

# Kinetic characteristics of cracks in polymers and composites based on mechanical and thermal loads

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**Abstract.** Theoretical ratios of important kinetic characteristics for brittle polymers with cracks under mechanical and thermal effects are obtained, which underlie the study of the thermokinetics of the process of polymer fracture in terms of the theory of the time dependence of strength-durability. Rectilinear (internal and surface) cracks in plate-type specimens and internal circular (disk-shaped) cracks in polymer fibers are considered. Two test modes are considered successively: constant tensile stress, constant absolute temperature, unchanging structure, inactive medium, as well as a more complex mode of purely thermal loading - the case least developed in the theory of destruction. The calculated ratios of a number of limiting characteristics and parameters of the destruction process are given: safe and critical stress; initial length of a microcrack and its relative critical length; safe and critical voltage; local stress at the crack tip (in the fluctuation volume); the value of the free surface energy. The above relations are the basis for the development of the theory of the time dependence of strength-durability. Key words: brittle polymers, thermokinetics, cracks.

## 1 Introduction

Modern structural and functional polymeric and composite materials are structurally sensitive materials [1-4]. Their creation on the basis of modern technologies is an important direction in the development of modern materials science. Structure-sensitive materials are obtained by various methods: nanopowder compaction, deposition on a substrate, crystallization of amorphous alloys, etc. [1-6] and have unique mechanical and thermal properties that allow them to be used in structures subject to various external influences [7,8]. An important step in the creation and use of these materials is the development of appropriate mathematical models to describe their behavior in a wide range of changes in external operational factors. The general methodology for constructing such models is still far from complete. First of all, this applies to models that describe the thermokinetics of the fracture process during durability testing. The main difficulty in developing such models is the need to mathematically describe the mutual influence of macro- and microstages of the destruction process, determine the main parameters and limiting characteristics of the process, establish

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a relationship between molecular constants that characterize the structure of materials on the one hand and macroscopic strength characteristics on the other, and, finally, to develop a methodology for calculating durability in certain test conditions. The traditional approach to the engineering assessment of the mechanical performance of polymeric and composite materials involves testing the sample in tension or compression (or torsion) up to failure. The stress at which failure occurs is a measure of strength. But this does not take into account the time factor, i.e., the finite lifetime of materials under the action of stress, a lower tensile strength, which is especially pronounced for polymers and composites based on them. It is more expedient to set not the voltage at which the structure should operate, but its life time, that is, the durability  $\tau = \tau(\sigma, T)$  [7,8] and from this ratio calculate the stress that this structure can withstand for a given period time at a given test temperature. Explaining this point of view is the fact that in real polymer and composite materials there are microdefects (cracks), which are responsible for premature failure. After applying a load that exceeds the safe stress  $\sigma_0$ , failure occurs by growing one, less often several, the most dangerous cracks from the initial length  $l_0$  to a certain critical length  $l_k$ , at which there is a transition to the athermal (fast) stage of the fracture process with a critical speed  $v_k$ , the value of which is determined by the propagation velocity of the elastic perturbations in a solid body. The process ends with the loss of the bearing capacity of the part or structure. To estimate the critical speed, the Roberts-Wells formula can be used

$$v_k = 0.38 \sqrt{\frac{E}{\rho}}, \quad (1)$$

Where E is the Young's modulus of the material,  $\rho$  is the density.

In the process of crack growth, the destruction of the material is localized in a small neighborhood of its tip  $V_a$  (fluctuation volume), where local stresses  $\sigma^*$  activating the process of rupture of strained chemical and intermolecular bonds significantly exceed the stresses in the rest of the sample volume. Finding the calculated engineering ratios of the indicated kinetic characteristics is one of the most important tasks of polymer materials science. The purpose of this article is to systematize the most important kinetic characteristics of brittle polymers with cracks under mechanical and thermal effects.

## 2 Ideological schemes of research

The registration of submicroscopic cracks in polymers by diffraction methods made it possible to determine their sizes (longitudinal and transverse), shape (linear in the form of a cut in plate-type samples, circular disk-shaped ones in polymer fibers), and position in the sample (surface, internal). Typical sizes of initial microcracks are for PMMA – 1700 Å, polyvinyl butyral - 3000 Å, polyethylene - 170 Å, polypropylene - 320 Å, polyvinyl chloride - 3000 Å, capron - 90 Å [7]. The data of fractographic studies of the fracture surface indicate that the critical crack length  $l_k$  is independent of the cross section of the sample, which varied by more than 100 times [7]. In all cases, fracture cracks grow from a defect of initial length  $l_0$  along the normal to the direction of maximum tensile stress. To characterize the crack, scale relations are fulfilled

$$\begin{aligned} \lambda \ll l_0 \leq l(\tau) \leq l_k \ll L, \\ 0 \leq \tau \leq \tau_f, \end{aligned} \quad (2)$$

Where  $\lambda$  is the fluctuation crack propagation upon rupture of one or a group of bonds (for organic polymers  $\lambda = 12A^\circ$  [5, 6]),  $\tau_f(\sigma, T)$  is the duration of the slow (thermofluctuation) stage of crack growth from the initial to the critical length. Scale relations (2) are the substantiating experimental result for the construction of mathematical models. Based on (2), a sample in the form of a plate or a cylindrical rod will be interpreted as an elastic plane ( $x, y$ ) with an internal crack  $|x| < l_0, y = 0$ , or as an elastic half-plane  $x > 0, |y| < \infty$  with a surface crack  $0 < x < l_0, y = 0$ , or as an elastic space ( $x, y, z$ ) with an internal circular axisymmetric crack  $z = 0, 0 \leq r < R_0$ .

In the mathematical model of the thermokinetics of the fracture process in specific cases of loading, an important role is given to the analytical formula for the crack growth rate as a function of its current length  $l(t)$ , of the stress field  $\sigma^*$  at its tip, more precisely in a small fluctuation volume  $V_a$ , where thermal fluctuation acts of rupture and recombination of strained chemical and intermolecular crack tip and molecular constants characterizing the structure of the polymer and the elementary act of breaking strained bonds:

$$V = V(l; \sigma^*; T_e; V_a; U; \dots), \quad (3)$$

Where  $U = U_0 - qT_e$  is the activation energy for breaking bonds, decreasing linearly with increasing temperature;  $U_0$  is the activation energy of the rupture process, extrapolated to absolute zero;  $q$  is the coefficient of temperature dependence of the activation energy (for polymeric organic glasses)  $q \sim (15 - 20)$  J/molK;  $\sigma_0^*$  – thermal fluctuation threshold of destruction (safe overvoltage at the crack tip). The fluctuation volume, an important molecular structural characteristic of polymers, is calculated on the basis of data on the structure of polymers and the mechanism of their destruction [5,6]:  $V_a = \lambda \lambda_\pi \lambda_m$ , where  $\lambda_\pi$  – prebreak bond elongation;  $\lambda_m$  – bond elongation. For non-oriented polymers, (polymer glasses formed by linear polymers), for oriented (fibers)  $V_a = 6\lambda_0^2 \cdot \lambda_m = 1.4 \cdot 10^{-28} \text{ M}^3$ ;  $\lambda_m = 1.5 A^\circ, \lambda_0 = 4A^\circ$  is the average intermolecular distance in the polymer) [7,8]. Local stress in formula (3)  $\sigma^* = \varphi(\sigma, \beta, l \dots)$  is one of the most important local kinetic characteristics of strength. The value of  $\sigma^*$  depends on the applied external stress  $\sigma$ , the stress concentration factor  $\beta$  at the crack tip, which plays an extremely important role in the study of defectiveness of materials, the current length of the crack  $l(t)$ , geometry of the sample, the configuration of the crack and its location in the sample (surface or internal). The value  $\sigma^*$  is calculated by the methods of fracture mechanics based on the solution of boundary problems of the mathematical theory of cracks. The calculated values of the quantity under mechanical and thermal loads make it possible to determine a number of limiting characteristics and parameters of brittle polymers with cracks in the overall picture of the thermokinetics of the fracture process.

For stresses  $\sigma$  not too close to safe and not exceeding critical  $\sigma_0 < \sigma < \sigma_k$ , the probability of recombination (restoration) of bonds at the crack tip is negligibly small compared to the probability of their rupture, and the average crack growth rate based on its molecular model [7,8] can be written in the following form

$$V(l, \sigma^*, T_e \dots) = \lambda v_0 \exp \left[ -\frac{(U - V_a \sigma^*)}{kT_e(l, t)} \right], \quad (4)$$

Where  $v_0$  – the frequency of thermal vibrations of kinetic units involved in breaking and restoring bonds ( $v_0 \sim 10^{-13} \text{ c}^{-1}$ ),  $k$  is the Boltzmann constant. The durability of a sample  $\tau = \tau(\sigma, T)$  in the form of a plate with a width  $L$  consists of the times of the rupture process at the first (fluctuation) stage  $\tau_f(\sigma, T)$  during crack growth at a rate (4) from the initial length  $l_0$  to the critical one  $l_k$  and the second (athermal)  $\tau_k$  at the limiting rate  $v_k$  (see formula (1))

$$\tau = \tau_{\phi} + \tau_k = \int_{l_0}^{l_k} \frac{dl}{V(l, \sigma^*, T_{g\dots})} + \frac{L-l_k}{v_k}. \quad (5)$$

### 3 Stress intensity factors under mechanical and thermal loads

As mentioned, the fracture of brittle polymers is localized in a small neighborhood of the crack tip (in the volume  $V_d$ ). To find the local stress  $\sigma^*$  at the crack tip in formula (4), it is necessary to use the methods of the mathematical theory of cracks. This will allow us to study the asymptotic stress distribution near the crack tip.

Based on scale relations (2), we will interpret a crack as a cut in a homogeneous and elastic isotropic continuum. Let us consider the stress-strain state in the vicinity of the crack tip  $|x| < l, y = 0$  in the elastic plane  $(x, y)$  under given arbitrary loads acting on the crack edges and constant loads at infinity. Simultaneously, there is a thermally stressed state caused by a stationary heat flux parallel to the plane of symmetry of the sample. Under plane tension, the stress distribution in the vicinity of the crack tip has the form [9]:

$$\begin{aligned} \sigma_{xx} &= \frac{K_1}{\sqrt{2r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) + \frac{K_2}{\sqrt{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}, \\ \sigma_{yy} &= \frac{K_1}{\sqrt{2r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) - \frac{K_2}{\sqrt{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}, \\ \sigma_{xy} &= \frac{K_1}{\sqrt{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \frac{K_2}{\sqrt{2r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right), \end{aligned} \quad (6)$$

Where  $(r, \theta)$  are polar coordinates with the pole at the crack tip;  $K_1, K_2$ , - stress intensity factors, which are found from the solution of the problem of the theory of elasticity as a function of the load and parameters characterizing the configuration of the body, the shape of the crack, its location in the sample, as well as the elastic and thermophysical constants of the material. The values  $K_1$  and  $K_2$  represent the asymptotics of the stress components in the vicinity of the crack tip, i.e., ultimately, the local stresses  $\sigma^*$  in relation (3). This article proposes the derivation of a generalized relationship for stress intensity factors in the presence of mechanical and thermal loads on the crack faces and outside it. An independent approach based on the complex potentials of N.I. Muskhelishvili. It turned out to be very effective for this case, a rather complicated problem of the mathematical theory of cracks.

Let us formulate the problem in terms of stresses using the known relations for plane static problems of thermoelasticity [10]. The problem is to determine the stress intensity factors in the asymptotic behavior of the stress tensor  $\sigma_{ij}(x, y)$  in (6) at  $z \rightarrow \pm l, (z = x + iy)$  based on the solution of the equations:

equilibrium

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} &= 0, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} &= 0; \end{aligned} \quad (7)$$

Compatibility

$$\Delta(\sigma_{xx} + \sigma_{yy}) = -\frac{2\beta r G}{(\chi-1)(\lambda^*+2G)} \Delta T(x, y) = 0; \quad (8)$$

Relations of stresses and displacements

$$\begin{aligned}\sigma_{xx} &= \lambda^* e + 2G \frac{\partial U}{\partial x} - \beta_T (\chi - 1) T(x, y), \\ \sigma_{yy} &= \lambda^* e + 2G \frac{\partial V}{\partial y} - \beta_T (\chi - 1) T(x, y), \\ \sigma_{xy} &= G \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right), \\ e &= \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\end{aligned}\quad (9)$$

In the region  $D \setminus B$ , where  $D = (|x, y| < \infty)$ ,  $B = (|x| < l, y = 0)$ . Here  $U(x, y)$ ,  $V(x, y)$  are the displacement vector components.

Omitting the details of cumbersome calculations, we present the result

$$\begin{aligned}K_1^\pm - iK_2^\pm &= \\ &= -\frac{1}{\pi\sqrt{l}} \left\{ \int_{-l}^{+l} \sqrt{\frac{l \mp x}{l \mp x}} p(x) dx \pm i \frac{\chi - 1}{\chi + 1} \int_{-l}^{+l} q(x) dx - [\sigma_{yy}^{(\infty)} - i\sigma_{xy}^{(\infty)}] \pi l \right\} \mp \\ &\mp \frac{\beta^*}{\pi(\chi+1)\sqrt{l}} \left\{ \int_{-l}^{+l} (l-x) \left( \frac{\partial T^+}{\partial y} - \frac{\partial T^-}{\partial y} \right)_{y=0} dx \pm i \int_{-l}^{+l} (T^+(x) - T^-(x)) dx \right\}.\end{aligned}\quad (10)$$

Here  $(\partial T / \partial y)_{y=0}^\pm$ ,  $T^\pm(x)$  are the heat flux and temperature on the crack faces, respectively; the sign (+) on the left refers to the right crack tip, the sign (-) to the left one. Under isothermal loading conditions and in the absence of loads at infinity ( $\sigma_{yy}^{(\infty)} = \sigma_{xy}^{(\infty)} = 0$ ), we arrive at the expression obtained in [10]. Assuming that the crack edges are free from stresses ( $p(x) = q(x) = 0$ ) and that there are no stresses at infinity, we find from (10) the intensity factors for purely thermal stresses. Relation (10) contains numerous special cases of mechanical and thermal loading, each of which can serve as the subject of independent research in the study of fracture kinetics.

## 4 local stresses at the crack tip

Let us now turn to the application of relation (10) to derive a number of important kinetic characteristics for polymers with cracks. Let us first consider a purely mechanical loading at a constant temperature.

### 4.1 Internal linear crack

With uniaxial tension of the sample under constant stress, we find from (10)

$$\begin{aligned}K_1 &= \sigma\sqrt{l}, \\ K_2 &= 0,\end{aligned}\quad (11)$$

and from relation (6) is the maximum tensile stress in the vicinity (for definiteness, the right one) of the crack tip, achieved in the crack plane:

$$(\sigma_{yy}(x, 0)) \frac{\sigma \sqrt{l}}{\sqrt{2(x-l)}_{max}} \quad (12)$$

Direct experiments (using IR spectrometry) to measure the true stresses on individual chemical bonds for solid polymers showed that as we approach the tip of the crack on the most stressed bonds, the load increases up to a certain value, after which it remains almost constant and exceeds the average stress on the bonds in the sample volume by several orders of magnitude. Such bonds are strongly deformed and broken in the first place; their rupture is due to the stress attributable to the bond spaced from the crack tip at the distance of its fluctuation propagation  $\lambda$ . Thus, the desired local stress at the crack tip  $a$  in the final form is

$$\sigma^* = \sigma \beta(l_0) \sqrt{\frac{l}{l_0}}, \quad (13)$$

Where the stress concentration factor appears for an internal rectilinear crack of initial length  $2l_0$

$$\beta(l_0) = 0.71 \sqrt{\frac{l_0}{\lambda}}. \quad (14)$$

Creep experiments (for  $\sigma = \text{const}$ ) show that the coefficient  $\beta$  practically does not change over the lifetime of the sample and is determined by the length of the initial defect in the sample. From (13) the value (half-length) of the initial microcrack is found

$$l_0 = 2\lambda\beta^2. \quad (15)$$

Numerical calculations based on relations (13–15) give results close to experimental ones [7,8].

## 4.2 Edge surface crack

Let us turn to the consideration of surface cracks. Such cracks are the most common and grow from the edge of the sample, where there are the most dangerous defects. When calculating  $\sigma^*$ , the sample according to (2) is considered as an elastic half-plane  $(x, y)$  with an edge crack  $0 < x < l, y = 0$ . This case is one of the rather complicated cracks in the mathematical theory. The approach developed in [6] (for the case of sample tension under constant stress at constant temperature) made it possible to obtain the following expression for the local stress at the crack tip

$$\begin{aligned} \sigma^* &= \sigma \beta(l_0) \sqrt{\frac{l}{l_0}}, \\ \beta(l_0) &= 0.79 \sqrt{\frac{l_0}{\lambda}}. \end{aligned} \quad (16)$$

Hence, the length of the initial surface crack is equal to

$$l_0 = 1.6\lambda\beta^2. \quad (17)$$

So for PMMA  $\lambda = 12 A^\circ, \beta = 11$  [5,6 and  $l_0 = 2.3 \cdot 10^{-7} \text{m}$  which is close to the estimate of the initial (initial) edge microcrack given in [7,8] ( $l_0 = 1.5 \cdot 10^{-7} \text{m}$ ) for inorganic glass  $\lambda = 5.4 A^\circ, \beta = 60$  [10] and from we have  $l_0 = 4 \mu\text{m}$ , which coincides with the experimental data in [7,8].

### 4.3 Internal disk fissure

The next question is internal disc-shaped cracks (circular) cracks. As mentioned above, along with linear submicrocracks, disc-shaped submicrocracks oriented perpendicular to the tensile force were found in polymers.

In [11], convenient for calculation ratios for stress intensity factors under mechanical and thermal loads were obtained:

$$\begin{aligned} K_1^{(M)} &= \frac{2}{\pi\sqrt{R}} \int_0^R \frac{y\sigma_0(y)dy}{\sqrt{R^2 - y^2}}, \\ K_1^{(T)} &= \frac{2(1+\nu)\alpha G}{\pi(1-\nu)\sqrt{R}} \int_0^R \frac{yT_0(y)dy}{\sqrt{R^2 - y^2}}. \end{aligned} \quad (18)$$

Here  $T_0(r)$  is the temperature at the crack  $0 \leq r < R, z = 0$ ; it is either given or found from the solution of the corresponding thermal problem. Under constant external load  $\sigma_0(r) = \sigma = \text{const}$  and isothermal test conditions, it follows from (18)

$$K_1 = \frac{2}{\pi} \sigma \sqrt{R} \quad (19)$$

and the local stress in the  $\lambda$  neighborhood of a circular crack has the form

$$\begin{aligned} \sigma^* &= \sigma \beta(R_0) \sqrt{\frac{R}{R_0}}, \\ \beta(R_0) &= 0.5 \sqrt{\frac{R_0}{\lambda}}, \end{aligned} \quad (20)$$

Where  $R$  is the variable radius of the growing crack,  $2R_0$  is the diameter of the initial circular crack. From (20) it follows

$$R_0 = 4\lambda\beta^2. \quad (21)$$

So, for oriented fibers (polyethylene; polypropylene; polycapromide), according to [10]  $\lambda = 4 A^\circ, \beta \sim (4 - 7)$ , from here and from (20) the radius of the initial microcrack is  $R_0 \approx (10^{-8} - 10^{-7}) \text{m}$ , which is confirmed by experiments in [7,8].

A number of important parameters and limiting characteristics of the fracture process should be added to these ratios. The characteristic  $\sigma_0$  requires special consideration. In the kinetic theory, the safe stress is introduced by the relation

$$\sigma_0 = \frac{\alpha_{nos}}{\beta\lambda_m}, \quad (22)$$

Where  $\alpha_{nog}$  is the free surface energy of the material (in vacuum). It was shown in [10] that the value coincides with the Griffith fracture threshold

$$\sigma_0 = \sigma_{G_0} = \sqrt{\frac{2E\alpha_{nog}}{\pi l_0}}. \quad (23)$$

Thus, from the above relations, the following calculated characteristics of the kinetics of the process of brittle fracture of polymers follow:  
safe stress

$$\sigma_{G_0} = \sqrt{\frac{2E\alpha_{nog}}{l_0}}, \quad (24)$$

initial length (half-length or radius) of microcracks

$$l_0 = \lambda \frac{\beta^2}{\chi^2}, \quad (25)$$

stress concentration factor

$$\beta(l_0) = \chi \sqrt{\frac{l_0}{\lambda}} \quad (26)$$

critical stress

$$\sigma_k = \frac{U_0 - qT}{V_a \beta} = \frac{(U_0 - qT)}{\chi V_a} \sqrt{\frac{\lambda}{l_0}}, \quad (27)$$

relative critical crack length

$$\sqrt{\frac{l_k}{l_0}} = \frac{U_0 - qT}{V_a \beta \sigma}; \quad (28)$$

local stress at the crack tip

$$\sigma^* = \sigma \beta(l_0) \sqrt{\frac{l}{l_0}}. \quad (29)$$

## 5 Local stresses under thermal loads

The calculation of local stresses under purely thermal loads is a practically undeveloped problem in the theory of brittle fracture. Of greatest interest are the cases of a steady thermal state in materials with a crack. Theoretical and experimental results in [12-15] show that, under a steady heat flow in a body with a crack, there is a significant increase in thermal stresses caused by a local increase in the temperature gradient in the vicinity of the crack tip. It can be assumed that thermoelastic expansion fields (as well as their mechanical counterparts) increase the stress intensity at the crack tip, causing it to grow. Experiments confirm this assumption [15]. The intensity factors of thermoelastic stresses are as follows

$$K_1 = 0,$$



$$K_2 = \frac{\alpha q_T}{4\lambda_T} E l^{3/2} \quad (30)$$

(Plane stress state) and then the desired local thermal stress at the crack tip

$$\sigma_T^* = \beta \sigma_T \left( \frac{l}{l_0} \right)^{\frac{3}{2}}, \quad (31)$$

Where

$$\begin{aligned} \sigma_T &= \alpha E l_0 \frac{q_T}{4\lambda_T}, \\ \beta(l_0) &= 0.71 \sqrt{\frac{l_0}{\lambda}}. \end{aligned} \quad (32)$$

The resulting relation for the stress  $\sigma_T$  in (32) is a fundamental result for the theory of thermal failure: the stress  $\sigma_T$  is a mechanical analogue under thermal loading and links the thermophysical, elastic, and structural characteristics of polymers, which makes it possible to trace the influence of each factor on the thermal response of a polymer material with initial microcrack. Let us add to the above relations a number of interesting characteristics, starting with the temperature  $T_\theta$  at the crack tip. First, we find the asymptotic temperature distribution near the vertex in the coordinates  $(r, \theta)$  (as in (6)) in the form

$$T(r, \theta) = \sqrt{2l} \frac{q_T}{\lambda_T} r^{1/2} \sin \frac{\theta}{2}. \quad (33)$$

Hence, as  $T_B$  we take the average integral value in the  $\lambda$ -vicinity of the right tip of the initial microcrack, which gives

$$T_\theta = \beta \lambda \frac{q_T}{\lambda_T}. \quad (34)$$

Here again, the relationship between macro and micro parameters and their influence on the thermal state of the polymer material at the crack tip is traced. Now we can write the crack growth rate as

$$V(l, \sigma_T^*, T_\theta, \dots) = \lambda v_0 \exp\left(-\frac{U - V_a \sigma_T^*}{k T_\theta}\right) \quad (35)$$

Where all the main quantities are calculated. The main external factor causing crack growth at the rate (35) is the thermal load with power  $q_T$ , which is one of the stress components. Relations (40) and (43) (at  $T = T_B$ ) define the stress range  $\sigma_T$  from safe  $\sigma_T^{(0)}$  to critical  $\sigma_T^{(k)}$ , which makes it possible to identify the characteristic values of external thermal loading from safe

$$q_T^{(0)} = \frac{3.2\lambda_T}{\alpha} \sqrt{\frac{\alpha_{nog}}{E}} l_0^{-3/2} \quad (36)$$

to critical

$$q_T^{(k)} = \frac{5.6\lambda_T\sqrt{\lambda}(U_0 - qT_0)}{\alpha EV_a} l_0^{-3/2} \dots \quad (37)$$

Other special cases can be considered similarly.

## 6 Conclusions

1. Theoretical ratios of important kinetic characteristics for brittle polymers with cracks under mechanical and thermal effects are obtained, which underlie the study of the thermokinetics of the process of polymer destruction in terms of the theory of the time dependence of strength-durability.

2. Rectilinear (internal and surface) cracks in plate-type specimens and internal circular (disk-shaped) cracks in polymer fibers are considered.

3. Two test modes are considered successively: constant tensile stress, constant absolute temperature, unchanging structure, inactive medium, as well as a more complex mode of purely thermal loading - the case least developed in the theory of destruction.

4. The calculated ratios of a number of limiting characteristics and parameters of the destruction process are given: safe and critical stress; initial length of a microcrack and its relative critical length; safe and critical voltage; local stress at the crack tip (in the fluctuation volume); the value of the free surface energy. The above relations are the basis for the development of the theory of the time dependence of strength-durability.

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