sigma}_{\alpha\beta}}{\bar{\sigma}_{\alpha\beta}^{(э)}} \right| \right)^{\frac{1}{2}} \to \min. \quad (38)$$

To solve minimization problems (38), the Hooke-Jeeves method was used.

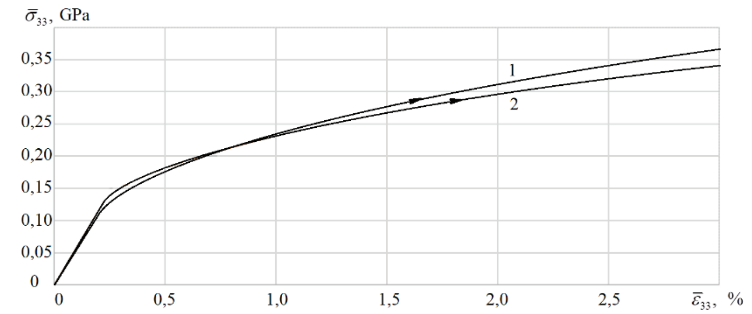
8 Results of numerical simulation of strain diagram of laminated composite

With the help of the developed algorithm, averaged strain diagrams $\bar{\sigma}_{ij} = F_{ij}\left(\bar{\varepsilon}_{mn}\right)$ of a layered composite were calculated and built under uniaxial loading in different directions for tension and compression. Also, using the problem of minimizing the functional (38), the constants E_3, G_{12}, G_{13}, H_2^0, H_3^0, H_4^0 and $n_2^0, n_3^0, n_4^0, \sigma_{\alpha s}^{\pm}$, $\alpha = 2,...,4$ were found. PC consisted of two layers – steel and aluminum, with a ratio of their relative thicknesses $h_S = 0,5$ and $h_A = 1 - h_S$. The composite layers were considered isotropic.

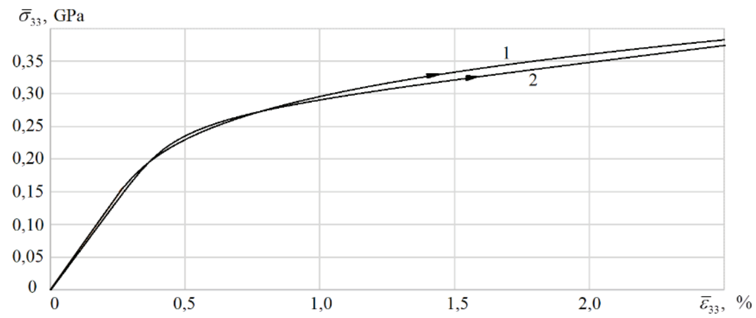
The strain diagrams are calculated for uniaxial loading in the transverse direction Ox_3, with interlaminar shear in the plane Ox_1x_3, and with shear in the plane of the layer Ox_1x_2.

Experimental and calculated strain diagrams $\bar{\sigma}_{33} \sim \bar{\varepsilon}_{33}$ for the composite under tension and compression are shown in Figs. 1, and the values of the optimal constants E_3, H_2^0, n_2^0, σ_{2s}^{+}, σ_{2s}^{-} are in Table 1. The calculation was carried out according to formula (29), and the unknown constants were preliminarily determined from the strain curve.

On Fig. 2 shows diagrams of deformation during interlaminar shear ($\bar{\sigma}_{13} \sim \bar{\varepsilon}_{13}$), the values for their constants G_{13}, n_3^0, H_3^0, σ_{3s} are given in Table 2; and in Fig. 3 – at a shift in the plane of the layer ($\bar{\sigma}_{12} \sim \bar{\varepsilon}_{12}$), and their unknown constants G_{12}, n_4^0, H_4^0, σ_{4s} – in Table 3.



a



b

Fig. 1. Diagrams of deformation $\bar{\sigma}_{33} \sim \bar{\varepsilon}_{33}$ of the composite under uniaxial loading in the transverse direction: 1 – experimental; 2 – calculated, a –tension; b – compression.

Table 1. Values of the constants of composite models in uniaxial tension and compression in the transverse direction.

Type of loading	Tension	Compression
E_3, GPa	0.58	0.58
H_2^0, GPa	0.27	0.25
$-n_2^0$	0.71	0.63
σ_{2s}^{+}, MPa	16.1	–
σ_{2s}^{-}, MPa	–	11.3

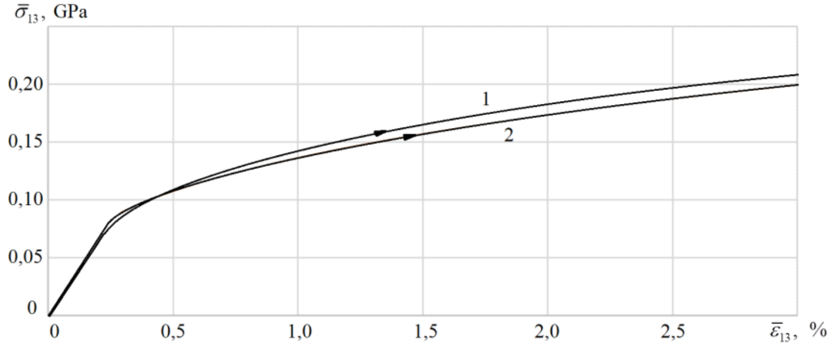


Fig. 2. Diagrams of deformation $\bar{\sigma}_{13} \sim \bar{\varepsilon}_{13}$ of the composite during interlayer shear: 1 – experimental; 2 – calculated.

Table 2. Values of constants of the composite model at interlayer shear.

Type of loading	Interlayer shear
G_{13}, GPa	0.165
H_3^0, GPa	0.19
$-n_3^0$	0.39
σ_{3s}, MPa	40

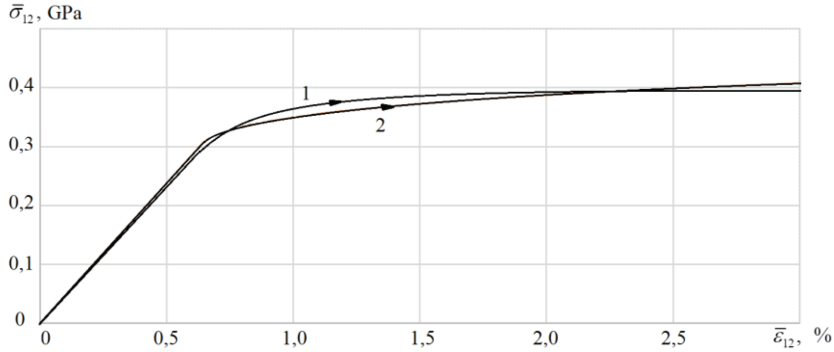


Fig. 3. Diagrams of deformation $\bar{\sigma}_{12} \sim \bar{\varepsilon}_{12}$ of the composite during shear in the plane of the layer: 1 – experimental; 2 – calculated.

Table 3. Values of the constants of the composite model during shear in the plane of the layer.

Type of loading	Shear in the plane of the layer
G_{12}, GPa	0.48
H_4^0, GPa	0.62
$-n_4^0$	0.45
σ_{4s}, MPa	5.86

The maximum relative error among the obtained diagrams is 14%, which indicates a quite satisfactory quality of the model of an effective transversally isotropic medium.

9 Conclusions

A model of an effective transversally isotropic linear elastic medium with small deformations, which belongs to the class of universal models, is proposed. The model is applied to deformation diagrams of layered composite materials with small deformations and a periodic structure using a universal representation of constitutive relations for composite layers.

A method for finding the effective constants of the composite model by solving the problem of minimizing the functional of the standard deviation of the experimental deformation diagrams obtained by numerically solving problems on the PC and theoretical deformation diagrams obtained by approximation is proposed.

Numerical modeling of deformation diagrams of layered composite materials with small deformations was performed using the AH method, which showed fairly good results.

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