Finite element modeling of thin-walled shell composite structures

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Abstract. An algorithm for solving the problem of elasticity theory for composite thin-walled shell composite structures with junction lines is proposed. The SMCM software was created based on the finite element method and the developed algorithm. A test problem was solved for a composite shell structural element of the box-plate type docked with 4 other plates. A comparative analysis of the obtained results was carried out with the reference solution of the three-dimensional elasticity problem in the ANSYS software, as well as with the two-dimensional shell solution of the ANSYS software. SMCM PC allows obtaining results that are closer to a three-dimensional solution. **Key words:** modeling of stresses, composite materials, shell, finite element method.

1 Introduction

Methods of two-dimensional theories of plates and shells are often used to calculate engineering thin-walled structures [1-14], which allow to reduce the dimension of the problems from 3D to 2D problems, which, require significantly lower characteristics for the computer technology used. However, when solving problems in the theory of thin-walled plates and shells, additional difficulties arise, in particular, the problem of the accuracy of approximating the solution of the problem in terms of the thickness of the shell [5], the influence of the finite element type on the solution, as well as the problem of correct conjugation of the solution in the junction zone of various shell structural elements In this paper, a numerical algorithm for solving the problem of elasticity theory for shell composite structures, including complex structures, is proposed. The paper compares the results of calculations obtained on the basis of the shell theory in the ANSYS software package and in the SMCM software package developed at the Bauman Moscow State Technical University Scientific Educational center for Supercomputer modeling and software engineering.

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2 Mathematical formulation of a 3-dimensional problem of the theory of elasticity for a composite structure

Consider the problem of linear elasticity in a limited area Ω with Lipschitz boundary $\partial\Omega = \Sigma_u \cup \Sigma_\sigma$ [15,16]:

$$\begin{gathered}
\nabla \cdot \boldsymbol{\sigma} = 0, \\
\boldsymbol{\sigma} = {}^4\mathbf{C}(\mathbf{x}) \cdot\cdot\, \boldsymbol{\varepsilon}, \\
\boldsymbol{\varepsilon} = \operatorname{def}(\mathbf{u}) = \frac{1}{2}(\nabla \otimes \mathbf{u} + \nabla \otimes \mathbf{u}^{\mathrm{T}}), \\
\mathbf{n} \cdot [\boldsymbol{\sigma}]\big|_{\Sigma_l} = 0, [\mathbf{u}]\big|_{\Sigma_l} = 0, \\
\boldsymbol{\sigma} \cdot \mathbf{n}\big|_{\Sigma^\pm} = -p_\pm \mathbf{n}, \\
\mathbf{u}\big|_{\Sigma^T} = \mathbf{u}_e,
\end{gathered} \tag{1}$$

$\boldsymbol{\sigma}$ – stress tensor; $\boldsymbol{\varepsilon}$ – small strain tensor; $\mathbf{u}$ – displacement vector; ${}^4\mathbf{C}(\mathbf{x})$ – variable symmetric positive-definite tensor field of elastic moduli (fourth rank); ∇ – nabla operator [17]; $\mathbf{n}$ – area normal vector; $p_\pm$ – set pressure on various surfaces $\Sigma^\pm$ construction, $\mathbf{u}_e$ – displacement vector.

For the finite element solution of the problem (1) consider a weak solution to this problem. Let $Y = \left[H^1(\Omega)\right]^3$ and $V_{\Sigma^T}(Y) = \left\{\mathbf{w} \in Y : Tr_{\Sigma^T}(\mathbf{w}) = 0\right\}$ $\boldsymbol{\omega}_e \in Y$ which vector, what $Tr_{\Sigma^T}(\boldsymbol{\omega}_e) = \mathbf{u}_e$ $p_\pm \in L_2(\Sigma_0)$. Weak problem solution (1) such a vector is called $\mathbf{u} \in Y$, what if $\boldsymbol{\omega} \in Y$ – such a vector that $\mathbf{u} - \boldsymbol{\omega} \in V_{\Sigma^T}(Y)$ anf $\forall \mathbf{w} \in V_{\Sigma^T}(Y)$ the variational equation for the problem is satisfied (2):

$$\int\limits_{\Omega} \operatorname{def}(\mathbf{w}) \cdot\cdot\, \boldsymbol{\sigma}(\mathbf{u})\, d\Omega = \int\limits_{\Sigma_\sigma} \mathbf{w} \cdot \mathbf{t}_n d\Sigma, \tag{2}$$

where $\boldsymbol{\sigma}(\mathbf{u}) = {}^4\mathbf{C} \cdot\cdot \operatorname{def}(\mathbf{u})$, $\mathbf{t}_n = \boldsymbol{\sigma} \cdot \mathbf{n}_{\Sigma_\sigma}$ and $\Sigma_\sigma = \Sigma^+ \cup \Sigma^-$.

3 Derivation of the variational formulation of the problem for a thin-walled shell

Consider the problem (2) for thin-walled complex shell:

$$\Omega = \bigcup_j^N \Omega_j, \tag{3}$$

where for which shell Ω_j we introduce orthogonal (curvilinear) coordinates X^i, in which this body is some neighborhood of a two-dimensional surface Σ_0:

$$\Omega_j = \{x = \boldsymbol{\rho}(X^1, X^2) + X^3\mathbf{n}(X^1, X^2) : (X^1, X^2) \in \Sigma_0,$$
$$X^3 \in \left[-\frac{h}{2}, \frac{h}{2}\right], \tag{4}$$

where $\boldsymbol{\rho}(X^1, X^2)$ – radius vector of points on the midsurface, $\mathbf{n}(X^1, X^2)$ – normal vector to midsurface, $\mathbf{x}$ – radius vector of an arbitrary point of the area.

We introduce for the local basis vectors:

$$\mathbf{r}_I = \partial_I \mathbf{x} = \partial_I \boldsymbol{\rho} + X^3 \partial_I \mathbf{n} = \boldsymbol{\rho}_I + X^3 \mathbf{n}, \mathbf{r}_3 = \partial_3 \mathbf{x} = \mathbf{n},$$
$$\partial_i = \frac{\partial}{\partial X^i}, \quad \boldsymbol{\rho}_I = \partial_I \boldsymbol{\rho}, \quad \mathbf{n} = \partial_I \mathbf{n}, \qquad i, j, k, l, \ldots \in \{1,2,3\}, \qquad I, J, K, L, \ldots \in \{1,2\}. \tag{5}$$

We will assume that the following assumptions are satisfied, which are usually accepted in shell theories of the Timoshenko type [1,10]:

1) Terms of relations having order $O\left(h^k\right)$ $k > 1$ negligible.
2) Instead of space $Y = \left[H^1(\Omega)\right]^3$ considered $\overset{\circ}{Y}$ space of vector functions:

$$\mathbf{u}\left(X^i\right) = \mathbf{U}\left(X^I\right) + X^3 \boldsymbol{\gamma}\left(X^I\right), \boldsymbol{\gamma} \cdot \mathbf{n} = 0,$$
$$\mathbf{U}, \boldsymbol{\gamma} \in \left[H^1(\Sigma_0)\right]^3, X^3 \in \left[-\frac{h}{2}, \frac{h}{2}\right]. \tag{6}$$

3) Normal deformations are negligible:

$$\varepsilon_{33} = \mathbf{r}_3 \cdot \boldsymbol{\varepsilon} \cdot \mathbf{r}_3 = \mathbf{n} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{n} = 0 \,. \tag{7}$$

Expressions for the covariant components of the strain tensor following from the relations (6) and (7) $(\nabla = \mathbf{r}^i \partial_i)$:

$$2\varepsilon_{IJ}(\mathbf{u}) = 2\mathbf{r}_I \cdot \boldsymbol{\varepsilon} \cdot \mathbf{r}_J = \partial_I \mathbf{u} \cdot \mathbf{r}_J + \partial_J \mathbf{u} \cdot \mathbf{r}_I = \partial_I \mathbf{U} \cdot \boldsymbol{\rho}_J + \partial_J \mathbf{U} \cdot \boldsymbol{\rho}_I +$$
$$+ X^3\left(\partial_I \boldsymbol{\gamma} \cdot \boldsymbol{\rho}_J + \partial_J \boldsymbol{\gamma} \cdot \boldsymbol{\rho}_I + \partial_I \mathbf{U} \cdot \mathbf{n}_J + \partial_J \mathbf{U} \cdot \mathbf{n}_I\right) + O\left(h^2\right) = 2e_{IJ}(\mathbf{u}) + 2X^3 \kappa_{IJ}(\mathbf{u}) + O\left(h^2\right), \tag{8}$$
$$2\varepsilon_{I3}(\mathbf{u}) = 2\mathbf{r}_I \cdot \boldsymbol{\varepsilon} \cdot \mathbf{r}_3 = \partial_I \mathbf{u} \cdot \mathbf{n} + \partial_3 \mathbf{u} \cdot \mathbf{r}_I = \boldsymbol{\gamma} \cdot \boldsymbol{\rho}_I + \mathbf{n} \cdot \partial_I \mathbf{U},$$

where

$$2e_{IJ}(\mathbf{u}) = \partial_I \mathbf{U} \cdot \boldsymbol{\rho}_J + \partial_J \mathbf{U} \cdot \boldsymbol{\rho}_I,$$
$$2\kappa_{IJ}(\mathbf{u}) = \partial_I \boldsymbol{\gamma} \cdot \boldsymbol{\rho}_J + \partial_J \boldsymbol{\gamma} \cdot \boldsymbol{\rho}_I + \partial_I \mathbf{U} \cdot \mathbf{n}_J + \partial_J \mathbf{U} \cdot \mathbf{n}_I. \tag{9}$$

Then the left side of the variational equation (2) can be represented as:.

$$\int_{\Omega} \operatorname{def}(\mathbf{w}) \cdot\cdot \boldsymbol{\sigma}(\mathbf{u})\, d\Omega = \int_{\Sigma} \int_{-h/2}^{-h/2} \varepsilon_{ij}(\mathbf{w}) \sigma^{ij}(\mathbf{u}) dX^3 d\Sigma = \int_{\Sigma} \int_{-h/2}^{-h/2} \varepsilon_{ij}(\mathbf{w}) \sigma^{ij}(\mathbf{u}) dX^3 d\Sigma \cdot \tag{10}$$

The integrand, taking into account (8) and (9), can be written as:

$$\varepsilon_{ij}(\mathbf{w}) \sigma^{ij}(\mathbf{u}) = e_{IJ}(\mathbf{w}) \sigma^{IJ}(\mathbf{u}) + X^3 \kappa_{IJ}(\mathbf{w}) \sigma^{IJ}(\mathbf{u}) + 2\varepsilon_{I3}(\mathbf{w}) \sigma^{I3}(\mathbf{u}). \tag{11}$$

Then (10) takes the form:

$$\int_{\Omega} \operatorname{def}(\mathbf{w}) \cdot\cdot \boldsymbol{\sigma}(\mathbf{u})\, d\Omega = \int_{\Sigma} e_{IJ}(\mathbf{w}) T^{IJ}(\mathbf{u}) + \kappa_{IJ}(\mathbf{w}) M^{IJ}(\mathbf{u}) + 2\varepsilon_{I3}(\mathbf{w}) Q^{I}(\mathbf{u})\, d\Sigma, \tag{12}$$

where the notation for forces, moments and shearing forces is introduced:

$$T^{IJ}(\mathbf{u}) = \int_{-h/2}^{-h/2} \sigma^{IJ}(\mathbf{u}) dX^3, \; M^{IJ}(\mathbf{u}) = \int_{-h/2}^{-h/2} \sigma^{IJ}(\mathbf{u}) X^3 dX^3, \; Q^{I}(\mathbf{u}) = \int_{-h/2}^{-h/2} \sigma^{I3}(\mathbf{u}) dX^3 \cdot \tag{13}$$

Functions $T^{IJ}(\mathbf{u})$, $M^{IJ}(\mathbf{u})$, $Q^{I}(\mathbf{u})$ taking into account the introduced assumptions, can be represented as:

$$\begin{aligned} T^{IJ}(\mathbf{u}) &= \int_{-h/2}^{-h/2} C^{IJkl} \varepsilon_{kl} dX^3 = \int_{-h/2}^{-h/2} C^{IJKL} \left(e_{KL}(\mathbf{u}) + X^3 \kappa_{KL}(\mathbf{u})\right) + 2C^{IJK3} \varepsilon_{K3}(\mathbf{u}) dX^3 = \\ &= \bar{C}^{IJKL} e_{KL}(\mathbf{u}) + B^{IJKL} \kappa_{KL}(\mathbf{u}) + 2\bar{C}^{IJK3} \varepsilon_{K3}(\mathbf{u}), \end{aligned}$$

$$M^{IJ}(\mathbf{u}) = B^{IJKL} e_{KL}(\mathbf{u}) + D^{IJKL} \kappa_{KL}(\mathbf{u}) + 2B^{IJK3} \varepsilon_{K3}(\mathbf{u}),$$

$$Q^{I}(\mathbf{u}) = \bar{C}^{I3KL} e_{KL}(\mathbf{u}) + B^{I3K3} \varepsilon_{K3}(\mathbf{u}), \tag{14}$$

where

$$\bar{C}^{IJkl} = \int_{-h/2}^{-h/2} C^{IJkl} dX^3, \; B^{IJkl} = \int_{-h/2}^{-h/2} C^{IJkl} X^3 dX^3, \; D^{IJKL} = \int_{-h/2}^{-h/2} C^{IJKL} \left(X^3\right)^2 dX^3 \cdot \tag{15}$$

The right side in (2) takes the following form ($\mathbf{w}$ such part of $\mathring{Y}$ represent in the form $\mathbf{w} = \mathbf{W} + X^3 \boldsymbol{\theta}$, $\boldsymbol{\theta} \cdot \mathbf{n} = 0$, $\mathbf{W}, \boldsymbol{\theta} \in \left[H^1(\Sigma_0)\right]^3$):

$$\begin{aligned} \int_{\Sigma_\sigma} \mathbf{w} \cdot \mathbf{t}_n d\Sigma &= -\int_{\Sigma^+} (\mathbf{W} + h/2\boldsymbol{\theta}) \cdot \mathbf{n} p_+ d\Sigma - \int_{\Sigma^-} (\mathbf{W} - h/2\boldsymbol{\theta}) \cdot \mathbf{n} p_+ d\Sigma = \\ &= -\int_{\Sigma} \mathbf{W} \cdot \mathbf{n} \Delta p d\Sigma = -\int_{\Sigma} \mathbf{w} \cdot \mathbf{n} \Delta p d\Sigma, \end{aligned} \tag{16}$$

where $\Delta p = p_+ - p_-$.

Thus, the variational equation can be written as:

$$\int_\Sigma e_{IJ}(\mathbf{w}) T^{IJ}(\mathbf{u}) + \kappa_{IJ}(\mathbf{w}) M^{IJ}(\mathbf{u}) + 2\varepsilon_{I3}(\mathbf{w}) Q^I(\mathbf{u}) d\Sigma = -\int_\Sigma \mathbf{W}\cdot\mathbf{n}\Delta p d\Sigma . \quad (17)$$

Introducing the notation for the column of generalized deformations:

$$\varepsilon(\mathbf{u}) = \begin{pmatrix} e_{11}(\mathbf{u}) & e_{22}(\mathbf{u}) & 2e_{12}(\mathbf{u}) & \kappa_{11}(\mathbf{u}) & \kappa_{22}(\mathbf{u}) & 2\kappa_{12}(\mathbf{u}) & 2\varepsilon_{13}(\mathbf{u}) & 2\varepsilon_{23}(\mathbf{u}) \end{pmatrix}^T,$$

variational equation (17), taking into account relations (14), is rewritten in the form:

$$\int_\Sigma \varepsilon^T(\mathbf{w}) C \varepsilon(\mathbf{u}) d\Sigma = -\int_\Sigma \mathbf{w}\cdot\mathbf{n}\Delta p d\Sigma , \quad (18)$$

where the matrix has a block form:

$$C = \begin{pmatrix} \bar{C} & B & \bar{C}_{ts} \\ B & D & B_{ts} \\ \bar{C}_{ts}^T & B_{ts}^T & B_s \end{pmatrix}, \bar{C} = \begin{pmatrix} \bar{C}^{1111} & \bar{C}^{1122} & \bar{C}^{1112} \\ \bar{C}^{1122} & \bar{C}^{2222} & \bar{C}^{2212} \\ \bar{C}^{1112} & \bar{C}^{2212} & \bar{C}^{1212} \end{pmatrix}, B = \begin{pmatrix} B^{1111} & B^{1122} & B^{1112} \\ B^{1122} & B^{2222} & B^{2212} \\ B^{1112} & B^{2212} & B^{1212} \end{pmatrix}$$

$$, \quad D = \begin{pmatrix} D^{1111} & D^{1122} & D^{1112} \\ D^{1122} & D^{2222} & D^{2212} \\ D^{1112} & D^{2212} & D^{1212} \end{pmatrix}, \quad \bar{C}_{ts} = \begin{pmatrix} \bar{C}^{1113} & \bar{C}^{1123} \\ \bar{C}^{2213} & \bar{C}^{2223} \\ \bar{C}^{1213} & \bar{C}^{1223} \end{pmatrix}, \quad B_{ts} = \begin{pmatrix} B^{1113} & B^{1123} \\ B^{2213} & B^{2223} \\ B^{1213} & B^{1223} \end{pmatrix},$$

$$B_s = \begin{pmatrix} B^{1313} & B^{1323} \\ B^{1323} & B^{2323} \end{pmatrix}.$$

4 Algorithm for the numerical solution of the problem for a thin-walled shell

We introduce a rectangular Cartesian coordinate system Ox^i, with basis vectors $\mathbf{e}_i$. Then each element $\mathbf{u} = \mathbf{U} + X^3\boldsymbol{\gamma} \in \mathring{Y}$ can be uniquely assigned to a column $u \in \left[H^1(\Sigma_0)\right]^6$:

$$u = \begin{pmatrix} U^1 & U^2 & U^3 & \gamma^1 & \gamma^2 & \gamma^3 \end{pmatrix}^T, \ \mathbf{U} = U^i\mathbf{e}_i, \ \boldsymbol{\gamma} = \gamma^i\mathbf{e}_i, \ \gamma^i n_i = 0 .$$

The column of generalized deformations can be represented as:

$$\varepsilon(u) = \varepsilon(\mathbf{u}) = Lu ,$$

The operator L looks like:

$$L=\begin{bmatrix} \mathrm{P}_{11}^1 & \mathrm{P}_{11}^2 & \mathrm{P}_{11}^3 & 0 & 0 & 0 \\ \mathrm{P}_{22}^1 & \mathrm{P}_{22}^2 & \mathrm{P}_{22}^3 & 0 & 0 & 0 \\ \mathrm{P}_{12}^1+\mathrm{P}_{21}^1 & \mathrm{P}_{12}^2+\mathrm{P}_{21}^2 & \mathrm{P}_{12}^3+\mathrm{P}_{21}^3 & 0 & 0 & 0 \\ \mathrm{N}_{11}^1 & \mathrm{N}_{11}^2 & \mathrm{N}_{11}^3 & \mathrm{P}_{11}^1 & \mathrm{P}_{11}^2 & \mathrm{P}_{11}^3 \\ \mathrm{N}_{22}^1 & \mathrm{N}_{22}^2 & \mathrm{N}_{22}^3 & \mathrm{P}_{22}^1 & \mathrm{P}_{22}^2 & \mathrm{P}_{22}^3 \\ \mathrm{N}_{12}^1+\mathrm{N}_{21}^1 & \mathrm{N}_{12}^2+\mathrm{N}_{21}^2 & \mathrm{N}_{12}^3+\mathrm{N}_{21}^3 & \mathrm{P}_{12}^1+\mathrm{P}_{21}^1 & \mathrm{P}_{12}^2+\mathrm{P}_{21}^2 & \mathrm{P}_{12}^3+\mathrm{P}_{21}^3 \\ \mathrm{N}_1^1 & \mathrm{N}_1^2 & \mathrm{N}_1^3 & \rho_1^1 & \rho_1^2 & \rho_1^3 \\ \mathrm{N}_2^1 & \mathrm{N}_2^2 & \mathrm{N}_2^3 & \rho_2^1 & \rho_2^2 & \rho_2^3 \end{bmatrix},$$

where

$$\mathrm{P}_{IJ}^k=\rho_I^k\frac{\partial}{\partial X^J},\ \mathrm{N}_{IJ}^k=n_I^k\frac{\partial}{\partial X^J},\ \mathrm{N}_I^k=n^k\frac{\partial}{\partial X^I},\ \boldsymbol{\rho}_I=\rho_I^k\mathbf{e}_k,\ \mathbf{n}_I=n_I^k\mathbf{e}_k.$$

Thus, we arrive at the formulation of the problem of determining a weak solution for the theory of Timoshenko shells $Y^1=\left[H^1\left(\Sigma_0\right)\right]^6$, $\omega\in Y^1$ – such a vector function that $Tr_\Sigma\left(\omega\right)=u_e=\left(U_e^1\quad U_e^2\quad U_e^3\quad \gamma_e^1\quad \gamma_e^2\quad \gamma_e^3\right)^T$, (where $\mathbf{u}_e=\mathbf{U}_e+X^3\boldsymbol{\gamma}_e\in\overset{\circ}{Y}$), and $\overset{\circ}{V}_{\partial\Sigma}\left(Y^1\right)=\left\{w\in Y^1:Tr_{\partial\Sigma}\left(w\right)=0,w^{3+i}n_i=0\right\}$.

Then a weak solution to the problem of finding the stress-strain state of the Timoshenko shell will be called such an element $u=\left(U^1\quad U^2\quad U^3\quad \gamma^1\quad \gamma^2\quad \gamma^3\right)^T\in\left[H^1\left(\Sigma_0\right)\right]^6$, $\gamma^i n_i=0$ that $u-w\in\overset{\circ}{V}_{\partial\Sigma}\left(Y^1\right)$ and $\forall w\in\overset{\circ}{V}_{\partial\Sigma}\left(Y^1\right)$ condition is met:

$$\int\limits_\Sigma\left(Lw\right)^T CLud\Sigma=-\int\limits_\Sigma w^T n\Delta pd\Sigma, \tag{19}$$

where

$$n=\left(n_1\quad n_2\quad n_3\quad 0\quad 0\quad 0\right)^T.$$

In this formulation of the problem, there is a somewhat inconvenient condition for the practical solution of such a problem on the elements $\overset{\circ}{V}_{\partial\Sigma}\left(Y^1\right)$: $\gamma^i n_i=0$. Instead of this condition, we introduce its weakened form:

$$\bar{C}^{1111}\int_{\Sigma}\left(u^{3+i}n_i\right)\left(w^{3+i}n_i\right)d\Sigma=0,\forall w\in V_{\Gamma}\left(Y^1\right)=\left\{w\in Y^1:Tr_{\Gamma}\left(w\right)=0\right\}.$$

Here, the coefficient $\bar{C}^{1111}$ is introduced for scaling in numerical calculations.

Then a weak solution to the problem of finding the stress-strain state of the Timoshenko shell will be called such an element $u=\begin{pmatrix}U^1 & U^2 & U^3 & \gamma^1 & \gamma^2 & \gamma^3\end{pmatrix}^T\in Y^1$, that $u-w\in V_{\Gamma}\left(Y^1\right)$ and $\forall w\in V_{\Gamma}\left(Y^1\right)$ condition is met:

$$\int_{\Sigma}\left(\tilde{L}w\right)^T\tilde{C}\tilde{L}ud\Sigma=-\int_{\Sigma}w^T n\Delta pd\Sigma. \tag{20}$$

Where $\tilde{L}$, $\tilde{C}$ – modified differential operator and elastic modulus matrix:

$$\tilde{L}=\begin{bmatrix}
\mathrm{P}_{11}^1 & \mathrm{P}_{11}^2 & \mathrm{P}_{11}^3 & 0 & 0 & 0\\
\mathrm{P}_{22}^1 & \mathrm{P}_{22}^2 & \mathrm{P}_{22}^3 & 0 & 0 & 0\\
\mathrm{P}_{12}^1+\mathrm{P}_{21}^1 & \mathrm{P}_{12}^2+\mathrm{P}_{21}^2 & \mathrm{P}_{12}^3+\mathrm{P}_{21}^3 & 0 & 0 & 0\\
\mathrm{N}_{11}^1 & \mathrm{N}_{11}^2 & \mathrm{N}_{11}^3 & \mathrm{P}_{11}^1 & \mathrm{P}_{11}^2 & \mathrm{P}_{11}^3\\
\mathrm{N}_{22}^1 & \mathrm{N}_{22}^2 & \mathrm{N}_{22}^3 & \mathrm{P}_{22}^1 & \mathrm{P}_{22}^2 & \mathrm{P}_{22}^3\\
\mathrm{N}_{12}^1+\mathrm{N}_{21}^1 & \mathrm{N}_{12}^2+\mathrm{N}_{21}^2 & \mathrm{N}_{12}^3+\mathrm{N}_{21}^3 & \mathrm{P}_{12}^1+\mathrm{P}_{21}^1 & \mathrm{P}_{12}^2+\mathrm{P}_{21}^2 & \mathrm{P}_{12}^3+\mathrm{P}_{21}^3\\
\mathrm{N}_{1}^1 & \mathrm{N}_{1}^2 & \mathrm{N}_{1}^3 & \rho_1^1 & \rho_1^2 & \rho_1^3\\
\mathrm{N}_{2}^1 & \mathrm{N}_{2}^2 & \mathrm{N}_{2}^3 & \rho_2^1 & \rho_2^2 & \rho_2^3\\
0 & 0 & 0 & n^1 & n^2 & n^3
\end{bmatrix}, \tag{21}$$

$$\tilde{C}=\begin{pmatrix}
\bar{C} & B & \bar{C}_{ts} & 0\\
B & D & B_{ts} & 0\\
\bar{C}_{ts}^T & B_{ts}^T & B_s & 0\\
0 & 0 & 0 & \bar{C}^{1111}
\end{pmatrix}.$$

To solve this problem based on the finite element method, it is more convenient to write using the variational equation of the Hellinger-Reisner variational principle, which has the following formA weak solution to the problem of finding the stress-strain state of the Timoshenko shell is a pair $\begin{pmatrix}u & \tilde{\varepsilon}\end{pmatrix}\in Y^1\times\left[H^1\left(\Sigma_0\right)\right]^9$ that $u-w\in V_{\partial\Sigma}\left(Y^1\right)$ and:

$$\begin{cases}B_{HR}^1\left(w,\tilde{\varepsilon}\right)=f\left(w\right),\forall w\in V_{\partial\Sigma}\left(Y^1\right),\\ B_{HR}^1\left(u,\mu\right)=B_{HR}^2\left(\mu,\tilde{\varepsilon}\right),\forall\mu\in\left[H^1\left(\Sigma_0\right)\right]^9,\end{cases} \tag{22}$$

where:

$$B_{HR}^{1}\left(w,\tilde{\varepsilon}\right)=\int_{\Sigma}\left(\tilde{L}w\right)^{T}\tilde{C}\tilde{\varepsilon}d\Sigma, B_{HR}^{2}\left(\mu,\tilde{\varepsilon}\right)=\int_{\Sigma}\mu^{T}\tilde{C}\tilde{\varepsilon}d\Sigma, f\left(w\right)=-\int_{\Sigma}w^{T}n\Delta pd\Sigma.$$

A software module was developed that implements the proposed numerical algorithm for solving the variational problem for a thin-walled shell based on the finite element method, using typical procedures of this method. [18]. The software module was developed as an integral part of the software package SMCM, created at Bauman Moscow State Technical University Scientific Educational center for Supercomputer modeling and software engineering.

5 Results of numerical simulation

To analyze the effectiveness of the developed numerical algorithm, 3 types of calculations of the elasticity problem were carried out:

1) three-dimensional calculation in the ANSYS software package;
2) shell calculation in the ANSYS software package,
3) shell calculation in the SMCM software package.

In the SolidWorks were built:

- shell geometry with dimensions 0.5 m * 0.3 m * 0.15 m (Fig. 1),
- three-dimensional geometry obtained by growing the thickness h=1 mm from the shell, while considering the shell as the midsurface.

Meshes were generated:

a) by ANSYS software package: tetrahedral mesh with linear approximation, mesh size was: 18 million finite elements, 4.7 million nodes, for solving the problem according to case 1;

b) by ANSYS software package: triangular mesh with quadratic approximation, the mesh size was: 57 thousand finite elements, 114 thousand nodes, for solving the problem according to case 2;

c) by SMCM software package: triangular mesh with quadratic approximation, the mesh size was: 57 thousand finite elements, 114 thousand nodes, for solving the problem according to case 3.

For all calculations, a material was chosen, the elastic properties of which are presented in table 1.

Table 1. Elastic properties of the material.

Elastic constants	Value
Modulus of elasticity E, GPa	70
Poisson's ratio ν	0.33

The following boundary conditions were set:

pressure is equal to 1 atm is set on the inner faces of the geometries. (Fig. 1, a)

at the open upper edges (cases 2, 3) and faces (case 1), fixed displacements are set equal to zero along the axes OX, OY, OZ (Fig. 1, b).

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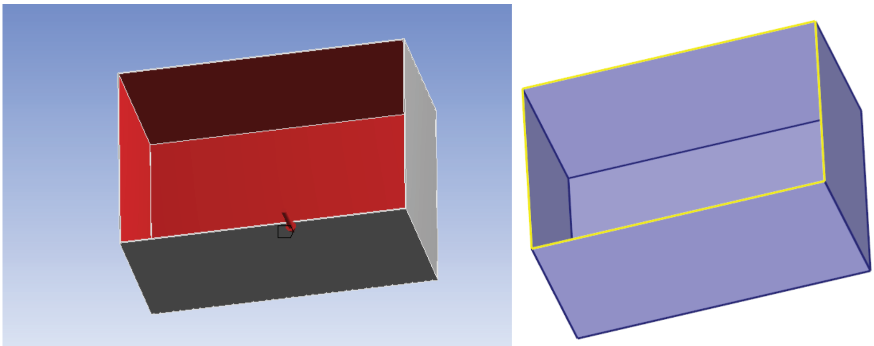

a b

Fig. 1. Boundary condition: a) pressure; b) displacement.

Comparative results of calculations of displacement fields – the midsurface in the global coordinate system, where OX – is directed along the normal to the surface of the bottom of the box (box) for all 3 options are presented in Figures 2, 3 and 4, as well as in Table 2. Analyzing the results of calculations for the maximum and minimum values of the displacement fields (tables 2, 3), we can conclude that the closest solution to the calculation case 1 – the solution of the three-dimensional elasticity problem in the Ansys software package – is the solution obtained in the SMCM software package. Shell case 2 of solving the problem by ANSYS software provides a lower accuracy of displacement simulation.

Table 2. Comparison of minimum displacement values for different calculation cases.

Dimensionless displacement fields (minimum in absolute value)	Case 1 (three-dimensional solution by ANSYS)	Case 2 (shell solution by ANSYS)	Case 3 (shell solution by SMCM)
$\lvert U_x \rvert$	$1.1553e^{-5}$	$5.5941e^{-8}$	0
$\lvert U_y \rvert$	0.00106	0.00334	0.00256
$\lvert U_z \rvert$	0.00097	0.00214	0.00164

Table 3. Comparison of the maximum displacement values for different calculation cases.

Dimensionless displacement fields (maximum in absolute value)	Case 1 (three-dimensional solution by ANSYS)	Case 2 (shell solution by ANSYS)	Case 3 (shell solution by SMCM)
$\lvert U_x \rvert$	0.00494	0.00594	0.00499
$\lvert U_y \rvert$	0.00105	0.00334	0.00292
$\lvert U_z \rvert$	0.00097	0.00214	0.00163

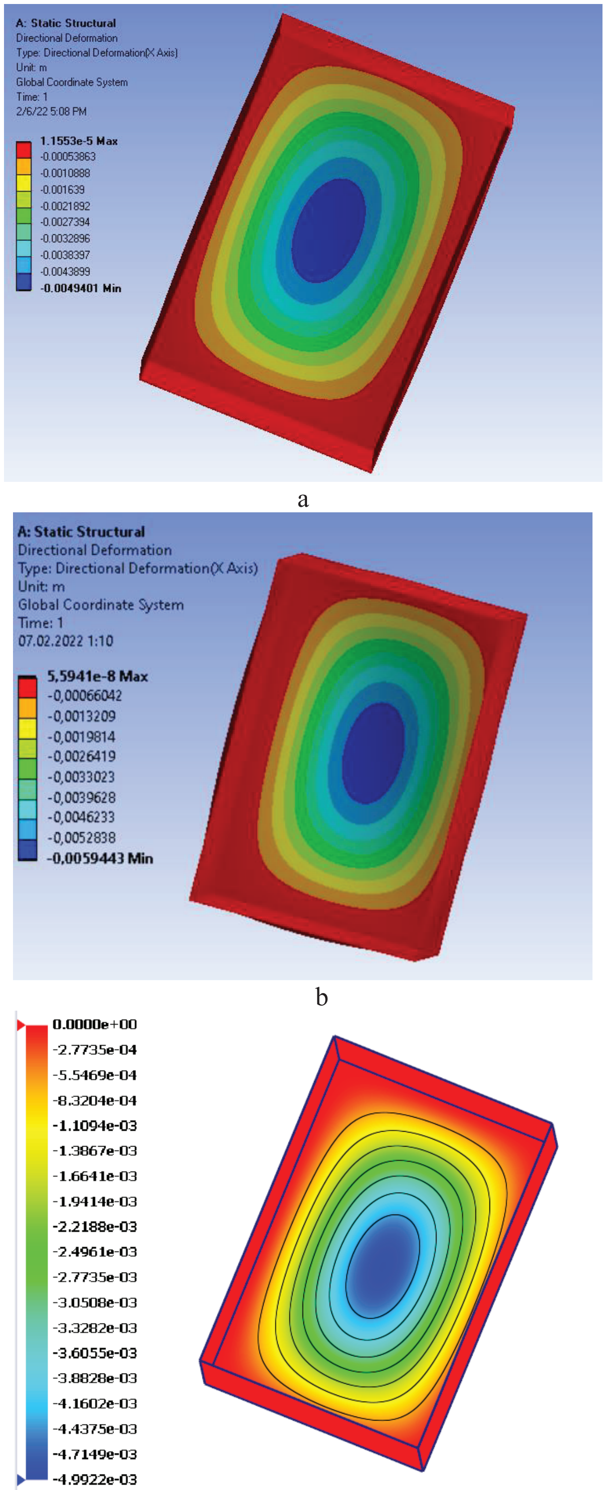

Fig. 2. Displacement fields U_x (m), a) case 1 (ANSYS), b) case 2 (ANSYS), c) case 3 (SMCM).

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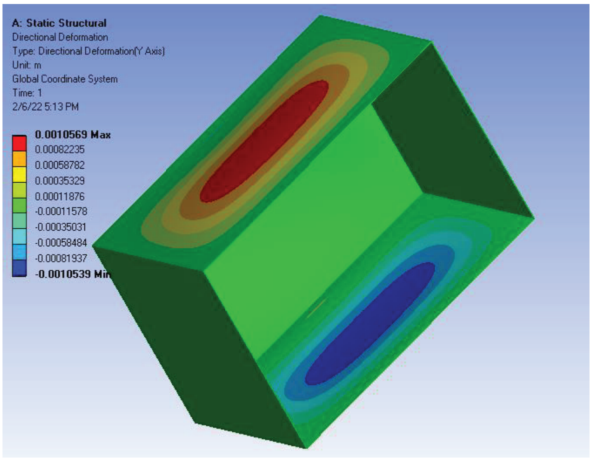
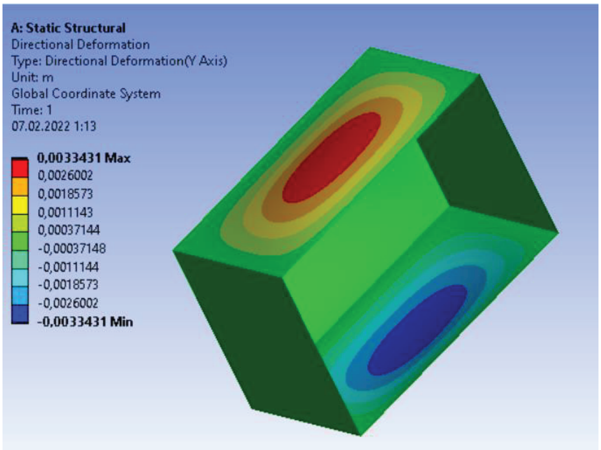
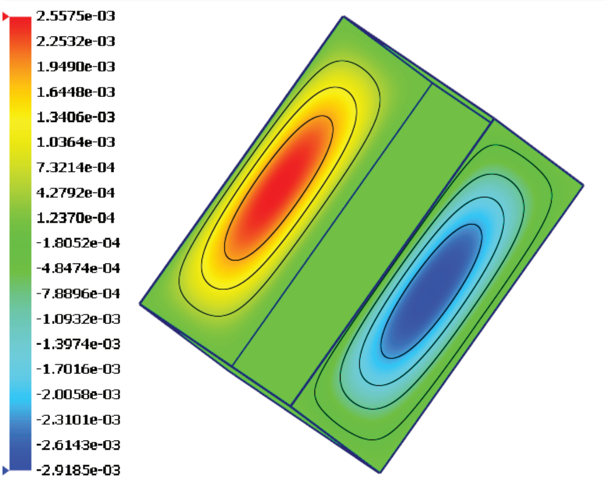


a

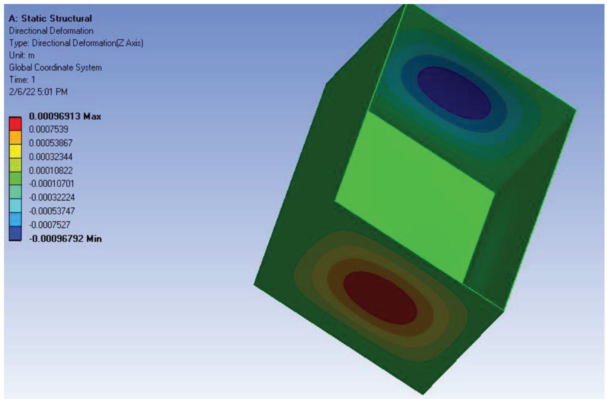


b

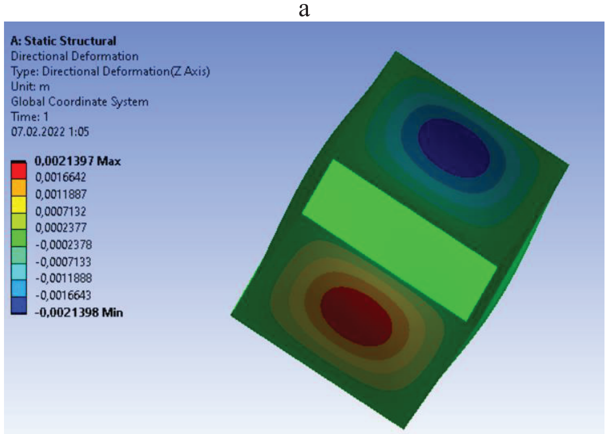


c

Fig. 3. Displacement fields U_y (m), a) case 1 (ANSYS), b) case 2 (ANSYS), c) case 3 (SMCM).



a



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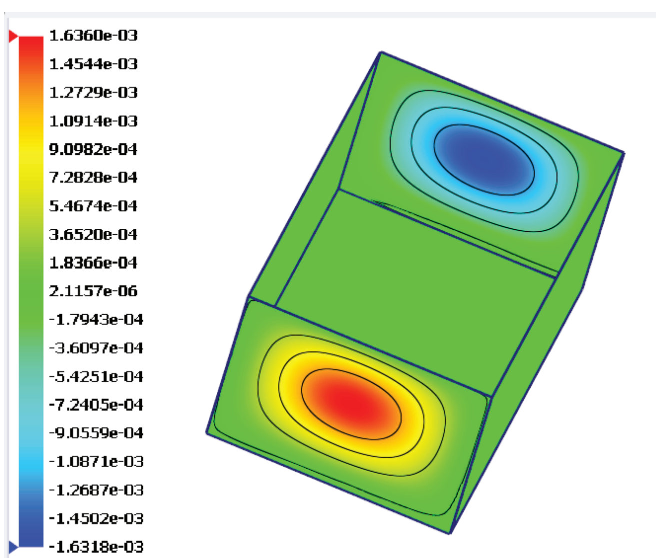

c

Fig. 4. Displacement fields U_z (m), a) case 1 (ANSYS), b) case 2 (ANSYS), c) case 3 (SMCM).

6 Conclusions

A numerical algorithm for solving the problem of elasticity theory for complex thin-walled shell composite structures was proposed, and SMCM software was created based on the finite element method and the developed numerical algorithm.

A test problem was solved for a complex shell structural element of the box type for 3 calculation options: solving the three-dimensional elasticity problem in the ANSYS software package, using the shell solution in the ANSYS software package and using the SMCM software package.

It has been established that the SMCM software allows obtaining results closer to a three-dimensional solution than the ANSYS shell solver.

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