Analysis of sensitivity of elastic characteristics of unidirectional carbon fiber specimens with thermoplastic matrix to loading rate at high temperature

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Abstract. The problem of assessing the sensitivity of unidirectional carbon fiber reinforced plastics to loading rate is considered. As a rule, when the loading rate changes, the stress-strain curves for unidirectional CFRP specimens under loading at different off-axis angles change due to the rheological properties and physical nonlinearity. An attempt is made to reveal the degree of anisotropy sensitivity of a unidirectional carbon fiber reinforced plastic with thermoplastic matrix at an elevated temperature in the range of changes in the loading rate by two orders of magnitude both in stress and in strain. **Key words:** high-rate strain, matrix analysis, least square method, residuals distribution, matrix norm, design matrix, off-axis loading, statistical analysis.

1 Introduction

Polymer-based composite materials are widely used as load-bearing elements of structures and are under time-varying loading conditions for a certain time of operation. In particular, such conditions can be those of high-rate loading in a wide range of rates. A range of the issues necessary for evaluating the limit state of elements are quite wide and may include conditions for buckling, vibrations, and failure by various mechanisms. To test the sensitivity of composite materials to the loading rate, the analysis of test results of unidirectional flat samples loaded at different off-axis angles, as well as angle-ply laminates, is usually carried out. It should also be noted a significant difference, especially in strength of specimens, between tensile and compressive loads. The analysis of test results shows the presence of a linear deformation area, and also, under certain conditions, the phenomenon of rheological and physically nonlinear properties takes place. The regularities of deformation and failure of carbon fiber and fiberglass plastics under high-rate loading are given in review papers [1, 2]. In particular, it is noted [1] that there are a number of contradictory experimental data on the dependence of rigidity and strength of specimens on the rate of loading. However, there is a general trend that with an increase in the loading rate, an increase in the strength and rigidity of specimens takes place. As for the ultimate strains, we can note a non-monotonic

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dependence on the loading rate, which, apparently, is associated with the predominance of certain deformation and fracture mechanisms. In the general case, the range of high-rate tests can be very wide and, in case of the influence of impact loads, leads to the appearance of wave processes that require special experimental equipment and methods of theoretical analysis [3].

The analysis of the stress-strain curves of unidirectional carbon fiber specimens loaded at different off-axis angles does not allow us to draw definite conclusions about the dependence of the elastic modulus on the strain rate in the area of elastic deformation (Fig. 1).

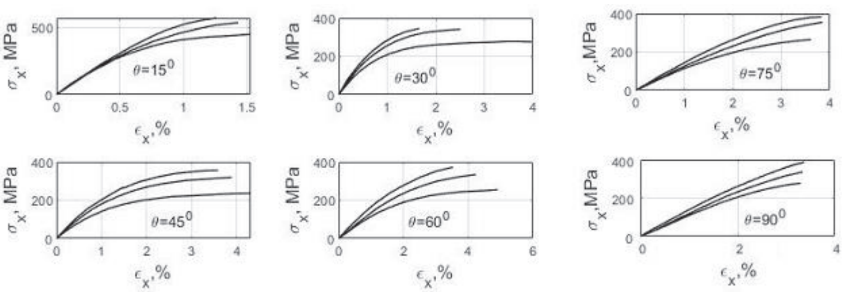


Fig. 1. Stress-strain curves for IM7-8552 carbon fiber plastic [4].

From a visual analysis of the stress-strain curves [4, 5], it can be concluded that a noticeable difference in the curves is observed in the area of physical nonlinearity, and with an increase in the strain rate, the degree of physical nonlinearity decreases. The dependence of the elastic modulus on the strain rate during loading of carbon fiber specimens at different off-axis angles requires additional analysis. To obtain an answer about the significance of the influence of the loading rate on the elastic characteristics of carbon fiber plastics, a statistical analysis of the experimental data should be carried out.

2 Experimental data

In this paper, an attempt is made to check the effect of both straining and loading rates varying by two orders of magnitude in stress and strain by using the results of tensile tests of unidirectional AS4/PEEK CFRP specimens at the temperature of 100°C [6]. Experimental data are shown in Table 1.

Table 1. Experimental elastic characteristics for unidirectional AS4/PEEK carbon fiber at 100°C [6].

Nominal rate	E_1 **. GPa**	E_2 **. GPa**	G_{12} **. GPa**	v_{12}
1.0 %/min	138	9.8	5.9	0.305
0.01 %/min	132	9.8	5.4	0.362
100 MPa/min	139	10.4	–	0.309
1 MPa/min	143	9.5	–	0.442

E_1 is the longitudinal modulus of elasticity, E_2 is the transverse modulus of elasticity, G_{12} is the in-plane shear modulus, v_{12} is the Poisson's ratio (Fig. 2).

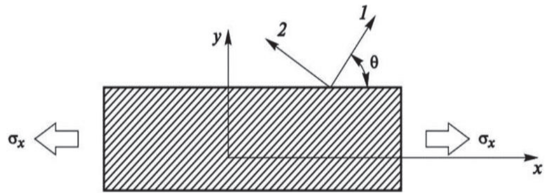


Fig. 2. Schematic illustration of off-axis loading [7].

The paper considers the testing results for specimens of AS4/PEEK unidirectional layered carbon fiber reinforced plastic made by the autoclave method and consisting of thermoplastic matrix and medium-modulus and medium-strength carbon fibers [6]. The volume fraction of fibers was about 61%. Standard specimens were cut from the laminated panel with the following laying: 0°, 10°, 30°, 45°, 60° and 90° (Fig. 3). The test procedure included two types of tensile tests at 100°C: tests at constant strain rates of 0.01%/min and 1%/min, and tests at constant loading rates of 1MPa/min and 100MPa/min. Specimens were tested at different off-axis angles using a servo-hydraulic machine MTS-810.

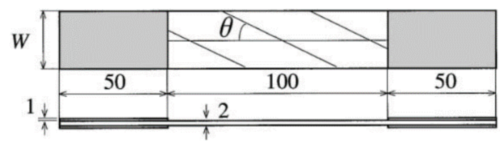
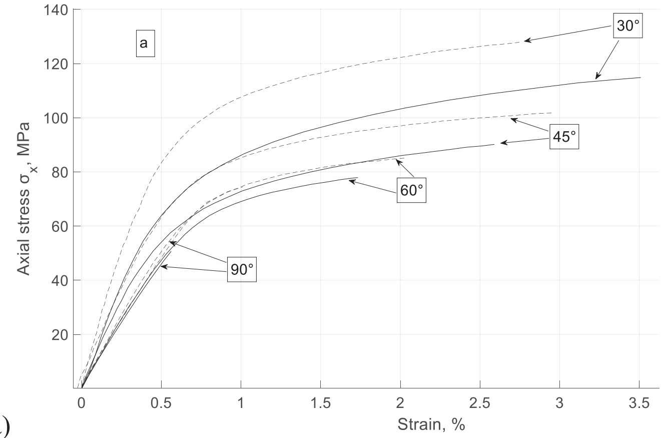
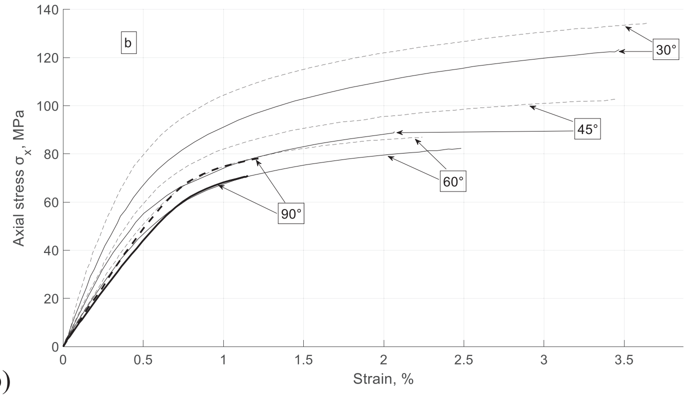


Fig. 3. AS4/PEEK specimen for testing [6].

From the data given in Table 1, noticeable differences in the numerical values of the Poisson's ratio draw attention, it can be seen that with an increase in the loading rate, the values of the Poisson's ratio decrease. With increasing the strain rates from 0.01%/min to 1%/min, an increase in the longitudinal elastic modulus and in-plane shear modulus takes place, and with changing the loading rates from 1 MPa/min to 100 MPa/min, the longitudinal elastic modulus decreases. The experimental strength values increase with increasing the loading rate (Fig. 4).



a)



b)

Fig. 4. Stress-strain curves for unidirectional AS4/PEEK specimens: (a) 0,01 % min^{-1} [solid] and 1 % min^{-1} [dashed]; (b) 1 MPa min^{-1}[solid] and 100 MPa min^{-1} [dashed].

3 Description of the method

3.1 Analysis of stress calculation error

The proposed method for assessing the sensitivity of mechanical properties to the strain rate is carried out using a matrix analysis of compliance matrices. In particular, for $\dot{\varepsilon} = 1\ \%/\min$, $\dot{\varepsilon} = 0.01\ \%/\min$, $\dot{\sigma} = 1\ \text{MPa}/\min$ and $\dot{\sigma} = 100\ \text{MPa}/\min$, the compliance matrices are denoted by $S_{1\sigma}$, $S_{100\sigma}, S_{001\varepsilon}$ and $S_{1\varepsilon}$, respectively, and their general form is as follows:

$$S = \begin{pmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{pmatrix}.$$

For the experimental data (Table 1), let us take the matrix $S_{1\sigma}$ as the base one and consider the effect of the compliance matrix on strains under the same force action. The relationship between stresses and strains in the main axes of orthotropy is determined by the following relation:

$$\{\varepsilon_{1-2}\} = [S] \cdot \{\sigma_{1-2}\},$$

where $\{\sigma_{1-2}\} = (\sigma_1 \quad \sigma_2 \quad \tau_{12})^T$, $\{\varepsilon_{1-2}\} = (\varepsilon_1 \quad \varepsilon_2 \quad \gamma_{12})^T$, T is transposition procedure.

Taking the compliance matrix for the calculation purpose corresponding to the stress rate of 1 MPa/min, as the initial one, and considering other compliance matrices as its perturbations, let us analyze the relative change in stresses at fixed strain values.

Estimates for the shear modulus at 1 and 100 MPa/min not presented in Table 1 were made using the following known relationship

$$\frac{1}{G_{12}}=\frac{4}{E_{45}}-\frac{1-2\nu_{12}}{E_1}-\frac{1}{E_2} \tag{1}$$

The obtained shear moduli determined using relation (1) turned out to be 5.2 GPa for 1 and 100 MPa/min.

In accordance with the accepted provisions, let us write the perturbations caused by changes in the loading rate up to 100 MPa/min as $\Delta S_{100}=S_{1\sigma}-S_{100\sigma}$, similarly, the perturbation of the compliance matrix for strain rates of 0.01%/min and 1%/min will have the form: $\Delta S_{001}=S_{1\sigma}-S_{001\varepsilon}$ $\Delta S_{1\varepsilon}=S_{1\sigma}-S_{1\varepsilon}$, respectively. When the condition for the norm $\left\|S^{-1}\Delta S\right\|<1$ is fulfilled, the expression for the relative change in stresses caused by the perturbation of the compliance matrix can be written as follows [8]

$$\frac{\left\|\delta\sigma_{1-2}\right\|}{\left\|\sigma_{1-2}\right\|}\le\frac{\left\|S^{-1}\Delta S\right\|}{1-\left\|S^{-1}\Delta S\right\|} \tag{2}$$

Estimation of the error in calculating stresses in the area of elastic behavior with respect to the initial compliance matrix corresponding to loading at the rate of 1 MPa/min, caused by change in the compliance matrix, is presented in Table. 2.

Table 2. Statistical analysis of the effect of loading rate.

Loading rate	$\left\|S^{-1}\Delta S\right\|$	**Error, %**
100 MPa/min	0.1222	13.9
1 % min^{-1}	0.1230	14.0
0,01 % min^{-1}	0.0937	10.3

3.2 Analysis of calculation errors

Let us consider a calculation case in which the least squares method (LSM) is used to determine the compliance matrices at different rates. The essence of the method is to minimize the discrepancy between the calculated and experimental values of the reciprocal values of the elastic moduli. Geometrically, this means projecting the column of free terms as a result of the experiment onto the column space of the design matrix, which contains the data of the experiments performed. The method is described in detail in [7].

A solution in matrix form is sought for an overdetermined system of linear equations, the general form of which is:

$$\begin{aligned}
&\varepsilon_x=S_{xx}\cdot\sigma_x=\left(S_{11}\cdot\cos^4\theta+\left(S_{66}+2S_{12}\right)\cdot\cos^2\theta\cdot\sin^2\theta+S_{22}\cdot\sin^4\theta\right)\cdot\sigma_x,\\
&\frac{\varepsilon_x}{\sigma_x}=\frac{1}{E_x}=\left(\cos^4\theta\quad\sin^4\theta\quad\cos^2\theta\cdot\sin^2\theta\right)\left(s_1\quad s_2\quad s_3\right)^T,\ \text{где}\\
&s_1=\frac{1}{E_1},\ s_2=\frac{1}{E_2},\ s_3=\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_1},\\
&s=\left(s_1\quad s_2\quad s_3\right)^T.
\end{aligned} \tag{3}$$

The system of linear equations, the solution of which makes it possible to determine the elasticity characteristics, can be written as follows:

$$Xs = b, \tag{4}$$

Where the design matrix X and the desired vector are defined as follows:

$$X = \begin{pmatrix} \cos^4 0 & \sin^4 0 & \cos^2 0 \cdot \sin^2 0 \\ \cos^4 10 & \sin^4 10 & \cos^2 10 \cdot \sin^2 10 \\ \cos^4 30 & \sin^4 30 & \cos^2 30 \cdot \sin^2 30 \\ \cos^4 45 & \sin^4 45 & \cos^2 45 \cdot \sin^2 45 \\ \cos^4 60 & \sin^4 60 & \cos^2 60 \cdot \sin^2 60 \\ \cos^4 90 & \sin^4 90 & \cos^2 90 \cdot \sin^2 90 \end{pmatrix} = \begin{pmatrix} 1.0000 & 0 & 0 \\ 0.9406 & 0.0009 & 0.0292 \\ 0.5625 & 0.0625 & 0.1875 \\ 0.2500 & 0.2500 & 0.2500 \\ 0.0625 & 0.5625 & 0.1875 \\ 0.0000 & 1.0000 & 0.0000 \end{pmatrix}, \ \mathrm{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{E_1} \\ \dfrac{1}{E_2} \\ \dfrac{1}{G_{12}} - \dfrac{2\nu_{12}}{E_1} \end{pmatrix},$$

The free vector is a column of inverse modules $b = \left(\dfrac{1}{E_x}\right)^T$ corresponding to the given angles. It should be noted that the value of the modulus of elasticity under loading at the angle of 10 degrees at rate of 1 MPa/min is absent, and therefore the design matrix for this rate will take the form:

$$X_1 = \begin{pmatrix} \cos^4 0 & \sin^4 0 & \cos^2 0 \cdot \sin^2 0 \\ \cos^4 30 & \sin^4 30 & \cos^2 30 \cdot \sin^2 30 \\ \cos^4 45 & \sin^4 45 & \cos^2 45 \cdot \sin^2 45 \\ \cos^4 60 & \sin^4 60 & \cos^2 60 \cdot \sin^2 60 \\ \cos^4 90 & \sin^4 90 & \cos^2 90 \cdot \sin^2 90 \end{pmatrix} = \begin{pmatrix} 1.0000 & 0 & 0 \\ 0.5625 & 0.0625 & 0.1875 \\ 0.2500 & 0.2500 & 0.2500 \\ 0.0625 & 0.5625 & 0.1875 \\ 0.0000 & 1.0000 & 0.0000 \end{pmatrix}.$$

Vector s is defined as:

$$s = \left(X^T X\right)^{-1} X^T b. \tag{5}$$

A comparison of the experimental and calculated values of the elastic moduli is given in Table 3.

Table 3. Experimental [6] and calculated values of the elastic moduli at different loading and strain rates at 100°C.

θ, degree	$E_\theta, (\dot{\varepsilon} = 1\ \%/\mathrm{min})$	$E_\theta, (\dot{\varepsilon} = 0.01\ \%/\mathrm{min})$	$E_\theta, (\dot{\sigma} = 100\ \mathrm{MPa}/\mathrm{min})$	$E_\theta, (\dot{\sigma} = 1\ \mathrm{MPa}/\mathrm{min})$
0	138 118.8	132 102.5	139 132.3	143 94
10	74.4 77.3	79.3 65.9	80 78.3	–
30	22.6 23.5	16.7 20.2	21 21.7	17 19.9
45	14.5 14.4	13.5 12.9	13.8 13.6	13.4 12.7
60	11.3 11.2	11.2 10.6	11.1 11.1	10.7 10.4
90	9.77 9.8	9.75 10.1	10.4 10.4	9.45 9.7

Estimates for the vector of reciprocal values of the elastic moduli can be obtained from (4):

$$\hat{b} = X\left(X^T X\right)^{-1} X^T b. \tag{6}$$

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To determine the residuals, let us use the following expression:

$$\varepsilon = b - \hat{b} = \left(I - X\left(X^T X\right)^{-1} X^T\right) b \tag{7}$$

The matrix $P = I - X\left(X^T X\right)^{-1} X^T$ is a design matrix and makes it possible to perform a dispersion analysis of experimental and calculated data [7]. It is known that the vector of residuals is a centered random variable obeying the normal law. For illustration, Fig. 5 shows the functions of the normal distribution of residuals for the studied loading rates.

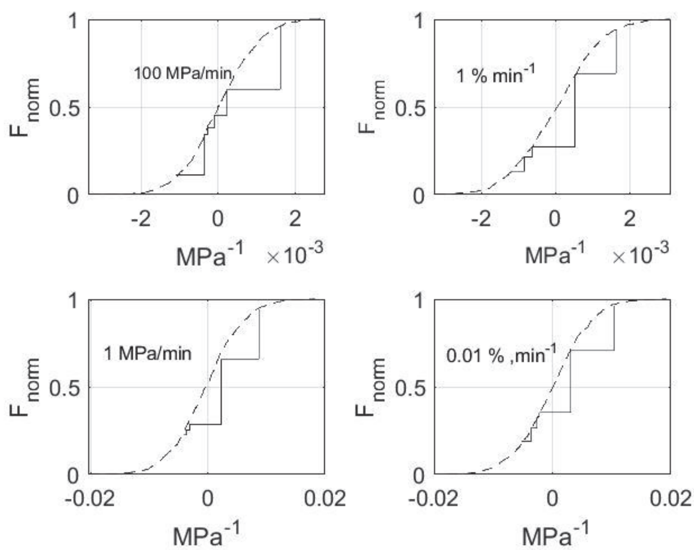

Fig. 5. Estimated empirical and theoretical residual distribution curves.

It can be seen from Fig. 5 that the ranges of variation of residuals at high loading rates are noticeably smaller than at low loading rates.

4 Conclusion

The analysis of the sensitivity of the elastic characteristics of AS4/PEEK to changes in the strain rate by two orders of magnitude at elevated temperature has been carried out and it has been shown that the errors caused by velocity perturbations in the compliance matrices introduce an error of the order of 10–14% into the calculation of stresses in the area of elastic behavior. Estimation of the elasticity characteristics determined using the least squares method shows good agreement with the experimental data, except for the values of the elastic moduli in the reinforcement direction, which may be due to the specifics of the least squares method computational algorithm. The error is especially noticeable at the loading rate of 1 MPa/min when it is equal to 30%, which, apparently, is due to the absence of the value of the elastic modulus at the angle of 10° and, possibly, an anomalously large value of Poisson's ratio. Statistical analysis of the residuals shows a noticeable, by about an order of magnitude, decrease in the dispersion with increasing loading rate. It can be noted that there is a close correlation between the arrays of experimental and calculated values of the elastic moduli for all studied values of loading rates.

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